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Your Roll No

6187

B.Sc.(Hons.) III Semester / II Yr. J

COMPUTER SCIENCE

Paper 301 : ALGORITHMS

(Admissions of 2001 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No on the top immediately on receipt of this question paper)

Note : Attempt **all** questions Parts of a question should be attempted together

1 (a) (i) Give an array of 7 elements which is the best case for Insertion Sort

(ii) Give an array of 7 elements which is the worst case of Insertion Sort

Assume that the elements are to be sorted in increasing order

The best/worst case(s) would be according to the number of shifts of one or more elements In each of the above cases, count the number of shifts 5

(b) An algorithm A divides a problem of input size n in the ratio $(1 - \alpha)$ to α where $0 < \alpha < 1$ in constant time Write a recurrence relation for the time taken by the algorithm A 2

(c) Give the worst case time and the best case time for the following algorithm "ABC(n)" 3

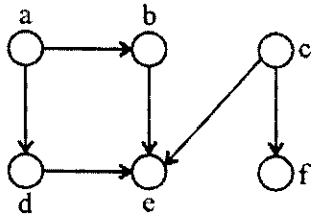
```
ABC (n)
{
    for (i = 2 to n/2)
    {
      if(n % i = = 0)
            break ,
      }
}
```

(d) (i) Create a Max-Heap using the elements A = <10, 15, 8, 28, 31, 65, 47> 2

(ii) Insert an element having value 24 in the Max-Heap 2

2 (a) In 0 – 1 knapsack problem, which approach among dynamic or greedy would be more useful and why ? 2

(b) An algorithm A works on an acyclic directed graph G(V, E) by iteratively removing from the graph a vertex with all incoming edges and inserting it at the front of a linked list Run it on the following graph Show all steps 3



(c) Show that the problem of finding a shortest path (assume that each edge has weight 1) in a graph has an optimal substructure property 4

3 (a) Consider the following array of elements

8, 4, 17, 21, 15, 52, 38

If we apply quick sort algorithm on the above array with the first element as the pivot, then compute the number of comparisons performed by the algorithm 3

(b) While applying Depth First Search in a directed graph, how do we differentiate between a cross edge and a forward edge with respect to the discovery time of its end vertices ? 2

(c) (i) Draw an undirected graph with the vertex set {A, B, C, D, E, F, G} and edges AB, AD, AC, BD, EF, CD, DE, FG, EG 1

(ii) Run Breadth First Search on the above graph starting from node A. 4

4 (a) Suppose that you are given Indian currency notes of denominations 1, 2, 5, 10, 20, 50 only Describe a Greedy algorithm to find the minimum number of currency notes to make an amount of Rs 379 (an informal discussion is sufficient) 4

(b) Compute the optimal number of scalar multiplications required to multiply the following matrices 4

A1 of order 20×25

A2 of order 25×15

A3 of order 15×10

(c) Analyse the worst case running time of Randomized Select algorithm 3

5 (a) An undirected graph having 5 nodes (A, B, C, D and E) is given below in the form of a weighted matrix

	A	B	C	D	E
A	0	15	10	3	7
B	15	0	2	0	5
C	10	2	0	2	0
D	3	0	2	0	6
E	7	5	0	6	0

(Weight 0 means no edge otherwise a +ve value represents cost of moving from one vertex to the other)

(i) Draw the corresponding simple, weighted, undirected graph 1

(ii) Find the minimum spanning tree using Kruskal's algorithm 4

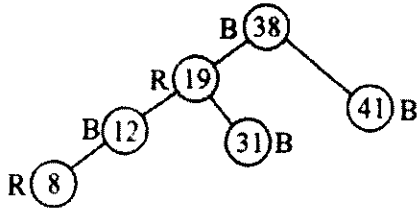
(b) Suppose that a stack is stored in an array, the initially capacity of a vector is 2 Every time we want to push an element into the stack and the array is full, then we must grow the array by doubling its capacity Use aggregate analysis to prove that each insert operation is Θ (1) amortized time 4

(c) In a university, the students roll numbers consist of three parts – Year of Admission, followed by course code, followed by sequence number

For example 2008 BCA 001, 2008 BCA 002 . 2009 BCS 010 2009 MCS 005

Give a linear time algorithm to sort the students records by their roll numbers Also, justify the time complexity (An informal discussion is sufficient) 3

6 (a) (i) Show the status of the tree after inserting a node having value 10 in the RB tree given below 2



(ii) Explain the functioning of RIGHT-ROTATE in an RB Tree with the help of an example 4

(b) Given an element x in an n-node order statistic tree and a natural number i, how can the i^{th} successor of x in the linear order of the tree be determined in O(lg n) time ? Justify the time complexity 4

7 (a) Create a binomial heap by inserting nodes with keys 10, 20, 30, 40, 50, 60 5

(b) Suppose that the following algorithm is used to compute the connected components of an undirected graph G = (V, E) where, vertex set V = {a, b, c, d, e, f, g, h, i, j, k} and the set of edges E is {(d, i), (f, k), (g, i), (b, g), (a, h), (i, j), (d, k), (b, j), (d, f), (g, j), (a, e)}

Connected – Components (G)

for each vertex V ∈ V [G]

{

MAKE-SET (V)

}

```
for each edge (U, V) ∈ E (G)
{
    if FIND-SET (U) ≠ FIND-SET (V)
    {
        UNION(u, v)
    }
}
```

(i) Compute the number of times FIND-SET operation is executed

(ii) Compute the number of times UNION is executed

(iii) Mention the data structure to be used

(iv) Give the time complexity of the algorithm 4
