INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

09th November 2011

Subject CT6 – Statistical Models

Time allowed: Three Hours (10.00 – 13.00)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***
2. ***Mark allocations are shown in brackets.***
3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***
4. ***In addition to this paper you will be provided with graph paper, if required.***
5. ***Please check if you have received complete Question Paper and no page is missing. If so, kindly get new set of Question Paper from the Invigilator.***

AT THE END OF THE EXAMINATION

Please return your answer book and this question paper to the supervisor separately.

Q. 1) On a multiple choice test, there are four alternatives to a question, only one of which is correct. A student either knows the answer with probability 0.7 or guesses with probability 0.3. Find the probability that a student, who answered correctly, did not actually know the answer. You may assume that a student who knows the answer would answer correctly, and one who does not know would select each alternative with equal probability.

[2]

Q. 2) The yearly claim amounts (in lacs of Rupees) from a portfolio of policies is assumed to be an exponential distribution with mean $1/\lambda$. The average of the last 100 claim amounts is Rs 0.5 lac. The prior distribution of λ has mean 1.95 and std. Dev 0.1.

i) Suggest a distribution that can be a conjugate prior for λ, with reasons. (3)

ii) Calculate the Bayes estimator of λ under the quadratic loss function, with the prior as in part (i). (4)

[7]

Q. 3) A student wishes to borrow books from a library over the next one year. The annual membership, which costs students Rs. 850, allows borrowers to borrow books without further charge. Alternatively, books are made available to non-members at a borrowing cost of Rs. 175 per book. The student anticipates that he would require 1 to 5 books over the next one year with the following probabilities.

No. of books	Probability
1	0.15
2	0.20
3	0.33
4	0.22
5	0.10

Determine the minimax and Bayes decision regarding becoming a member.

[3]

Q. 4) Under a reinsurance treaty for a large portfolio, the reinsurer pays the amount by which a claim (X) exceeds the retention limit of Rs. 50000. The reinsurer assumes that the random variable X has the distribution

$F(x) = 1 - \exp\left(-ax^{1/3}\right)$ for $x > 0$,

where a is a positive valued parameter. In the last year, there were 50 claims involving the reinsurer, and for those claims, $\sum_{i=1}^{50} x_i^{1/3} = 2600$, where x_i is the gross amount of the i^{th} claim, inclusive of the retention amount.

i) Calculate the maximum likelihood estimate of a. (5)

ii) Give an appropriate estimate of the standard error of this estimator. (2)

iii) The reinsurer learns from the direct insurer that in the last year, there were 600 claims that did not involve the reinsurer. Provide an equation from which one can calculate the modified maximum likelihood estimate of *a*, in view of the additional information. There is no need to actually calculate the estimate. (3)

[10]

Q. 5) The table below shows the cumulative claim payments (in nominal/monetary amounts) from a portfolio of household insurance policies.

Accident Year	*Development Year* 0	1	2	3	4
2007	100	110	120	125	130
2008	110	120	125	130	
2009	105	115	125		
2010	95	105			
2011	120				

Claims are assumed to be 98% paid by development year 4 and fully run-off at the end of development year 5. It may be assumed that payments are made in the middle of a calendar year.

Suppose that the annual claim payment inflation rates over the 12 months up to the middle of the given year are as follows:

2008	5.0%
2009	5.0%
2010	6.0%
2011	8.5%

i) Use the inflation adjusted chain ladder method to determine the reserve to be held at the end of 2011 (in monetary amounts) for future payments on this portfolio assuming a future annual rate of inflation of 10% (as of 30th June). (11)

ii) Estimate the monetary amount of payments to be made in 2012 on this portfolio. (1)

iii) State the assumptions underlying the inflation adjusted chain ladder method (2)

[14]

Q. 6) The number of claims (X) from a health insurance policy over a one year period has the following probabilities:

$P(X = 0) = 0.70$, $P(X = 1) = 0.15$, $P(X = 2) = 0.08$,
$P(X = 3) = 0.05$, $P(X = 4) = 0.02$, $P(X > 4) = 0$.

i) Describe how you would use the acceptance/rejection method to generate discrete pseudo-random samples from this distribution, by using as reference the discrete uniform distribution that has probability 0.2 for each of the values 0,1,2,3 and 4. You may assume that you can also generate samples from the Bernoulli distribution, i.e., a sample that is equal to 1 with probability p and 0 with probability $1 - p$, for any specified value of p in the range (0,1). (4)

ii) What proportion of values generated as above will be accepted? Explain. (2)

iii) Describe an alternative method that could be used to generate pseudo-random samples from the above distribution. (3)

[9]

Q. 7) **i)** Which, if any, of the following statements are true?

a) An ARIMA (1,1,1) can never be Markov.

b) An ARIMA (1,1,1) can never be stationary.

c) No linear combination of a pair of co-integrated processes can be stationary. (3)

ii) An ARMA (2,2) model is fitted to a time series data set consisting of 800 consecutive samples. The residuals exhibit 499 turning points. By performing an approximate statistical test, indicate whether the residuals can be said to have originated from a white noise process. (3)

[6]

Q. 8) Claims for a particular risk arrive as a Poisson process with rate λ. An insurer covers these risks through premium with loading factor θ. The claim sizes follow a distribution with probability density function $f(x)$ given by

$$f(x) = 0.5e^{-x}(1+2e^{-x}),\ x > 0.$$

i) State the usual assumptions that govern the insurer's surplus process. (2)

ii) Calculate the expected claim size. (1)

iii) Assuming $\theta = 0.25$, calculate the numerical value of the adjustment coefficient R, and show that it satisfies the inequality $R < 2\theta\frac{m_1}{m_2}$, where θ is the premium loading factor and m_k denotes the k^{th} order moment of the distribution of claim amounts. (5)

iv) Determine an upper bound for the probability of ruin with an initial surplus of 15. (1)

v) Determine the probability of ruin, starting with the same initial surplus of 15, assuming the claim sizes follow an exponential distribution with the same mean as calculated in part (i). Comment on your answer. (4)

[13]

Q. 9) An actuary wishes to estimate the pure premium for a certain risk. Claim amounts from the risk follow the Normal distribution $N(\theta, \sigma_1^2)$. He proposes to use $N(\mu, \sigma_2^2)$ as the prior distribution of θ. Past values of claim amounts from the risk have been observed as $x_1, x_2, \ldots, x_n$.

i) Show that the Bayes estimate for θ under the quadratic loss function can be written in the form of a credibility estimate and write down a formula for the credibility factor. (5)

ii) The actuary decides to use the prior parameter values μ = 500 and σ_2^2 = 250^2. Sample data randomly observed for 10 claim amounts are 100, 200, 350, 700, 525, 930, 100, 25, 105, and 160. Determine the Bayes credibility estimate of the pure premium when σ_1^2 = 100^2 and when σ_1^2 = 350^2. (2)

iii) Comment on the results. (3)

[10]

Q. 10) The table below shows the claim amounts (in Rs.'000s) over a four year period for three fleets of buses, A, B and C.

	Year 1	Year 2	Year 3	Year 4
A	5	7	5	6
B	25	25	30	40
C	100	110	105	125

Estimate the credibility premium for each fleet using EBCT Model 1.

[6]

Q. 11) In the context of Generalised Linear Models, consider the exponential distribution with density function $f(x)$, where

$$f(x) = \frac{1}{\mu} e^{-x/\mu} \quad , x > 0.$$

i) Show that $f(x)$ can be written in the form of an exponential family of distributions. (1)

ii) Determine the canonical parameter θ, the variance function and the dispersion parameter. (3)

[4]

Q. 12) The number of deaths (S) in a railway division in a particular year is the aggregate over the number of deaths in different fatal accidents. The number of fatal accidents in a year (N), and the number of deaths in the i^{th} fatal accident (U_i) for i = 1,2, …, N have the following distributions.

$$P(N = n) = 0.25 \times (0.75)^n,\ n = 0,1,2,\dots,$$
$$P(U_i = u) = 0.75 \times (0.25)^{u-1},\ u = 1,2,3,\dots.$$

Further, the numbers of deaths in different fatal accidents are independent of one another, and are also independent of N.

i) Calculate the moment generating function of N, and hence its mean and variance. (4)

ii) Calculate the mean and variance of U_i. (2)

iii) Calculate the mean and variance of S. (2)

iv) The railway authorities provide accidental death cover for up to two deaths per fatal accident per year, and engage a reinsurer for covering the remaining deaths. Calculate the mean and variance of Y, the number of deaths covered by the reinsurer over a year. (8)

[16]

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