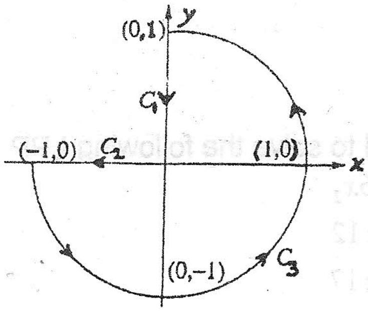
(3 Hours) [Total Marks : 100

N.B. : (1) Question No.1 is compulsory.
(2) Attempt any four questions out of the remaining six questions.
(3) Figures to right indicate full marks.

1. (a) If $A=\begin{bmatrix}2 & 4\\ 0 & 3\end{bmatrix}$ then find the eigen values of $6A^{-1}+A^{3}+2I$ 5

(b) State and prove C-R equations in polar coordinates. 5

(c) Integrate $f(z)=z$ around the closed contour shown in figure. 5



(d) Find all the basic solutions to the following problem which of them are basic feasible, non-degenerate, infeasible basic and optimal basic feasible solutions. 5

$$\text{Maximise } z=x_1-2x_2+4x_3$$
$$\text{subject to } x_1+2x_2+3x_3=7$$
$$3x_1+4x_2+6x_3=15$$

2. (a) Find the characteristic equation of the matrix A and hence find the matrix represented by $A^7-4A^6-20A^5-34A^4-4A^3-20A^2-33A+I$ 6

$$\text{where } A=\begin{bmatrix}1 & 3 & 7\\ 4 & 2 & 3\\ 1 & 2 & 1\end{bmatrix}$$

(b) Show that $v=e^{-x}(x\cos y+y\sin y)$ is harmonic and find the corresponding analytic function $f(z)=u+iv$ 6

(c) Using the penalty (Big M) method solve the following LPP. 8

$$\text{Minimise } z=4x_1+x_2$$
$$\text{subject to } 3x_1+x_2=3$$
$$4x_1+3x_2\ge 6$$
$$x_1+2x_2\le 4$$
$$x_1, x_2\ge 0$$

3. (a) Show that the matrix $A=\begin{bmatrix}-17 & 18 & -6\\ -18 & 19 & -6\\ -9 & 9 & 2\end{bmatrix}$ is diagonalisable. Find the diagonal form D, and the diagonalising matrix M. 6

(b) Solve the following LPP by simplex method 6

$$\text{Maximise } z=10x_1+x_2+2x_3$$
$$\text{subject to } x_1+x_2-3x_3\le 10$$
$$4x_1+x_2+x_3\le 20$$
$$x_1, x_2, x_3\ge 0$$

(c) State and prove Cauchy's Integral formula and hence evaluate $\oint_C \frac{z+2}{z^3-2z^2}\,dz$ 4+4

where c is $|z-2-i|=2$

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4. (a) Find the orthogonal Trajectory of the family of curves $3x^2y+2x^2-y^3-2y^2=c$ **6**

(b) Find all possible Laurent's expansions of the functions $f(z)=\dfrac{7z-2}{z(z-2)(z+1)}$ about $z=-1$. **6**

(c) Using the principle of Duality solve the following LPP **8**

$$\begin{aligned} &\textit{Minimise } z=4x_1+14x_2+3x_3 \\ &\textit{subject to } x_1-3x_2-x_3\le -3 \\ &2x_1+2x_2-x_3\ge 2 \\ &x_1,x_2,x_3\ge 0 \end{aligned}$$

5. (a) If. $A=\begin{bmatrix}3 & 1\\ 1 & 3\end{bmatrix}$ find e^A , 5^A **6**

(b) Use the dual simplex method to solve the following LPP **6**

$$\begin{aligned} &\textit{Minimise } z=20x_1+16x_2 \\ &\textit{subject to } x_1+x_2\ge 12 \\ &2x_1+x_2\ge 17 \\ &x_1\ge 2.5 \\ &x_2\ge 6 \\ &x_1,x_2\ge 0 \end{aligned}$$

(c) Find the bilinear transformation which maps the points $1, i, -1$ of z-plane onto the points $i, 0, -i$ respectively of w-plane. Hence find the image of $|z|<1$ on to the w-plane. **8**

6. (a) Show that $A=\begin{bmatrix}1 & -6 & -4\\ 0 & 4 & 2\\ 0 & -6 & -3\end{bmatrix}$ is derogatory, and find it's minimal polynomial. **6**

(b) Evaluate the following using residue theorem **3+3**

(i) $\oint_C \dfrac{12z-7}{(z-1)^2(2z+3)}dz$ where c is $|z+i|=\sqrt{3}$

(ii) $\oint_C \tan z\, dz$ where c is $|z|=2$

(c) Using Kuhn-Tucker conditions, solve the following NLPP. **8**

$$\begin{aligned} &\textit{Maximise } z=2x_1+3x_2-x_1^2-x_2^2 \\ &\textit{subject to } x_1+x_2\le 1 \\ &2x_1+3x_2\le 6 \\ &x_1,x_2\ge 0 \end{aligned}$$

7. (a) Show that the map of the real axis of the z-plane is a circle under the transformation $w=\dfrac{2}{z+i}$. Find it's centre and radius. **6**

(b) Evaluate **6**

(i) $\displaystyle\int_{-\infty}^{\infty}\frac{dx}{(x^2+1)^3}$ (ii) $\displaystyle\int_0^{2\pi}\frac{\cos 2\theta}{5+4\cos\theta}d\theta$

(c) Using the method of Lagrangian multipliers solve the following problem **8**

$$\begin{aligned} &\textit{Optimise } z=4x_1^2-x_2^2-x_3^2-4x_1x_2 \\ &\textit{subject to } x_1+x_2+x_3=15 \\ &2x_1-x_2+2x_3=20 \\ &x_1,x_2,x_3\ge 0 \end{aligned}$$