Actuarial Society of India

EXAMINATIONS

15th November 2005

Subject CT3 – Probability and Mathematical Statistics

Time allowed: Three Hours (10.30 – 13.30 pm)

Total Marks : 100

INSTRUCTIONS TO THE CANDIDATES

1. **Do not write your name anywhere on the answer scripts. You have only to write your Candidate's Number on each answer script.**
2. **Mark allocations are shown in brackets.**
3. **Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.**
4. **Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.**
5. **In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.**

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q.1) A set of claim amounts in (lakhs of Rs.) is given below.

192	136	253	138	87
112	221	176	336	203
159	55	308	165	254

Present these data graphically using a box plot. **[2]**

Q.2) The mean of 5 observations is 15 and the variance is 9. If two more observations having values -3 and 10 are combined with these 5 observations, what will be the new mean and variance of 7 observations? **[3]**

Q.3) An actuary applies for a job in two insurance companies X and Y. The probability of his being selected in X is 0.7 and being rejected at Y is 0.5. The probability of at least one of his applications being rejected is 0.6. What is the probability that he will be selected by one of the companies? **[3]**

Q.4) In an insurance company, three actuarial assistants A, B and C are assigned to process loan application forms against the policies. A processes 40%; B processes 35%; and C processes 25% of the loan applications. The error rate while processing the loan applications by A, B and C are 0.04, 0.06 and 0.03 respectively. A loan application selected at random by the Branch Manager is found to have an error. What is the probability that it was processed by A? **[4]**

Q.5) The probability density for damage claims X paid by the Automobile insurance company on collision insurance is given below.

$$f(x) = \frac{2a}{p}\left(\frac{1}{a^2 + x^2}\right); \quad |x| \le a$$

$$= 0 \text{ otherwise}$$

Obtain the mean and variance of X. **[3]**

Q.6) Let X_i , $i = 1,2,\ldots,k$ be independent geometric random variables with parameter q. Show that $Y = X_1+X_2 +\ldots+X_k$ follows a Negative Binomial distribution with parameters(k,q). **[2]**

Q.7) Among 120 applicants for a job in an insurance company, only 80 are actually qualified. If five of the applicants are randomly selected for an in-depth interview, find the probability that only two of the five will be qualified?

[2]

Q.8)

a) State the postulates of Poisson process. **(1)**

b) If $N_1(t)$ and $N_2(t)$ are two independent Poisson processes with parameters λ_1 and λ_2 respectively show that

$$P\left[N_1(t) = k / (N_1(t) + N_2(t) = n)\right] = \binom{n}{k} p^k q^{n-k}$$

where $p = \dfrac{l_1}{l_1 + l_2}$ and $q = \dfrac{l_2}{l_1 + l_2}$ **(4)**

Total [5]

Q.9) The joint *pmf* of X and Y is given below.

	X		
Y	0	1	2
0	1/6	1/3	1/12
1	2/9	1/6	0
2	1/36	0	0

Find

a) Mean values of X and Y **(2)**

b) Find the covariance between X and Y **(1)**

c) Conditional distribution of X given Y = 1 **(2)**

d) Correlation between X and Y **(3)**

Total [8]

Q.10) If the probability is 0.20 that a certain bank will refuse a loan application, using normal approximation (to three decimals), find the probability that the bank will refuse at most 40 out of 225 loan applications. **[4]**

Q.11) Find the probability that the variance of a random sample of size 5 from a Normal population with unknown mean μ and known variance $\sigma^2 = 25$ will fall between 20 and 30. **[3]**

Q.12) The number of claims, which arise in a year under each individual policy of a certain type, has a Poisson distribution with unknown mean λ. Last year's experience for a group of independent policies of this type is examined and it is found that no claims arose under 78% of the policies (and claims did arise under 22% of the policies).
Find the maximum likelihood estimate of λ **[3]**

Q.13) A random sample of 200 unrelated motor policies with identical risk profiles from a certain company's business gave rise to a total of 52 claims in 2001. Note that each policy can give rise to more than one claim in any one year. Assuming a Poisson model with mean λ for the number of claims made on such a policy in 2001, use a normal approximation to calculate a lower 95% confidence interval for λ **[5]**

Q.14) Consider testing the hypothesis $H_0: p=0.1$ *vs* $H_1: p=0.04$
using as test statistic Y, where $Y \sim$ binomial (100,p) .
Using a normal approximation, determine the Type II error probability for a test with a Type I error probability of 0.05. **[4]**

Q.15) In the surgical treatment of duodenal ulcers there are three different operations corresponding to the removal of various amounts of the stomach. The three operations are denoted A, B and C, with A being the least traumatic and C the most traumatic.

It is known that these operations have an undesirable side-effect for some patients. In cases where the side effect is present, it can be classified as being of "slight degree" or of "moderate degree".

The data in the following table relate to a group of 417 patients and specify the operation received and the degree of the side effects suffered.

Operation	Existence /degree of side effects			Total
	None	Slight	Moderate	
A	63	26	7	96
B	126	63	25	214
C	51	40	16	107
Total	240	129	48	417

a) Perform a χ^2 test on this table to investigate independence between level of operation and degree of side-effects **(5)**

b) Also examine whether the operation has any significance on the presence of side effects. **(3)**

Total [8]

Q.16) The following table gives the length of time required to assemble the device using standard procedure and new procedure. Two groups of nine employees were selected randomly, one group using the new procedure and the other following standard procedure.

	Length of time (in minutes)								
Standard procedure (X_1)	32	37	35	28	41	44	35	31	34
New procedure (X_2)	35	31	29	25	34	40	27	32	31

It is given that

$$\sum_{i=1}^{9}(X_1 - \overline{X_1})^2 = 195.5556$$

$$\sum_{i=1}^{9}(X_2 - \overline{X_2})^2 = 160.2222$$

a) Do the data present sufficient evidence to indicate that the mean time to assemble the device under standard procedure is less than the mean time under new procedure? **(3)**

b) Obtain the 95% confidence interval for the difference in mean. **(2)**

c) Test for the equality of the variances of the two procedures. **(3)**

Total [8]

Q.17) It is thought that a suitable model for a plumber's charges when called out for a job is a linear one based on a fixed call-out charge and an hourly rate.

A random sample of 10 of his invoices gave the following results:

Duration of job (hours) x	0.5	1.0	1.5	2.0	2.5	3.0	3.5	4.5	5.0	5.5
Cost of Job (Rs) y	40	55	45	65	80	75	95	100	120	130

$x = 29$ $x^2 = 110.5$ $y = 805$ $y^2 = 73,225$ $xy = 2,795$

a) Plot these data and comment on the suitability of the proposed model. **(1)**

b) Calculate the least squares estimates of the plumber's call-out charge, and the plumber's hourly rate charge. **(6)**

c) Determine a 90% confidence interval for the plumber’s hourly rate charge. **(2)**

d) Compute Pearson correlation co-efficient and test for its significance. **(4)**

Total [13]

Q.18) An insurance company issues house buildings policies for houses of similar size in four different post-code regions A, B, C and D. An insurance agent takes independent random samples of 10 house buildings policies for houses of similar size in all the four regions. The annual premiums (Rs) were as follows:

Region A: 229 241 270 256 241 247 261 243 272 219
($\Sigma x = 2{,}479$; $\Sigma x^2 = 617{,}163$)

Region B: 261 269 284 268 249 255 237 270 269 257
($\Sigma x = 2{,}619$; $\Sigma x^2 = 687{,}467$)

Region C: 253 247 244 245 221 229 245 256 232 269
($\Sigma x = 2{,}441$; $\Sigma x^2 = 597{,}607$)

Region D: 279 268 290 245 281 262 287 257 262 246
($\Sigma x = 2{,}677$; $\Sigma x^2 = 718{,}973$)

(a) Perform a one-way analysis of variance at the 5% level to compare the premiums for all four regions. **(7)**

(b) Present the data in respect of Region C and Region D in a simple diagram and hence comment briefly on the validity of the assumptions required for the analysis of variance. **(1)**

(c) Calculate a 95% confidence interval for the underlying common standard deviation σ of such premiums in the four regions. **(2)**

Total [10]

Q.19) Consider the model for aggregate claim amount S_N :

$$S_N = X_1 + X_2 + \ldots + X_N$$

where X_i are *i.i.d.* random variables representing individual claim amounts and N is a random variable, independent of the X_i and representing the number of claims. Let X have mean μ_X and variance s^2_X and N have mean μ_N and variance s^2_N.

a) Show that $E(NS_N) = \mu_X(m^2_N + s^2_N)$ by considering expected values conditional on the value of N. **(3)**

b) Hence drive an expression for the covariance between N and S_N. **(2)**

Total [5]

Q.20) Let the joint *pdf* of (X,Y) be

$$f(x,y) = \frac{2}{3}(x+2y) \; ; \; 0 < x , y < 1$$
$$= 0 \text{ otherwise.}$$

Find

a) Conditional mean of X given Y = 1/2 **(2)**

b) Conditional variance of X given Y = 1/2 **(3)**

Total [5]

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