This question paper contains 3 printed pages]

Your Roll No

6189

B.Sc.(Hons.) Computer Science / III Sem. J

Paper 303 : ALGEBRA

(Admissions of 2001 and onwards)

Time : 3 Hours **Maximum Marks : 75**

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt **all** questions

All questions carry equal marks

Use of calculator is permitted

1 Define a group Let G = {0, 1, 2} and define an operation $*$ on G by $a * b = |a - b|$ for $a, b \in G$ Is G a group with respect to $*$? Justify your answer

2 Define a monoid Prove that if in a monoid every element x different from the identity e satisfies $x^2 = e$, then the monoid is commutative

3 Let $f : R \to S$ and $g : S \to T$ be morphism of rings Then show that the composition $g \circ f : R \to T$ is a morphism and further show that $\text{Ker}(g \circ f) = \text{Ker}(f)$ if g is an isomorphism

4 Let $R = \{a + ib\sqrt{3} / a, b \in \mathbb{Z}\}$, where $\mathbb{Z}$ is set of all integers Is R a subring of $\mathbb{C}$, ring of complex numbers ?

5 Draw the Hasse diagram representing the partial ordering relation $\{(a, b) \quad a \text{ divides } b\}$ on $\{1, 2, 3, 4, 6, 8, 12\}$ Identify the maximal and minimal elements Give chains and anti-chains and find maximum length of chain

6 Determine the dimension of $n \times n$ symmetric matrices over $\mathbb{R}$ Justify your answer

7 Define a convex set in $\mathbb{R}^n$. Show that the set of all elements $(x, y) \in \mathbb{R}^2$ which satisfies $3x + 5y \leq 4$, is a convex set

8 Let $L \quad \mathbb{R}^2 \to \mathbb{R}^3$ be linear map such that

$$L(1, 2) = (1, 3, -1) \text{ and}$$

$$L(2, -3) = (2, -1, 4).$$

Find $L(1, 0)$ and $L(0, 1)$

9 What is the dimension of the space of solutions of the following system of linear equations ?

$$2x + 7y = 0$$

$$x - 2y + z = 0$$

10 Prove that a mapping $F \quad V \to W$ has an inverse iff it is both injective and surjective

11 Let V be a vector space with a scalar product <, > Let $V_1, V_2, \ldots, V_n$ be vectors which are mutually perpendicular and such that $\|V_i\| \neq 0$ $1 \leq i \leq n$ Let V be an element of V, and let C_i be the component of V along V_i Let $a_1, a_2, \ldots, a_n$ be numbers Then show that

$$\|V - \sum_{k=1}^{n} C_k V_k\| \leq \|V - \sum_{k=1}^{n} a_k V_k\|$$

12 Find an orthonormal basis of the subspace W of C^3 spanned by $V_1 = (1, i, 0)$ and $V_2 = (1, 2, 1 - i)$

13 Find the eigen values and a basis for the eigenspaces of the matrix

$$\begin{pmatrix} 1 & 1 & 2 \\ 0 & 5 & -1 \\ 0 & 0 & 7 \end{pmatrix}$$

14 Find the volume of the parallelopiped spanned by the following vectors in 3-space (–1, 2, 1), (2, 0, 1), (1, 3, 0).

15 Classify and sketch the curve

$$4x^2 + 2\sqrt{2}xy + 3y^2 = 1$$