BACHELOR IN COMPUTER APPLICATIONS

Term-End Examination

December, 2007

CS-601 : DIFFERENTIAL AND INTEGRAL CALCULUS WITH APPLICATIONS

Time : 2 hours *Maximum Marks : 60*

Note : *Question number 1 is **compulsory**. Answer any **three** questions from the rest. Use of calculator is **not** allowed.*

1. (a) Explain with reasons whether or not the statements given below are true. *3×5=15*

(i) Product of two odd functions is an even function.

(ii) The domain of definition of $f(x) = \sqrt{4-x^2}$ is $-2 < x < 2$.

(iii) If $f'(a)$ exists and is finite then $f(x)$ is continuous at $x = a$.

(iv) $f(x) = (x-1)e^x$ is an increasing function for $x > 0$.

(v) The function $f(x) = \tan x$ has a finite number of vertical asymptotes.

(b) Evaluate :

$$\underset{\theta \to \pi/3}{\text{Lt}} \frac{2\left(\cos\frac{\pi}{3} - \cos\theta\right)}{\sin\left(\theta - \frac{\pi}{3}\right)}$$ *4*

(c) Evaluate :

$$\int_0^{\pi} \cos^{13} x \, dx$$ 3

(d) Prove that

$$\frac{d}{dx}\left[\int_0^{\sin x} \tan \theta \, d\theta\right] = \cos x \, (\tan \sin x)$$ 5

(e) Verify the truth of Rolle's Theorem for the function, $y = \cos^2 x$ on the interval $\left[-\frac{\pi}{4}, +\frac{\pi}{4}\right]$. 3

2. (a) Evaluate :

$$\int e^x \frac{x^2+1}{(x+1)^2} \, dx$$ 4

(b) Using the concept of differential obtain the approximate value of sin 61°, correct upto three places of decimal. 3

(c) Find $\frac{dy}{dx}$, when $y = \tan^{-1}\left(\frac{ax+b}{a-bx}\right)$. 3

3. (a) If $y = e^{a \sin^{-1} x}$, prove that, $(1-x^2) y_2 - xy_1 = a^2 y$ where y_1 is first derivative and y_2 is second derivative of y. 5

(b) Find the area enclosed between the parabola, $y^2 = 4ax$ and its latus rectum. 5

4. (a) Given $e^1 = 2{\cdot}72$, $e^2 = 7{\cdot}39$, $e^3 = 20{\cdot}09$, $e^4 = 54{\cdot}60$, evaluate $\int_0^4 e^x dx$ using Simpson's one-third rule taking four equal intervals. 4

(b) Find the curvature of the curve $y = x^2$ at the point (1, 1). 3

(c) If the curves $y = a^x$ and $y = b^x$ intersect at an angle α, then prove that

$$\tan \alpha = \frac{\ln a - \ln b}{1 + \ln a \, . \ln b}$$ 3

5. (a) Prove that,

$$\int_0^1 \frac{\log x \, dx}{\sqrt{1 - x^2}} = \frac{\pi}{2} \ln \frac{1}{2}$$ 5

(b) Show that the tangent to the curve,

$$\left(\frac{x}{a}\right)^n + \left(\frac{y}{b}\right)^n = 2$$

at the point (a, b) is $\frac{x}{a} + \frac{y}{b} = 2$. 5