Actuarial Society of India

EXAMINATIONS

15th May 2007

Subject CT3 – Probability and Mathematical Statistics

Time allowed: Three Hours (10.00 – 13.00 Hrs)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. **Do not write your name anywhere on the answer sheets. You have only to write your Candidate's Number on each answer sheets.**

2. **Mark allocations are shown in brackets.**

3. **Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.**

4. **Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.**

5. **In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.**

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Please return your answer scripts and this question paper to the supervisor separately.

Q. 1)   The following data represents the noise levels measured, in decibels, at 36 different times in a city.

82, 89, 94, 110, 74, 122, 112, 95, 100, 78, 65, 60, 90, 83, 87, 75, 114, 85
69, 94, 124, 115, 107, 88, 97, 74, 72, 68, 83, 91, 90, 102, 77, 125, 108, 65

a) Construct a stem and leaf plot of the data   (2)
b) Determine the quartiles   (3)
c) Draw a box plot   (1)

[6]

Q. 2)   A fair coin is tossed three times and the following events are considered:
   A – Toss 1 and Toss 2 produce different outcomes.
   B – Toss 2 and Toss 3 produce different outcomes.
   C – Toss 3 and Toss 1 produce different outcomes.
Show that
   a) $P(A) = P(A/B) = P(A/C)$ (2)
   b) A, B and C are pairwise independent events.   (2)

[4]

Q. 3)   A claim size distribution is modeled using a simple distribution with density of the form

$$f(x) = \begin{cases} k(50-x), & 0 \le x \le 50 \\ 0, & \text{otherwise} \end{cases}$$

a) Find k   (1)
b) Determine the mean of this claim size distribution.   (2)
c) Calculate the probability that an individual claim size is greater than 25   (1)
d) Calculate the probability that an individual claim size is less than 30 given that
   it is greater than 25   (3)

[7]

Q. 4)   A man wins a rupee for head and loses a rupee for tail when a fair coin is tossed. Suppose that he tosses once and quits if he wins; but tries once more if he losses on the first toss. What are his expected winnings?   **[4]**

Q. 5)   The incidence of hospitalization tends to vary with the flu season over the years. Data on a particular group suggests that one should expect
   one hospitalization in September;
   one hospitalization in October;
   two hospitalizations in November and
   three hospitalizations in December.
Determine the probability that there are less than five hospitalizations in September to December period. Assume that hospitalizations each month are independent and follow Poisson distribution.   **[4]**

Q. 6)   Consumer demand for milk X, in a Metropolitan area, is known to follow a Gamma distribution with *pdf*

$$f(x) = \frac{\lambda^{\alpha} e^{-\lambda x} x^{\alpha-1}}{\Gamma \alpha} \quad ; \alpha > 0, \lambda > 0, 0 \le x < \infty.$$

It is given that the average demand is 'a' liters and the modal demand is 'b' liters ($b < a$).

a) Compute the mode in terms of α and λ (4)

b) Given that $EX = \dfrac{\alpha}{\lambda}$, what is the variance of the demand in terms of *a* and *b*? (4)

c) Compute the skewness of the demand when $\alpha = 2$ and $\lambda = 2$ (2)

[10]

Q. 7) An auto insurance company is analyzing the claim frequency on a block of 250 policies. Historical data suggest that 10% of policy holders in this block will file at least one claim in the coming coverage period. What is the probability that more than 12% policy holders file at least one claim in the coming average period? Assume that claim occurrences are independent for distinct policy holders. **[4]**

Q. 8) A random sample of size n is taken from distribution with pdf

$$f(x) = 2x/\theta^2 ; 0 < x < \theta$$
$$= 0 \qquad otherwise$$

a) Write down the likelihood function and hence by drawing the rough sketch of the likelihood function, obtain the maximum likelihood estimator (mle) for θ . (4)

b) Examine if the mle is unbiased (2)

[6]

Q. 9) A Newspaper reported the results of a National survey conducted on a sample of 1034 parents of whom 848 reported that pizza was their children's favourite food. Let *p* be the proportion of all parents whose children's favourite food was pizza (at the time of survey).

a) In the light of these survey can we conclude $p = 0.5$, setting level of significance at 0.05? (3)

b) Obtain 90% confidence interval for *p*. (2)

[5]

Q. 10) An insurance company offers three types of policies, namely (a) Whole life policy, (b) Endowment policy with profit and (c) Endowment policy without profit. A random sample of 75 newly insured in a year revealed that 45 were from urban area and remaining from rural areas. The choices of plans chosen by those from urban areas were 10 whole life policies, 18 endowment policies with profit and 17 endowment policies without profit, respectively. The choices of plans chosen by those from rural areas were 14 whole life policies, 8 endowment policies with profit and 8 endowment policies without profit respectively. Using 5% level of significance

a) test whether the nativity and the choice of policy are independent. (3)

b) if the two endowment policies are grouped as one policy, called endowment policy, test whether the nativity and the choice of policy are independent. (3)

[6]

Q. 11) A training Manager of an Insurance company wishes to see if there has been any change in the ability of his trainees after they have been on a course. The trainees take an aptitude test before they start the course and equivalent one after they have completed it. The scores are recorded below:

Scores before Training	42	35	37	46	53	38	44	40	43
Scores after Training	47	28	26	54	42	17	44	31	44

a) Has any change taken place? Test your claim at 5% level. (3)
b) Obtain 95% confidence interval for the mean change in ability of trainees (4)
c) Compute Pearson's correlation coefficient between the scores before and after training
   and test its significance using t-test at 5% level of significance. (6)

[13]

Q. 12) The following measurements, Air velocity (cm/secs) and Evaporation coefficients
   (mm^2/secs) of burning fuel droplets in an impulse engine.

Air Velocity: x	20	60	100	140	180	220	260	300	340	380
Evaporation Coefficient :y	0.18	0.37	0.35	0.78	0.56	0.75	1.18	1.36	1.17	1.65

For the above data, a regression model $y = \alpha + \beta x + \epsilon$ is to be fitted
 a) State the assumptions on this model. (1)
 b) Obtain the least squares estimate of α and β (4)
 c) Compute an unbiased estimate of the error variance (2)
 d) Obtain 95% confidence interval for α (2)
 e) Test the hypothesis $\beta = 0$ against $\beta \neq 0$ at *5%* level (3)
 f) 95% confidence interval for evaporation coefficient when air velocity is 190 cm/sec (2)

[14]

Q. 13) A fire insurance company intending to alter its logo, sent questionnaires to a sample of its
employees asking their opinions about three possibilities. The employees were asked to
award each of the three logos (A,B and C), a score out of five on six different aspects of
design, color and so on. The total scores awarded by the eight employees are given below.

Employees								
Logos	1	2	3	4	5	6	7	8
A	22	19	14	18	21	18	23	20
B	24	22	18	20	21	21	18	19
C	20	16	16	19	18	15	19	18

Test the hypothesis that the three logos are equally preferred by the employer, at 5% level
of significance.

[6]

Q. 14) Suppose (X,Y) have joint *pdf*

$$f(x,y) = 2\,exp[-(x+y)]\,;\,0 \leq y \leq x < \infty$$
$$= 0 \qquad elswehere$$

Derive the conditional expectation *E(X/Y=y)*

[4]

Q. 15)   Let $C = W_1 + W_2 + \dots + W_N$, where N represents the random number of claims received during the time period, W_j represents the size of the jth claim and C represents the aggregate claim. Assuming $W_j^{'s}$ are *iid* and are also independent of N. Derive

a) The distribution function of C.   (4)
b) Moment generating function of C.   (3)

[7]

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