Actuarial Society of India

EXAMINATIONS

31st October 2006

Subject CT3 – Probability and Mathematical Statistics

Time allowed: Three Hours (10.30 – 13.30 pm)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1.   Do not write your name anywhere on the answer sheet/s. You have only to write your Candidate's Number on each answer sheet/s.

2.   Mark allocations are shown in brackets.

3.   Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.

4.   Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.

5.   In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q1) The following frequency table gives the starting salaries (in '0000' Rupees/annum) of a sample 42 graduating seniors of a liberal arts college.

Starting salary	Frequency
47	4
48	1
49	3
50	5
51	8
52	10
54	5
56	2
57	3
60	1

    i) Sketch the Box plot for this data. (3)

   ii) Find the sample mean. (1)

  iii) Find the sample variance. (2)

  iv) Find IQR. (1)

[7]

Q2) Let A, B, C be events with $P(B)$ and $P(C) > 0$. If B and C are independent, show that
$P(A/B) = P(A/B ? C).P(C) + P(A/B ? C^c)P(C^c)$.
(C^c denotes complement of C) **[3]**

Q3) A box contains 8 bulbs of which 3 are defective. A bulb is selected from the box and tested. If it is defective another bulb is selected and tested until a non-defective bulb is chosen. Find the expected number of bulbs chosen? **[4]**

Q4) The joint pdf X and Y is
$$f(x,y) ? 4y(x ? y)e^{?(x?y)}; 0 ? x ?? , 0 ? y ? x$$
$$= 0 \quad otherwise$$
Compute $E(X/Y=y)$. **[7]**

Q5) A random variable X has exponential distribution with $pdf : f(x) = e^{-x}$; $x > 0$

  (i) Obtain the *mgf* of $Y = 1-e^{-X}$ (3)
 (ii) What is the distribution of Y? Comment. (2)

[5]

Q6) State the postulates of Poisson process. Show that the difference of two independent Poisson process is not a Poisson process. **[4]**

Q7) If s_1^2 and s_2^2 are the variances of independent random samples of size $n_1 = 10$ and $n_2 = 15$ from Normal populations with equal variances. Find $P? \dfrac{s_1^2}{s_2^2} ? 4.03 ?$.

[3]

Q8) A fair die is tossed 180 times. Determine the Probability that the face 6 will appear
    i)   between 29 and 39 times inclusive. (2)
    ii)   less than 22 times. (2)

[4]

Q9) Twenty electronic tubes were put to test and the test continued till of all them failed. The failure times (in hours) were recorded.

9.9	35.5	57.9	94.6	141.4	154.4	163.3	226.7	244.3	337.2
391.8	417.2	444.6	461.2	497.1	582.6	606.8	616.3	672.0	784.7

Total hours:6939.5

The failure times are assumed to be independent and follow exponential distribution with density $f(x ; \lambda) = \lambda e^{-\lambda x}$; $\lambda, x > 0$.

  i)   Determine the Maximum Likelihood Estimate (*MLE*) $\hat{\lambda}$ of λ . (4)

  ii) Obtain the large sample variance of $\hat{\lambda}$. (3)

 iii) Hence calculate an approximate 95% confidence interval for λ. (2)

 iv) Supposing the test was terminated at 600 hours, what would be the *MLE* $\tilde{\lambda}$ of λ ? (5)

[14]

Q10) In a random sample of 200 stomach cancer patients yielded 92 having blood type *A*, 20 having blood type *B*, 4 having blood type *AB* and 84 having blood type *O*.
Are these data significant enough, at 5% level of significance to enable us to reject the null hypothesis that the blood type distribution of stomach cancer sufferers is the same as that of the general population?

[5]

Q11) A software company has developed a new software package to help the system analysts working in insurance industries to reduce the time required to design, develop and implement an information system. To evaluate the benefits of this new software the insurance company has selected 24 system analysts, out of which 12 of them were instructed to produce the information system using current technologies and the rest of them were trained and then were asked to produce the information system using new software. The data set is as given below.
Time required completing the information system using

Current Technology (x_1)	300	280	344	385	372	360	288	321	376	290	301	283
New Software (x_2)	276	222	310	338	200	302	317	260	320	312	334	265

$\sum x_1 = 3,900$ $\sum x_2 = 3,456$ $\sum x_1^2 = 12,85,096$ $\sum x_2^2 = 10,16,622$

 (i) Stating the hypothesis test that the new software package will provide a shorter mean project completion time than the current technology? Use a=0.05. (5)

 (ii) Test the hypothesis that the variances of the project completion times are equal. Use a = 0.05. (4)

(iii) Obtain 90% confidence interval for the difference between the means of two populations. (3)

[12]

Q12) A private insurance company has three offices at Delhi, Mumbai and Chennai. To measure how much the employees at these places know about total quality management, a random sample of six employees was selected from each place and given a quality awareness examination. The examination scores are given in the following table.

Examination Scores

Observation	Delhi	Mumbai	Chennai
1	85	71	59
2	75	75	64
3	82	73	62
4	76	74	69
5	71	69	75
6	85	82	67

Test the hypothesis that the mean examination score is the same for all three places at 5% level by stating the model the hypothesis and the assumptions involved for this test. **[8]**

Q13) In a correlation analysis, based on a sample of size 10 from a bivariate normal distribution, a test of $H_0: \rho = 0$ against $H_1 : \rho > 0$ results in a probability value of 0.025. Compute the value of the sample correlation coefficient. **[5]**

Q14) The following data shows the student population and quarterly sales data for ten fast food restaurants. The manager believes that quarterly sales for these restaurants(y) is related positively to the size of the student population(x).

Restaurant	Student population (in '000)	Quarterly sales (in Rs.'000)
i	x_i	y_i
1	2	58
2	6	105
3	8	88
4	8	118
5	12	117
6	16	137
7	20	157
8	20	169
9	22	149
10	26	202

A simple linear regression model $y = \beta_0 + \beta_1 x + \epsilon$ is fitted.
  (i) Develop a scatter plot for these data. (1)
 (ii) Develop the estimated regression equation. (3)
(iii) Test the Hypothesis $H_0 : \beta_1 = 0$ against $H_1 : \beta_1 \neq 0$ using $\alpha = 0.01$. (4)
(iv) Compute the 99% confidence interval for β_1 (4)

[12]

Q15) Let X_1, X_2, ... be i.i.d. non-negative integer valued random variables with common pgf p(s). Suppose N is also a non-negative integer valued random variables that is independent of X_i's . Let $S_N = \sum_{j=1}^{N} X_j$

  (i)   Find the *pgf* of S_N.   (5)

  (ii) Hence find the mean of S_N.   (2)

[7]

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