

INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

18th May 2011

Subject CT1 – Financial Mathematics

Time allowed: Three Hours (15.00 – 18.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. *Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception*
2. *Mark allocations are shown in brackets.*
3. *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*
4. *In addition to this paper you will be provided with graph paper, if required.*

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

- Q. 1) a.** Prove from first principles that

$$(I\ddot{a})_{\overline{n}|} = (Ia)_{\overline{n}|} + \ddot{a}_{\overline{n+1}|} - (n+1)v^n \quad (3)$$

- b.** A finance company plans to launch the following types of annuity products. The initial amount of annuity is ₹1,000/- per annum.

Find the price of each product assuming an effective rate of interest as 6% per annum

- i. Deferred annuity payable monthly in advance for 25 years, the first annuity being paid at the end of 3 years from the date of purchase. (2)

- ii. Annuity payable annually in advance for 30 years increasing at the rate of 5% after every three installments. (3)

- iii. Annuity payable in arrear for 25 years decreasing by a fixed amount of ₹50/- after every 5 installments. (3)

[11]

- Q. 2)** An investment firm offers a product to its customers which gives continuous rate of return at the following rate, where t is measured in years:

$$\delta(t) = \begin{cases} 0.005 + 0.01t & \text{for } 0 < t \leq 10 \\ 0.002t & \text{for } 10 < t \leq 15 \\ 0.03 + 0.02t & \text{for } t > 15 \end{cases}$$

- a.** Find the effective continuous rate of return to a customer for a 20 year product. (4)

- b.** The product gives a continuous income of ₹ $0.5e^{0.155t+0.005t^2}$ from $t = 5$ to $t = 10$ and a lump sum amount of ₹ 100 at $t = 20$. What should be price of the product? (6)

[10]

- Q. 3) a.** Why may returns on fixed interest government bonds be uncertain? (2)

- b.** What is the difference between buying a call option and selling a put option? When would these get exercised? (4)

[6]

- Q. 4)** Calculate the value of a forward contract on 1st May 2011 for a security which is priced at ₹5,800 on 1st May 2011. The forward contract was entered into on 1st November 2009 to pay a price of ₹6,800 for this security in five years' time. The security pays continuous dividends with the dividend yield and price at the time of agreement being 3% p.a. and ₹5,000 respectively. Assume no arbitrage and the risk free interest to be constant during the period.

[4]

- Q. 5)** A company Arbitrage Ltd offers a financial product with an investment guarantee at maturity @ 9.5% p.a. for 10 years. It invests the money at a floating rate i_t , with $(1+i_t)$ following a log-normal distribution with the following parameters

$$\mu = 0.1, \quad \sigma = 0.05 \quad \text{for } t < 5$$

and $\mu = 0.1, \quad \sigma = 0.015 \quad \text{for } t \geq 5.$

Where t is the time in years since initial investment.

The product was offered to a company Profit Booker Ltd which agrees to invest ₹9,000 initially and ₹6,000 at the end of the 5th year. The maturity amount will be payable at the end of the 10th year.

- a. Calculate, with a 99.5% confidence level, the minimum accumulated amount that company Arbitrage Ltd. will have at the end of 10 years, if it invests the money received from Profit Booker Ltd. at this floating rate. (9)
- b. Calculate the amount that Arbitrage Ltd need to invest now in a risk free bond in order to mitigate the possible amount of loss, if any, at the 99.5% confidence level assuming the interest rate for the risk free bond to be 6% p.a. (3)

[12]

- Q. 6)** The asset and liability cash-flows of a company over the next 7 years are given below :

Year	Asset	Liability
1	0	34
2	150	0
3	0	140
4	0	0
5	15	31.5
6	20	0
7	62	40.331

Assume an effective rate of interest of 8% p.a. and that all the cash-flows occur at the end of the year.

- a. Calculate the effective duration of the assets and liabilities. (4)
- b. Show that with a 1% increase in the interest rate, the present values of the assets and liabilities are almost equal. (3)

[7]

- Q. 7)** The Government of a country issued at par index linked bonds in April 2008 with maturity after exactly 3 years and coupons @8.25% p.a. payable yearly in arrear. The coupons and redemption amount are linked to an inflation index with a one year time lag.

The index value at 1st April of each year were as follows:

Year	2007	2008	2009	2010	2011
Index	158	167	175	190	230

- a.** Find the inflation rates p.a. for each year implied by the above table of indices. (2)
- b.** Calculate the coupon and redemption payments made each year per ₹ 100 nominal. (3)
- c.** Calculate the coupon and redemption payments, expressed in April 2008 money units, for an investor who bought the stock in April 2008. (4)
- d.** Based on the calculations above, explain whether the investor is fully protected against inflation. (2)
- e.** If an investor had expectations of the inflation index being the same as in the above table, calculate the price he would have paid in order to achieve a gross redemption yield of 7% p.a. (2)

[13]

- Q. 8)** Mr. Preferred Customer wants to purchase a house. He takes a loan of ₹1,000,000, from a bank, repayable over a period of 9 years through level annual installments. The rate of interest charged by the bank is 9.5% per annum convertible quarterly.

- a.** Calculate the amount of annual installment payable by him. (2)

Mr. Preferred Customer will get a lump sum of ₹ 500,000 after 1.5 years which he wants to use to repay a part of outstanding loan amount. The bank allows such a repayment of outstanding loan subject to following limits:

- Such a repayment can be made only at the end of a year
- Maximum of 30% of original loan amount (i.e. amount borrowed at beginning) can be repaid in one such installment.

Mr. Preferred Customer earns an interest of 6% per annum convertible half-yearly, on his excess fund until it is used to pay the outstanding loan.

Calculate the loan outstanding amount at the end of the 3rd year after payment of the installment then due, assuming that the maximum eligible pre-payment of loan was made by him at the end of each of 2nd and 3rd years. (7)

[9]

Q. 9) a. A company sells the following products:

Type1: Pay ₹ 100,000 now and receive ₹ 14,402 per annum payable monthly in arrears for 15 years

Type 2: Pay ₹ 100,000 now and receive ₹ 11,400 per annum payable monthly in arrears for 15 years together with the original capital outlay at the end of 15 years

An investor is considering buying either one of these products for which he needs to borrow money from the market @ 0.5% per month effective interest.

- i. Calculate the discounted payback period under both the products. (6)
- ii. Calculate the NPV for the two products. (4)
- iii. Which product should he invest in? (1)

b. Calculate, to the nearest 0.1%, the effective annual Money Weighted Rate of Return of the fund whose value at $t=0$ is 300 and at $t=5$ is 2624 with the following intermediate cash flows:

time (t)	Cash in flow	Cash out flow
0.5	1,500	-
1	500	-
2.5	-	700
4	200	-

What extra information you require to calculate the effective Time Weighted Rate of Return for the period? (5)
[16]

Q. 10) A bank is going to issue a bond with coupons of 8% per annum payable quarterly in arrear and redeemable at 105% of issue price with optional redemption period from 10 to 15 years from the date of issuance of the bond. The issue price is ₹ 100/- per unit.

Mr. Intelligent wants to purchase 100 bonds for which he wants to take a loan from another bank which charges interest at 6.5% per annum. Mr. Intelligent is subject to an income tax of 10% deferred for a period of 1.5 years.

Calculate the present value of the maximum profit Mr. Intelligent can make from this transaction.

[5]

Q. 11) The price of Zero Coupon Bonds at different time points is given below.

Time (t in years)	Zero Coupon Bond Price (P_t)
1	0.92280
2	0.81460
3	0.71940
4	0.64760
5	0.59790
6	0.56580
7	0.54670
8	0.53690
9	0.53320
10	0.53340

a. Calculate the spot rate for 5 years.

(2)

Mr. A requires ₹ 20,000 at time $t=7, 8$ and 9 and ₹ 105,000 at time $t=10$. He currently has only ₹75,000. He wants to invest it for 5 years. After 5 years, he plans to use the proceeds of the aforesaid investment and some additional capital, if required, to purchase suitable bonds to meet his cash-flow requirements.

b. Calculate the additional capital he may require at time $t=5$ to meet his cash-flow requirements assuming the yields curve implied by the above table applies.

(5)

[7]
