Actuarial Society of India
EXAMINATIONS

06th November 2006

Subject CT8 – Financial Economics

Time allowed: Three Hours (02.30 – 05.30 pm)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1) *Do not write your name anywhere on the answer sheet/s. You have only to write your Candidate's Number on each answer sheet.*

2) *Mark allocations are shown in brackets.*

3) *Attempt all questions, beginning your answer to each question on a separate sheet. However, answers to objective type questions could be written on the same sheet.*

4) *Fasten your answer sheets together in numerical order of questions. This, you may complete immediately after expiry of the examination time.*

5) *In addition to this paper you should have available graph paper, Actuarial Tables and an electronic calculator.*

Professional Conduct:

"It is brought to your notice that in accordance with provisions contained in the Professional Conduct Standards, If any candidate is found copying or involved in any other form of malpractice, during or in connection with the examination, Disciplinary action will be taken against the candidate which may include expulsion or suspension from the membership of ASI."

Candidates are advised that a reasonable standard of handwriting legibility is expected by the examiners and that candidates may be penalized if undue effort is required by the examiners to interpret scripts.

AT THE END OF THE EXAMINATION

Hand in both your answer scripts and this question paper to the supervisor.

Q.1) List the principal assumptions required for the Capital Asset Pricing Model to hold. **[5]**

Q.2)

(i) Describe briefly the meaning of:
(a) specific risk
(b) beta (4)

(ii) Define the systematic or market risk of a share. (1)
[5]

Q.3)

(i) Explain what is meant by the multifactor model. You should define any notation you use. (4)

(ii) Briefly describe three different types of factor that can be used in a multifactor model. (3)
[7]

Q.4)

(i) Describe briefly what is meant by a short-rate model of interest rates and how such models are used to price bonds and interest rate derivatives. (4)

(ii) Explain what is meant by a one-factor model of interest rates. (3)
[7]

Q.5)

(i) State what is meant by put-call parity. (1)

(ii) Derive an expression for the put-call parity of a European option that has a dividend payable prior to the exercise date. (6)

(iii) If the equality in (ii) does not hold, explain how an arbitrageur can make a riskless profit. (3)
[10]

Q.6) The price of a non-dividend paying stock at time 1, S_1, is related to the price at time 0, S_0, as follows:

$S_1 = uS_0$ with probability p and
$S_1 = dS_0$ with probability $(1\text{-}p)$

The continuously compounded rate of return on a risk-free asset is r.

(i) Derive an expression for the replicating portfolio for a European call option written on the stock that expires at time 1 and has a strike price of k, where $dS_0 < k < uS_0$ (6)

(ii) Show that the price of the option in (i) can be written as the discounted expected payoff under a probability measure Q. Hence find an expression for the probability, q, of an upward move in the stock price under Q. (2)

(iii) Explain the relationship between the Q probability measure in (ii) and the real world probability measure. Explain what relationship you would expect q and p to have if all investors are (a) risk averse, (b) risk seeking, or (c) risk-neutral. (5)
[13]

Q.7) The price St of a share, paying no dividends, follows geometric Brownian motion:

$$dS_t = S_t\,(?\,dt + ?\,dZ_t)$$

where Z_t is a standard Brownian motion. A derivative is available on this share that can only be exercised at time T. The price of the derivative at time t, $f(t,S_t)$, depends on the time and the current share price. A cash bond is also available that offers a risk-free rate of return of r (continuously compounded). The price of the bond is B_t.

You wish to set up a replicating portfolio for the derivative made out of shares and cash, so that:

$$?\,S_t + ?\,B_t = f(t,S_t) \qquad (*)$$

(i) Write down the stochastic differential equation that is satisfied by B_t. (1)

(ii) Explain what it means for such a portfolio to be self-financing. Give a differential equation that must be satisfied by the portfolio considered above in order that this is the case. (2)

(iii) Explain what it means for a process to be previsible. (1)

(iv) By applying Ito's lemma to the right-hand side of equation (*) and using your answer to part (ii), deduce that:

(a) $$?_t \; ? \; \frac{?f}{?S_t}$$

(b) $$\frac{?f}{?t} \; ? \; rS_t \frac{?f}{?S_t} \; ? \; \frac{1}{2} ?^2 S_t^2 \frac{?^2 f}{?S_t^2} \; ? \; rf$$

(9)

[13]

Q.8) Assets A and B have the following distribution of returns in various states:

State	*Asset A*	*Asset B*	*Probability*
1	*10%*	*-2%*	*0.2*
2	*8%*	*15%*	*0.2*
3	*25%*	*0%*	*0.3*
4	*-14%*	*6%*	*0.3*

(i) Calculate the correlation between the returns on asset A and asset B. (5)

(ii) Calculate the proportion of assets that should be invested in asset A to obtain the minimum risk portfolio. (4)

(iii) An investor decides to hold 25% of his wealth in asset A and 75% in asset B. He is concerned that the correlation between the assets may not remain constant over time. Calculate the lowest value of the correlation between assets A and B for which the investor still gets diversification benefits from holding 25% in asset A. Assume that the variances are unchanged. (4)

[13]

Q.9)

(i) By first applying Itó's lemma to the function f (St) = logSt, solve the stochastic differential equation defining geometric Brownian motion:
dS_t = ? ? S_t dt + ? s S_t dB_t (4)

(ii) St , the price of a share at time t, is modelled as geometric Brownian motion. If ? ? 20% pa and s = 10% pa, calculate the probability that the share price will exceed 110 in six months' time given that its current price is 100. (5)

[9]

Q.10)

(i) Describe briefly the Vasicek one-factor model of interest rates and its key statistical properties.

In the Vasicek model, the spot rate of interest is governed by the stochastic differential equation:

$$dr_t = a\,(b - r_t)\,dt + ?\ dB_t$$

where Bt is a standard Brownian motion and a, b > 0 are constants. (4)

(ii) A stochastic process {$Ut : t$? 0} is defined by $U_t = e^{at} r_t$

(a) Derive an equation for dU_t .
(b) Hence solve the equation to find U_t .
(c) Hence show that:

$$r_t = b + (r_0 - b)\,e^{-at} + \underset{0}{\overset{t}{?}} e^{a(s-t)} dB_s$$

(5)

(iii) State the probability distribution of r_t and its limit for large t. (4)

(iv) Derive, in the case where $s < t$, the conditional expectation E [r_t/F_s], where {Fs : s=0} is the filtration generated by the Brownian motion B_s.

(5)

[18]

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