PART III – MATHEMATICS

1. If A and B are two square matrices such that $B = -A^{-1}BA$, then $(A+B)^2$ is equal to

A) 0 B) $A^2 + B^2$ C) $A^2 + 2AB + B^2$ D) $A + B$

2. Let $A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & k \\ 0 & -2 & 3 \end{bmatrix}$. If $B = 10A^{-1}$, then k equals

A) 2 B) –1 C) –2 D) 5

3. The value of λ for which the system of equations $x + y - 2z = 0$, $2x - 3y + z = 0$, $x - 5y + 4z = \lambda$ is consistent is

A) 1 B) –1 C) 0 D) 2

4. The region in the Argand plane defined by $|z - 2i| + |z + 2i| < 5$ is the interior of the ellipse with major axis along

A) the real axis B) $y = -x$
C) $y = x$ D) the imaginary axis

5. The argument of the complex number $(1+i)^4$ is

A) 135° B) 180° C) 45° D) 90°

6. If $\omega = \alpha + i\beta$, where $\beta \neq 0$ and $z \neq 1/2$ satisfies the condition that $\dfrac{\omega - 2\overline{\omega}z}{1 - 2z}$ is purely real, then the set of values of z is

A) $\{z : z = \bar{z}\}$ B) $\left\{z : |z| = \dfrac{1}{2}\right\}$
C) $\{z : |z| = 1\}$ D) $\left\{z : |z| = \dfrac{1}{2} \text{ and } z \neq \dfrac{1}{2}\right\}$

7. The point (a, $2a$) is an interior point of the region bounded by the parabola $y^2 = 16x$ and the double ordinate through the focus. Then a belongs to the interval

A) $a < 4$ B) $a > 4$ C) $0 < a < 2$ D) $0 < a < 4$

8. If the foci of the ellipse $\frac{x^2}{16}+\frac{y^2}{b^2}=1$ and the hyperbola $\frac{x^2}{144}-\frac{y^2}{81}=\frac{1}{25}$ coincide, then the value of b^2 is

A) 1 B) 5 C) 7 D) 9

9. A tetrahedron has vertices at O(0,0,0), A(1,2,1), B(2,1,3) and C(−1,1,2) then the angle between the faces OAB and ABC will be

A) $\cos^{-1}\left(\frac{19}{35}\right)$ B) 90° C) 30° D) $\cos^{-1}\left(\frac{17}{31}\right)$

10. If the normal to the curve $y=f(x)$ at the point (3, 4) makes an angle $\frac{3\pi}{4}$ with the positive x-axis, then $f'(3)$ is equal to

A) –1 B) 1 C) $\frac{4}{3}$ D) $\frac{-3}{4}$

11. The maximum value of $(1/x)^x$ is

A) e^e B) $e^{1/e}$ C) $(1/e)^e$ D) e

12. A spherical balloon is expanding. If the radius is increasing at the rate of 2 inches per minute, the rate at which the volume increases (in cubic inches per minute) when the radius is 5 inches is

A) 10π B) 50π C) 200π D) 100π

13. The maximum degree of x by which determinant is $\begin{vmatrix} x+y & x & x \\ 5x+4y & 4x & 2x \\ 10x+8y & 8x & 3x \end{vmatrix}$ divisible

A) 0 B) 1 C) 2 D) 3

14. If ω is cube root of unity, then $\left(1+\omega-\omega^2\right)^7$ equals

A) 128ω B) −128ω C) $128\omega^2$ D) $-128\omega^2$

15. The region in the Argand plane defined by $|z-2i|+|z+2i|<5$ is the interior of the ellipse with major axis along

A) $y=x$ B) $y=-x$
C) the real axis D) the imaginary axis

16. If P(x, y) is any point on $16x^2 + 25y^2 = 400$ and F_1 = (3, 0), F_2 = (–3, 0) and then $PF_1 + PF_2$ equals

A) 6 B) 8 C) 10 D) 12

17. If P is a point on the rectangular hyperbola $x^2 - y^2 = a^2$, C is its centre and S,S′ are the two foci, then SP.S′P =

A) $(CS)^2$ B) $(CP)^2$ C) 2 D) $(SS')^2$

18. If $|a| = 6$, $|b| = 8$, $|c| = 2$ and $\vec{a} + \vec{b} + \vec{c} = 0$, then the value of $\left|\vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a}\right|$ is

A) –52 B) 104 C) 52 D) –104

19. Let $\bar{a}$, $\bar{b}$ and $\bar{c}$ be three non-zero and non-coplanar vectors and $\bar{p}$, $\bar{q}$ and $\bar{r}$ be three vectors given by $\bar{p} = \bar{a} + \bar{b} - 2\bar{c}$, $\bar{q} = 3\bar{a} - 2\bar{b} + \bar{c}$ and $\bar{r} = \bar{a} - 4\bar{b} + 2\bar{c}$. If the volume of the parallelopiped determined by $\bar{a}$, $\bar{b}$ and $\bar{c}$ is V_1 and that of the parallelopiped determined by $\bar{p}$, $\bar{q}$ and $\bar{r}$ is V_2, then $V_2 : V_1$ is equal to

A) 2 : 3 B) 5 : 7 C) 15 : 1 D) 1 : 1

20. The value of $\int_0^{\frac{\pi}{2}} (\cos x)^{\sin x} \left(\cos x - \sec x + \log(\cos x)^{\cos x}\right) dx$ is

A) 1 B) $\frac{\pi}{2}$ C) 0 D) –1