AB-3361

Seat No. _______

M. Phil. Examination

April / May – 2003

Mathematics : Paper - II

Time :   Hours]   [Total Marks : **75**

Q.1. Attempt any three of the following: (15)

(a) Describe the general form of making a Prism. Using this process define a polytope of each dimension n. For this polytope of dimension n, verify Euler's generalized formula. Also find the value of N_2^4.

(b) What is a Schlegel Diagram? Draw Schlegel Diagrams of all the five Platonic Solids.

(c) Give a proof of Euler's Formula.

(d) Show that there is no regular polyhedron all faces of which are hexagons.

Q.2. Attempt any three of the following: (15)

(a) Define a partially ordered set and a lattice. Give three examples of lattice. Also give an example of a partially ordered set which is not a lattice.

(b) Define a Boolean Algebra. Simplify the following Boolean expressions:

(i) $(a * b)' \oplus (a \oplus b)'$

(ii) $(a' * b' * c) \oplus (a * b' * c) \oplus (a * b' * c')$

(iii) $(1 * a) \oplus (0 * a')$

(c) Define a Boolean Ring. In a Boolean ring A, prove that

(i) p + p = 0 for all p in A

(ii) pq = qp for all p, q in A

(d) Put $P^{\perp} = \overline{P}'$. Show that

(i) $P \subseteq Q \Rightarrow Q^{\perp} \subseteq P^{\perp}$

(ii) If P is open, then $P \subseteq P^{\perp\perp}$

(iii) If P is open, then $P^{\perp} = P^{\perp\perp\perp}$

Q.3. Attempt any three of the following: (15)

(a) Show that $x \in \overline{A}$ iff there exists a filter $\mathfrak{F}$ containing A which converges to x

(b) Show that a filter $\mathfrak{F}$ on X is an ultrafilter iff for each subset E of X either $E \in \mathfrak{F}$ or $X - E \in \mathfrak{F}$.

(c) What is βN? Show that it is a compact, Hausdorff space. Write a most important property of βN.

(d) Show that there are uncountably many distinct ultrafilters on N.

Q.4. Attempt any three of the following: (15)

(a) For an infinite set X, show that $|Xx\{0,1\}| = |X|$. Using this show that every set X can be partitioned into sets A, B such that $|A| = |B| = |X|$.

(b) Define an ordinal. Write down the first three elements of any ordinal. Describe the difference between 2ω and $\omega 2$.

(c) Show that $|\mathfrak{P}(X)| = |2^X|$.

(d) Describe the topology of $[0,\omega]$, where ω is the first infinite ordinal number. Show that $[0,\Omega]$, where Ω is the first uncountable ordinal, is not separable.

Q.5. Attempt any three of the following: (15)

(a) Show that a metric space X is compact iff every real valued continuous function on X is bounded.

(b) State and prove Scroder-Bernstein Theorem

(c) Describe two instances, where one can describe geometrically a 1-1 correspondence.

(d) Let $Q = \{r_1, r_2, ...\}$ and $f: R \to R$ be defined as $f(r_n) = \frac{1}{n}$ and $f(x) = 0$ if x is not a rational. Describe the continuity of f.
