6. (a) How many complex additions and complex multiplications are required for the direct computation of an N-point DFT ? 4

(b) Discuss the radix–2 DIT FFT algorithm. 6

7. Determine the direct form I and direct form II realization of an LTI system given as $2y(n)+y(n-1)-4y(n-3)=x(n)+3x(n-5)$. How can you realize a unit delay in hardware ? 10

8. (a) Using the impulse invariance method with $T=1$, determine $H(z)$ if $H(s)=\frac{1}{s^2+\sqrt{2}s+1}$. What is the order of the system ? 4+1

(b) Derive the variance of the Bartlett power spectrum estimate. 5

Total number of printed pages – 8 B. Tech

CPEC 5302

Sixth Semester Examination – 2009

DIGITAL SIGNAL PROCESSING

Full Marks – 70

Time : 3 Hours

Answer either from Set-A or Set-B, but not from both.

SET – A

*Answer Question No. **1** which is compulsory and any **five** from the rest.*

The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Find out if a system with the input-output relation given by $y(n)=\alpha x(n)+\beta$ is linear. Justify.

(b) Determine whether a system described with $y(n) = nx(n)$ is time invariant.

(c) Draw the transfer function of an ideal low pass filter with proper labeling.

(d) What is the period of $x(t) = 20\sin 25\pi t$?

(e) Give an example of a recursive system and explain why it is recursive ?

(f) What is the magnitude response of a system described with the following input-output relationship $y(n)=0.5[x(n)+x(n-1)]$?

(g) What is the z-transform of a sequence $x(n) = \{1, 2, 4, 6\}$?

(h) What is the DFT of a real and even sequence ?

(i) If a finite duration sequence of length M_1 excites an FIR filter with a length of M_2, what is the length of the output sequence ?

(j) When an FIR filter is said to have a linear phase ?

2. (a) Determine the range of values of α for which an LTI system described with the following impulse response is stable.

$$h(n) = \alpha^n \cos(n\omega_0)u(n)$$ 7

(b) Express the sequence defined by

$$x(n)=\begin{cases}-2, n=-1,0,1\\4, n=-2,2\\0 \text{ otherwise}\end{cases}$$

as a weighted sum of unit sample sequences. 3

3. (a) Find out the response of the relaxed system having an impulse response of $\left(\frac{1}{2}\right)^n u(n)$ to the input signal $2^n u(n)$. 7

(b) Give a block schematic of a system having the following input-output relationship $y(n) = x(n) + ay(n-1) + by(n-2)$. How many additions and multiplications are performed per sample ? 3

4. (a) Find out the z-transform and ROC of the signal $x(n) = \left(\frac{1}{3}\right)^n u(n)$. 6

(b) Determine the transfer function of a system described by $y(n) = \frac{3}{2}y(n-1) + 2x(n)$. 4

5. (a) Compute the DFT of a sequence defined by $x(n) = \{-2, 2, 1, -1\}$ 5

(b) Compute the N point DFT a sequence given as $x(n) = e^{-n}$, $0 \le n \le 4$. 5

6. (a) How many complex additions and complex multiplications are required for the direct computation of an N-point DFT ? 4

(b) Discuss the radix–2 DIT FFT algorithm. 6

7. Determine the direct form I and direct form II realization of an LTI system given as $2y(n) + y(n-1) - 4y(n-3) = x(n) + 3x(n-5)$. How can you realize a unit delay in hardware ? 10

8. (a) Using the impulse invariance method with $T = 1$, determine $H(z)$ if $H(s) = \frac{1}{s^2 + \sqrt{2}s + 1}$. What is the order of the system ? 4+1

(b) Derive the variance of the Bartlett power spectrum estimate. 5

SET – B (IT Branch)

*Answer Question No. **1** which is compulsory and any **five** from the rest.*
The figures in the right-hand margin indicate marks.

1. Answer the following questions : 2×10

(a) Find out if a system with the input-output relation given by $y(n) = \alpha x(n) + \beta$ is linear. Justify.

(b) Determine whether a system described with $y(n) = nx(n)$ is time invariant.

(c) Draw the transfer function of an ideal low pass filter with proper labeling.

(d) What is the period of $x(t) = 20\sin 25\pi t$?

(e) Give an example of a recursive system and explain why it is recursive ?

(f) What is the magnitude response of a system described with the following input-output relationship $y(n)=0.5[x(n)+x(n-1)]$?

(g) Prove the periodicity property of DFT.

(h) What is the DFT of a real and even sequence ?

(i) If a finite duration sequence of length M_1 excites an FIR filter with a length of M_2, what is the length of the output sequence ?

(j) When an FIR filter is said to have a linear phase ?

2. (a) Determine the range of values of α for which an LTI system described with the following impulse response is stable.

$$h(n) = \alpha^n \cos(n\omega_0)u(n)$$ 7

(b) Express the sequence defined by

$$x(n)=\begin{cases}-2, n=-1,0,1\\4, n=-2,2\\0 \text{ otherwise}\end{cases}$$

as a weighted sum of unit sample sequences. 3

3. (a) Find out the response of the relaxed system having an impulse response of $\left(\frac{1}{2}\right)^n u(n)$ to the input signal $2^n u(n)$. 7

(b) Give a block schematic of a system having the following input-output relationship $y(n) = x(n) + ay(n-1) + by(n-2)$. How many additions and multiplications are performed per sample ? 3

4. (a) Determine the frequency response and magnitude spectrum of a system characterized by $y(n) = 1.8y(n-1) - 0.81y(n-2) + x(n) + 0.95x(n-1)$ 5

(b) Convert the analog filter with system function $H_a(s)=\frac{s+0.1}{(s+0.1)^2+16}$ into a digital IIR filter by means of bilinear function. Give the location of poles and zeros. 5

5. (a) Compute the DFT of a sequence defined by $x(n) = \{-2, 2, 1, -1\}$ 5

(b) Compute the N point DFT a sequence given as $x(n) = e^{-n}$, $0 \le n \le 4$. 5