AIEEE 2010

Answers by Aakash IIT-JEE

(Division of Aakash Educational Services Ltd.)

Q.N.	CODE A	CODE B	CODE C	CODE D
01.	1	1	1	2
02.	2	2	1	3
03.	3	3	1	1
04.	3	1	4	4
05.	4	1	4	4
06.	1	3	2	3
07.	2	2	**2**	2
08.	3	1	4	2
09.	2	2	4	3
10.	3	1	4	3
11.	4	3	3	4
12.	3	3	4	2
13.	2	2	3	1
14.	2	2	3	2
15.	2	3	2	1
16.	3	1	2	1
17.	3	1	**2**	1
18.	1	4	1	3
19.	3	2	3	4
20.	1	3	1	4
21.	4	4	1	4
22.	3	2	2	2
23.	1	4	3	3
24.	1	4	3	2
25.	4	4	3	2
26.	4	1	4	3
27.	4	1	2	1
28.	1	1	3	1
29.	4	4	4	2
30.	1	4	4	2

Q.N.	CODE A	CODE B	CODE C	CODE D
31.	4	4	4	2
32.	3	4	4	3
33.	1	3	1	1
34.	4	4	2	4
35.	1	2	4	2
36.	3	4	3	4
37.	3	1	4	1
38.	2	4	2	2
39.	2	1	2	2
40.	1	1	1	3
41.	4	4	2	2
42.	3	1	1	2
43.	2	2	2	3
44.	3	2	3	1
45.	2	1	4	2
46.	4	3	3	1
47.	1	1	4	1
48.	4	1	3	4
49.	1	**3**	3	2
50.	1	4	1	3
51.	4	1	4	3
52.	3	1	1	4
53.	3	**3**	2	4
54.	3	2	1	4
55.	3	3	2	3
56.	2	3	4	2
57.	2	3	1	4
58.	4	4	3	1
59.	2	2	2	3
60.	3	2	4	4

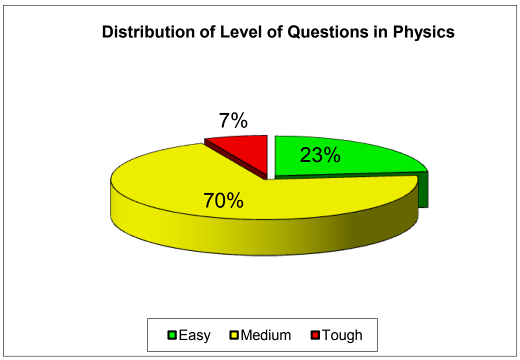
Q.N.	CODE A	CODE B	CODE C	CODE D
61.	3	2	3	1
62.	2	3	4	2
63.	3	2	1	4
64.	4	3	4	3
65.	4	4	4	4
66.	2	1	2	**4**
67.	1	4	1	2
68.	2	3	3	4
69.	2	1	4	2
70.	4	2	1	3
71.	**1**	2	3	3
72.	1	1	2	2
73.	1	3	2	**4**
74.	1	1	4	2
75.	3	1	2	2
76.	4	4	4	4
77.	3	3	1	2
78.	3	1	1	2
79.	2	2	1	1
80.	2	2	4	2
81.	2	3	2	3
82.	2	2	4	1
83.	3	1	2	4
84.	4	4	1	1
85.	4	1	4	1
86.	3	3	4	3
87.	3	1	3	3
88.	**1**	4	3	1
89.	1	3	3	1
90.	3	4	3	1

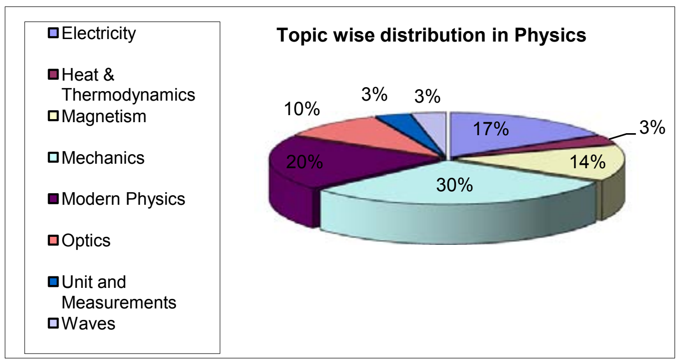
Though every care has been taken to provide the answers correctly but the Institute shall not be responsible for error, if any.

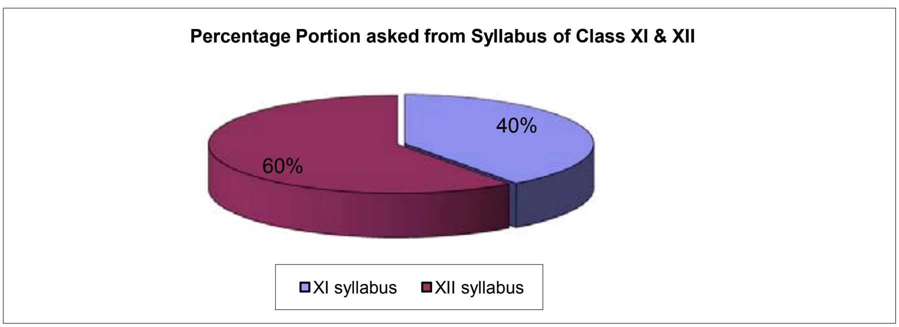
ANALYSIS OF PHYSICS PORTION OF AIEEE 2010

	XII	XI	XII	XI	XII	XII	XI	XI	
	Electricity	Heat & Thermodynamics	Magnetism	Mechanics	Modern Physics	Optics	Unit and Measurements	Waves	*Total*
Easy	1	0	0	3	3	0	0	0	*7*
Medium	4	0	4	5	3	3	1	1	*21*
Tough	0	1	0	1	0	0	0	0	*2*
Total	*5*	*1*	*4*	*9*	*6*	*3*	*1*	*1*	***30***

XI syllabus	**12**	**XII syllabus**	**18**







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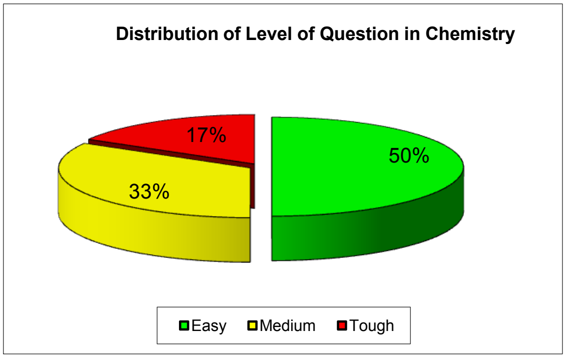
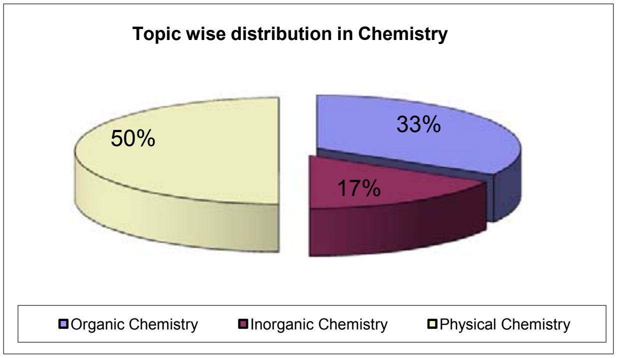
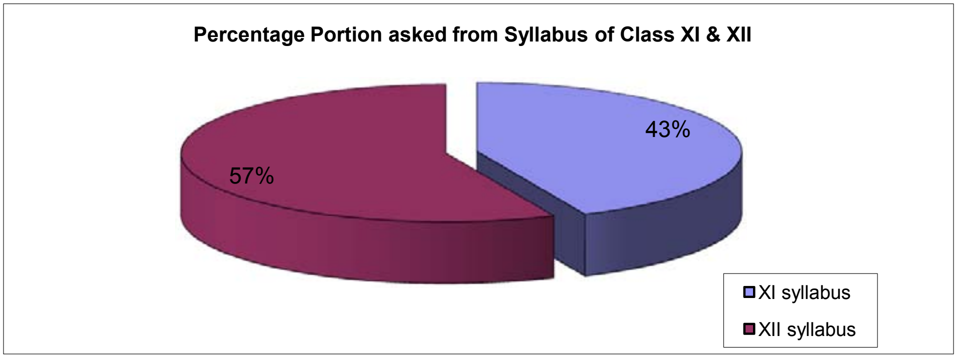
ANALYSIS OF CHEMISTRY PORTION OF AIEEE 2010

	Organic Chemistry	Inorganic Chemistry	Physical Chemistry	*Total*
Easy	7	2	6	*15*
Medium	2	2	6	*10*
Tough	1	1	3	*5*
Total	*10*	*5*	*15*	**30**

XI syllabus	*13*	**XII syllabus**	**17**







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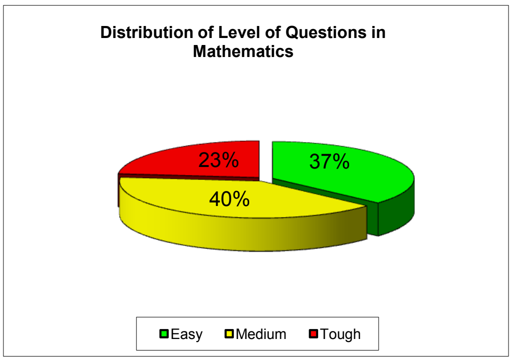
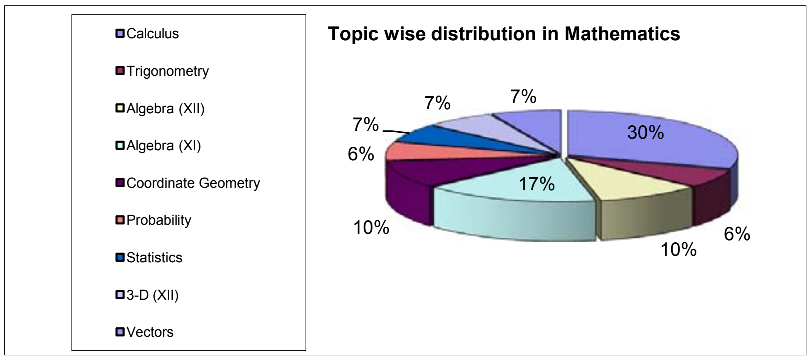
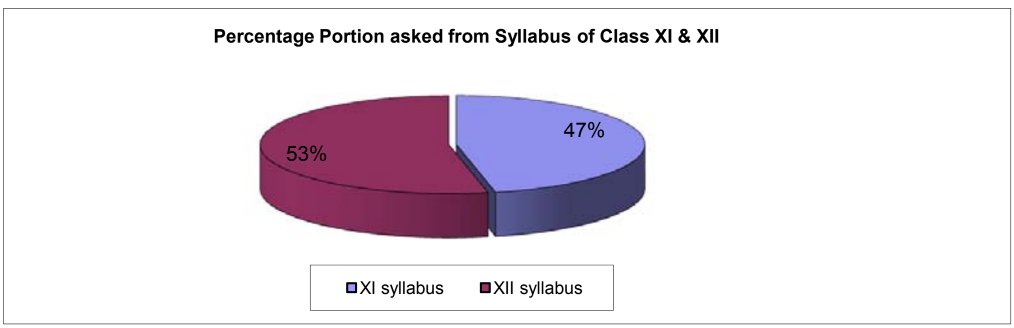
ANALYSIS OF MATHEMATICS PORTION OF AIEEE 2010

	XII	XI	XII	XI	XI	XI	XI	XII	XII	
	Calculus	**Trigonometry**	**Algebra (XII)**	**Algebra (XI)**	**Coordinate Geometry**	**Probability**	**Statistics**	**3-D (XII)**	**Vectors**	***Total***
Easy	4	0	0	0	2	1	1	1	2	*11*
Medium	2	1	1	4	1	1	1	1	0	*12*
Tough	3	1	2	1	0	0	0	0	0	*7*
Total	*9*	*2*	*3*	*5*	*3*	*2*	*2*	*2*	*2*	***30***

XI syllabus	**14**
XII syllabus	**16**







DATE : 25/04/2010

Code - A

Aakash IIT-JEE

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Ph.: 011-47623456 Fax : 011-47623472

Time : 3 hrs. **Max. Marks: 432**

Solutions

for

AIEEE 2010

(Physics, Chemistry & Mathematics)

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. *Use of pencil is strictly prohibited*.

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions**.** The maximum marks are **432.**

5. There are ***three*** parts in the question paper. The distribution of marks subjectwise in each part is as under for each correct response.

Part A – PHYSICS (144 marks) –Questions No. 1 to 20 and 23 to 26 consist of **FOUR (4)** marks each and Questions No. 21 to 22 and 27 to 30 consist of **EIGHT (8)** marks each for each correct response.

Part B – CHEMISTRY (144 marks) – Questions No. 31 to 39 and 43 to 57 consist of **FOUR (4)** marks each and Questions No. 40 to 42 and 58 to 60 consist of **EIGHT (8)** marks each for each correct response.

Part C – MATHEMATICS (144 marks) – Questions No. 61 to 66, 70 to 83 and 87 to 90 consist of **FOUR (4)** marks each and Questions No. 67 to 69 and 84 to 86 consist of **EIGHT (8)** marks each for each correct response

6. *Candidates will be awarded marks as stated above in Instructions No. 5 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question.* ***No deduction*** *from the total score will be made* ***if no response*** *is indicated for an item in the answer sheet.*

7. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.

8. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However the candidates are allowed to take away this Test Booklet with them.

9. The **CODE** for this Booklet is **A.** Make sure that the **CODE** printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet

10. Do not fold or make any stray marks on the Answer Sheet.

PART–A : PHYSICS

Directions : *Questions number 1-3 are based on the following paragraph.*

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

1. The initial shape of the wavefront of the beam is

(1) Planar

(2) Convex

(3) Concave

(4) Convex near the axis and concave near the periphery

Ans. (1)

Sol. As the beam is initially parallel, the shape of wavefront is planar.

2. The speed of light in the medium is

(1) Maximum on the axis of the beam

(2) Minimum on the axis of the beam

(3) The same everywhere in the beam

(4) Directly proportional to the intensity I

Ans. (2)

Sol. Given $\mu = \mu_0 + \mu_2 I$

Also, $\mu = \dfrac{c}{v} \Rightarrow v = \dfrac{c}{\mu_0 + \mu_2 I}$

As intensity is maximum at centre, so v is minimum on the axis.

3. As the beam enters the medium, it will

(1) Travel as a cylindrical beam

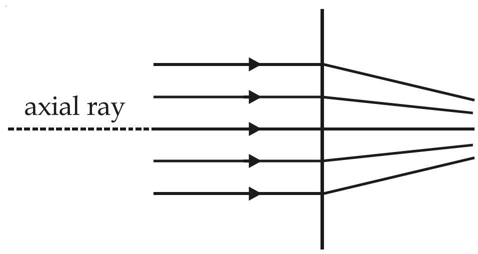
(2) Diverge

(3) Converge

(4) Diverge near the axis and converge near the periphery

Ans. (3)

Sol. As the beam enters the medium, axial ray will travel slowest. So, it will lag behind. To compensate for the path, the rays will bend towards axis.



Directions : *Questions number 4-5 are based on the following paragraph.*

A nucleus of mass $M + \Delta m$ is at rest and decays into two daughter nuclei of equal mass $\dfrac{M}{2}$ each. Speed of light is c.

4. The speed of daughter nuclei is

(1) $c\sqrt{\dfrac{\Delta m}{M + \Delta m}}$ (2) $c\dfrac{\Delta m}{M + \Delta m}$

(3) $c\sqrt{\dfrac{2\Delta m}{M}}$ (4) $c\sqrt{\dfrac{\Delta m}{M}}$

Ans. (3)

Sol. Energy released $Q = \Delta mc^2$

$$Q = \frac{1}{2}\left(\frac{M}{2}\right)v^2 + \frac{1}{2}\left(\frac{M}{2}\right)v^2$$

$$\Delta mc^2 = \frac{M}{2}v^2$$

$$v = \sqrt{\frac{2\Delta m}{M}}\,c$$

5. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(1) $E_1 = 2\,E_2$ (2) $E_2 = 2\,E_1$

(3) $E_1 > E_2$ (4) $E_2 > E_1$

Ans. (4)

Sol. As energy is released, binding energy per nucleon of products is more than that of reactants.

$\Rightarrow$ $E_2 > E_1$.

Directions : *Questions number 6-7 contain Statement-I and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.*

6. **Statement-1 :** When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement-2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is the ***not*** the correct explanation of Statement-1

(4) Statement-1 is false, Statement-2 is true

Ans. (1)

Sol. X-rays frequency is more than that of UV rays. So, KE_{max} and stopping potential increase. Statement-2 is incorrect. Photoelectrons are emitted with a range of kinetic energies because different electrons have different binding energies.

7. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

(1) Statement-1 is true, Statement-2 is false

(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1

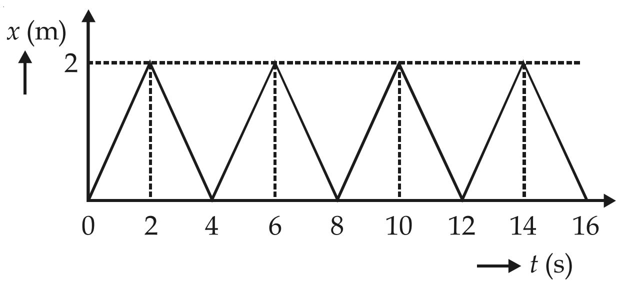
(3) Statement-1 is true, Statement-2 is true; Statement-2 is the ***not*** the correct explanation of Statement-1

(4) Statement-1 is false, Statement-2 is true

Ans. (2)

Sol. If the particle moving in same direction lose all their energy, final momentum will become zero, whereas initial momentum is not zero.

8. The figure shows the position–time (x-t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



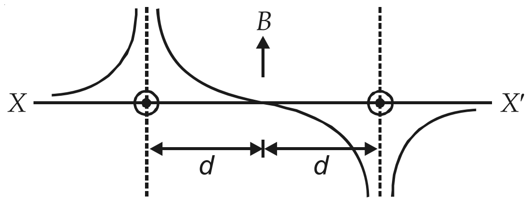
(1) 0.2 Ns (2) 0.4 Ns

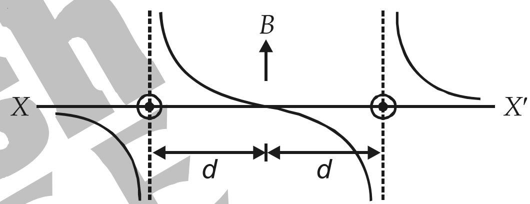
(3) 0.8 Ns (4) 1.6 Ns

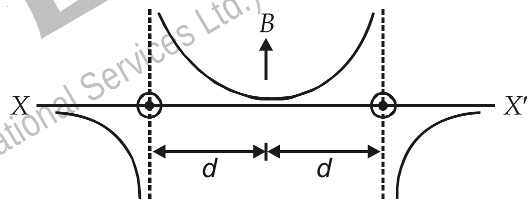
Ans. (3)

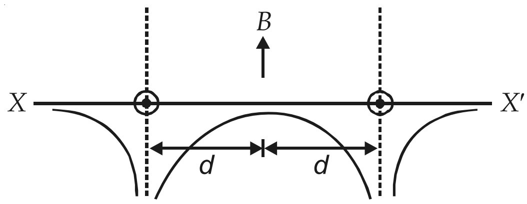
Sol. $I = \Delta p = m|\Delta v| = 0.4 \times (1 + 1) = 0.8$ Ns

9. Two long parallel wires are at a distance $2d$ apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by

(1)


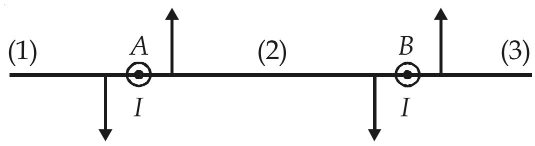
(2)


(3)


(4)


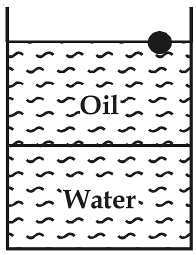
Ans. (2)

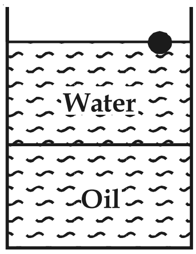
Sol. Taking up as positive, in region 1, field will remain negative, and as one moves from $-\infty$ to A, field increases in magnitude from zero to large value.

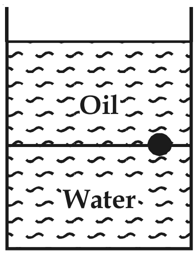


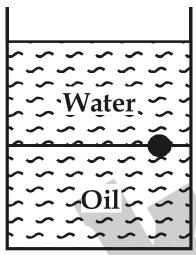
As one moves from A to B, field changes sign from positive to negative, becoming zero at mid point. As one moves in region 3, from B to $+\infty$, field decreases from a large value to zero.

10. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium positions?

(1) 

(2) 

(3) 

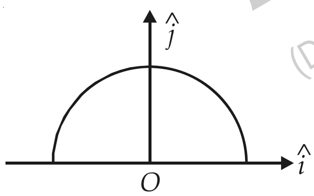
(4) 

Ans. (3)

Sol. $\rho > \rho_{oil}$, ball must sink in oil alone.

As $\rho < \rho_{water}$, ball must float in water.

11. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(1) $\dfrac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $\dfrac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$

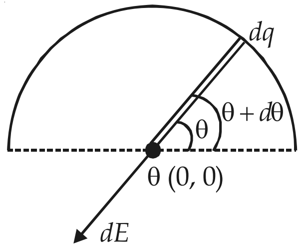
(3) $-\dfrac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $-\dfrac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

Ans. (4)

Sol. By symmetry, $\int dE\cos\theta = 0$

$$E = -\int dE\sin\theta\,\hat{j}$$

$$E = -\left[\int \frac{dq}{4\pi\varepsilon_0 r^2}\sin\theta\right]\hat{j}$$



Now, $dq = \dfrac{q}{\pi}\,d\theta$

$$E = -\int_0^{\pi}\frac{q}{4\pi^2\varepsilon_0 r^2}\sin\theta\,d\theta\,\hat{j} = \frac{-q}{2\pi^2\varepsilon_0 r^2}\hat{j}$$

12. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to $32\,V$, the efficiency of the engine is

(1) 0.25 (2) 0.5

(3) 0.75 (4) 0.99

Ans. (3)

Sol. For adiabatic expansion

$$T_1V_1^{\gamma-1} = T_2V_2^{\gamma-1}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1} = (32)^{\frac{7}{5}-1} = (32)^{2/5} = 4$$

$$\eta = 1 - \frac{T_2}{T_1} = 1 - \frac{1}{4} = 0.75$$

13. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(1) 4, 4, 2 (2) 5, 1, 2

(3) 5, 1, 5 (4) 5, 5, 2

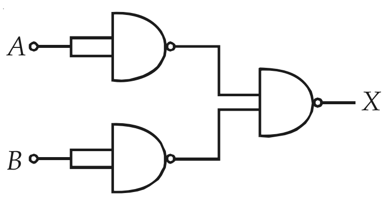
Ans. (2)

Sol. 23.023 $\rightarrow$ 5

0.0003 $\rightarrow$ 1

$2.1 \times 10^{-3} \rightarrow 2$

14. The combination of gates shown below yields

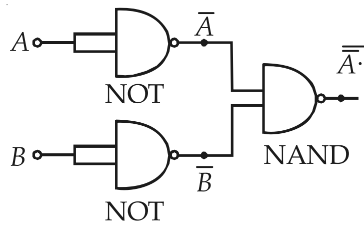


(1) NAND gate (2) OR gate

(3) NOT gate (4) XOR gate

Ans. (2)

Sol.



$\overline{\overline{A}\cdot\overline{B}} = \overline{\overline{A}} + \overline{\overline{B}}$

$= A + B$

15. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(1) γ-rays (2) X-rays

(3) Ultraviolet rays (4) Microwaves

Ans. (2)

Sol. $P = nh\nu$

$4 \times 10^3 = 10^{20} \times 6.63 \times 10^{-34} \times \nu$

$\nu = \frac{4}{6.63} \times 10^{17}\,\text{Hz}$. This is range of X-rays.

16. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-4}$

(3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$

Ans. (3)

Sol. For each α emission, 2 proton and 2 neutron are lost. For each position emission, 1 proton is lost, 1 neutron is increased

$n_p = Z - 2 \times 3 - 2 \times 1 = Z - 8$

$n_n = (A - Z) - 2 \times 3 + 2 = A - Z - 4$

17. Let there be a spherically symmetric charge distribution with charge density varying as

$\rho(r) = \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)$ upto $r = R$, and $\rho(r) = 0$ for $r > R$,

where r is the distance from the origin. The electric field at a distance $r(r < R)$ from the origin is given by

(1) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$ (2) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$

(3) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3} - \frac{r}{R}\right)$ (4) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4} - \frac{r}{R}\right)$

Ans. (3)

Sol. Charge enclosed by a Gaussian sphere of radius $r(< R)$ is

$$Q_{in} = \int \rho\, dV = \int_0^r \rho_0\left(\frac{5}{4} - \frac{r}{R}\right)4\pi r^2 dr$$

$$= \rho_0\left[\frac{5}{4} \times 4\pi\frac{r^3}{3} - \frac{4\pi r^4}{4R}\right]_0^r$$

$$= \rho_0\left[\frac{5}{3}\pi r^3 - \frac{\pi r^4}{R}\right]$$

$$E = \frac{Q_{in}}{4\pi\varepsilon_0 r^2} = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3} - \frac{r}{R}\right]$$

18. In a series LCR circuit $R = 200\ \Omega$ and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(1) 242 W (2) 305 W

(3) 210 W (4) 0 W

Ans. (1)

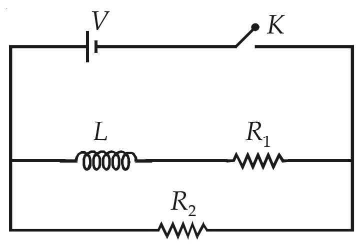
Sol. The series LCR will be in resonance

So, $P = \varepsilon_v I_v \cos\phi$

$$= \frac{\varepsilon_v^{\ 2}}{Z}\cos\phi = \frac{\varepsilon_v^{\ 2}}{R}$$

$$= \frac{(220)^2}{200} = \frac{48400}{200} = 242\,\text{W}$$

19. In the circuit shown below, the key K is closed at $t = 0$. The current through the battery is



(1) $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(2) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = 0$ and $\frac{V}{R_2}$ at $t = \infty$

(3) $\frac{V}{R_2}$ at $t = 0$ and $\frac{V(R_1 + R_2)}{R_1 R_2}$ at $t = \infty$

(4) $\frac{V}{R_2}$ at $t = 0$ and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

Ans. (3)

Sol. At $t = 0$, no current flows through inductor

So, $I = \dfrac{V}{R_2}$

At $t = \infty$, inductor behaves as a conductor

So, $I = \dfrac{V}{\dfrac{(R_1R_2)}{(R_1+R_2)}}$

20. A particle is moving with velocity $v = K\left(y\hat{i} + x\hat{j}\right)$, where K is a constant. The general equation for its path is

(1) $y^2 = x^2$ + constant

(2) $y = x^2$ + constant

(3) $y^2 = x$ + constant

(4) xy = constant

Ans. (1)

Sol. $\dfrac{dx}{dt} = Ky$; $\dfrac{dy}{dt} = Kx$

$\dfrac{dy}{dx} = \dfrac{x}{y} \Rightarrow ydy = xdx$

$\Rightarrow y^2 = x^2 + constant$

21. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio $\dfrac{t_1}{t_2}$ will be

(1) 2 (2) 1

(3) $\dfrac{1}{2}$ (4) $\dfrac{1}{4}$

Ans. (4)

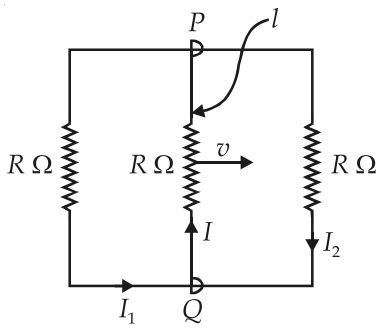
Sol. $U = \dfrac{q^2}{2C} = \dfrac{(q_0)^2}{2C}e^{-2t/RC} = U_0e^{-2t/RC}$

$q = q_0e^{-t/RC}$

When charge becomes $\dfrac{1}{4}$ times, energy becomes $\dfrac{1}{16}$ times.

So, t_1 = one half life, while t_2 = 4 half lives

22. A rectangular loop has a sliding connector PQ of length l and resistance $R\ \Omega$ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are



(1) $I_1 = I_2 = \dfrac{Blv}{6R}$, $I = \dfrac{Blv}{3R}$

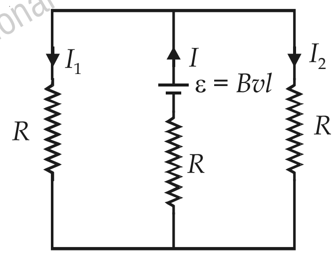
(2) $I_1 = -I_2 = \dfrac{Blv}{R}$, $I = \dfrac{2Blv}{R}$

(3) $I_1 = I_2 = \dfrac{Blv}{3R}$, $I = \dfrac{2Blv}{3R}$

(4) $I_1 = I_2 = I = \dfrac{Blv}{R}$

Ans. (3)

Sol.



$I = \dfrac{\varepsilon}{R + \dfrac{R}{2}} = \dfrac{2\varepsilon}{3R}$

$I_1 = \dfrac{\varepsilon}{3R}$, $I_2 = \dfrac{\varepsilon}{3R}$

where $\varepsilon = Bvl$

23. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by

$$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right].$$

The tension in the string is

(1) 6.25 N (2) 4.0 N

(3) 12.5 N (4) 0.5 N

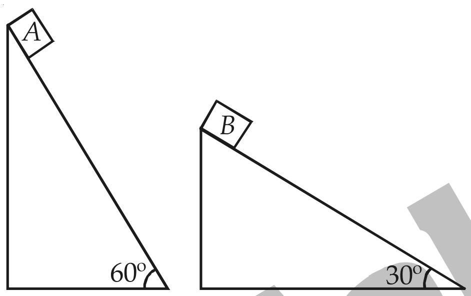
Ans. (1)

Sol. $v = \dfrac{\omega}{k} = \dfrac{\left(\dfrac{2\pi}{0.04}\right)}{\left(\dfrac{2\pi}{0.50}\right)} = \dfrac{0.50}{0.04} = 12.5\ \text{m/s}$

$v = \sqrt{\dfrac{T}{\mu}} \Rightarrow T = \mu v^2 = (12.5)^2 \times 0.04$

$= 6.25$ N

24. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks *A* and *B* are placed on the two planes. What is the relative vertical acceleration of *A* with respect to *B*?



(1) 4.9 ms^{-2} in vertical direction

(2) 4.9 ms^{-2} in horizontal direction

(3) 9.8 ms^{-2} in vertical direction

(4) Zero

Ans. (1)

Sol. $a_{A(\text{along vertical})} = g\sin^2 60^\circ$

$a_{B(\text{along vertical})} = g\sin^2 30^\circ$

$\Rightarrow a_{(A/B)\ \text{along vertical}} = g\left(\dfrac{3}{4} - \dfrac{1}{4}\right) = \dfrac{g}{2} = 4.9\ \text{m/s}^2$

25. For a particle in uniform circular motion, the acceleration a at a point $P(R, \theta)$ on the circle of radius *R* is (Here θ is measured from the *x*-axis)

(1) $\dfrac{v^2}{R}\hat{i} + \dfrac{v^2}{R}\hat{j}$

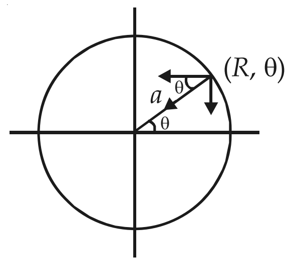
(2) $-\dfrac{v^2}{R}\cos\theta\,\hat{i} + \dfrac{v^2}{R}\sin\theta\,\hat{j}$

(3) $-\dfrac{v^2}{R}\sin\theta\,\hat{i} + \dfrac{v^2}{R}\cos\theta\,\hat{j}$

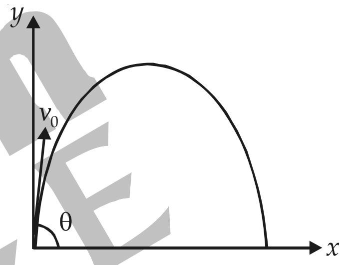
(4) $-\dfrac{v^2}{R}\cos\theta\,\hat{i} - \dfrac{v^2}{R}\sin\theta\,\hat{j}$

Ans. (4)

Sol. $a = \dfrac{-v^2}{R}\cos\theta\,\hat{i} - \dfrac{v^2}{R}\sin\theta\,\hat{j}$



26. A small particle of mass *m* is projected at an angle θ with the *x*-axis with an initial velocity v_0 in the *x*-*y* plane as shown in the figure. At a time $t < \dfrac{v_0 \sin\theta}{g}$, the angular momentum of the particle is



(1) $\dfrac{1}{2}mgv_0\,t^2\cos\theta\,\hat{i}$

(2) $-mgv_0\,t^2\cos\theta\,\hat{j}$

(3) $mgv_0\,t\cos\theta\,\hat{k}$

(4) $-\dfrac{1}{2}mgv_0\,t^2\cos\theta\,\hat{k}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along *x*, *y* and *z*-axis respectively

Ans. (4)

Sol. Angular momentum, $L = \int \tau\, dt$

$L = -\int mg\,x\,dt\,\hat{k}$

$= -\int mg\,v_0\cos\theta\,t\,dt\,\hat{k}$

$= -\dfrac{mg\,v_0\cos\theta\,t^2}{2}\hat{k}$

27. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

(1) 1

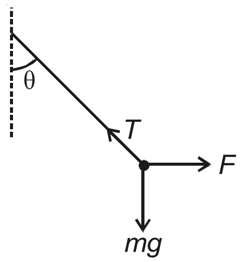
(2) 4

(3) 3

(4) 2

Ans. (4)

Sol.



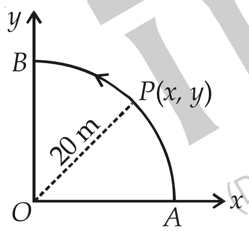
For equilibrium, $F = mg\tan\theta$

in oil $F' = mg'\tan\theta$

$$\frac{F}{F'} = \frac{g}{g'} \Rightarrow k = \frac{1}{\left(1 - \frac{\rho}{\sigma}\right)}$$

$$= \frac{1}{1 - \frac{0.8}{1.6}} = 2$$

28. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of P is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of P when $t = 2$ s is nearly



(1) 14 m/s² (2) 13 m/s²

(3) 12 m/s² (4) 7.2 m/s²

Ans. (1)

Sol. $S = t^3 + 5$

$$\frac{dS}{dt} = 3t^2$$

$v = 3t^2$

$$\frac{dv}{dt} = 6t$$

At $t = 2$ s

$v = 12$ m/s

$$\Rightarrow a_c = \frac{(12)^2}{20} = \frac{144}{20} = 7.2 \text{ m/s}$$

$$\frac{dv}{dt} = 12 \text{ m/s}^2$$

$$a = \sqrt{a_c^2 + a_t^2} = \sqrt{12^2 + (7.2)^2}$$

$\approx$ 14 m/s²

29. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(1) $\frac{b^2}{6a}$ (2) $\frac{b^2}{2a}$

(3) $\frac{b^2}{12a}$ (4) $\frac{b^2}{4a}$

Ans. (4)

Sol. $U = \frac{a}{x^{12}} - \frac{b}{x^6}$

At equilibrium $\frac{dU}{dx} = 0$

$$\Rightarrow \frac{-12a}{x^{13}} + \frac{6b}{x^7} = 0$$

$$\Rightarrow \frac{12a}{x^{13}} = \frac{6b}{x^7}$$

$$\Rightarrow x^6 = \frac{2a}{b}$$

$$\Rightarrow U_{\text{(at equilibrium)}} = \frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\left(\frac{2a}{b}\right)}$$

$$= \frac{b^2}{4a} - \frac{b^2}{2a} = \frac{-b^2}{4a}$$

At $x = \infty$, $U = 0$

$$\Rightarrow D = \frac{b^2}{4a}$$

30. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(1) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

(2) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(3) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$

(4) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

Ans. (1)

Sol. $R_S = R_1 + R_2$

$$\frac{dR_S}{dT} = \frac{dR_1}{dT} + \frac{dR_2}{dT}; \quad R\alpha_s = R_1\alpha_1 + R_2\alpha_2$$

As $R_1 = R_2 \Rightarrow R = R_1 + R_2 = 2R_1$

$$\Rightarrow \quad \alpha = \frac{\alpha_1+\alpha_2}{2}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\frac{1}{R_P^2}\frac{dR_P}{dT} = \frac{1}{R_1^2}\frac{dR_1}{dT} + \frac{1}{R_2^2}\frac{dR_2}{dT}$$

$$\frac{\alpha_P}{R_P} = \frac{\alpha_1}{R_1} + \frac{\alpha_2}{R_2}$$

$$\Rightarrow \quad \alpha_P = \frac{\alpha_1+\alpha_2}{2}$$

PART–B : CHEMISTRY

31. In aqueous solution the ionisation constants for carbonic acid are

$K_1 = 4.2 \times 10^{-7}$ and $K_2 = 4.8 \times 10^{-11}$

Select the correct statement for a saturated 0.034 M solution of the carbonic acid.

(1) The concentration of H^+ is double that of CO_3^{2-}

(2) The concentration of CO_3^{2-} is 0.034 M

(3) The concentration of CO_3^{2-} is greater than that of HCO_3^-

(4) The concentrations of H^+ and HCO_3^- are approximately equal

Ans. (4)

Sol. Since $K_2 << K_1$

$\therefore$ Conc. of H^+ and HCO_3^- are approximately same.

32. Solubility product of silver bromide is 5.0×10^{-13}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is

(1) 5.0×10^{-8} g (2) 1.2×10^{-10} g

(3) 1.2×10^{-9} g (4) 6.2×10^{-5} g

Ans. (3)

Sol. $[Ag^+] = 0.05$, $[Br^-] = x$ M

$K_{sp} = [Ag^+]\,[Br^-]$

$5 \times 10^{-13} = 0.05 \times x$

$x = 10^{-11}$ M

$\therefore$ Solubility of KBr is (120×10^{-11}) or 1.2×10^{-9} g/L

33. The correct sequence which shows decreasing order of the ionic radii of the elements is

(1) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

(2) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$

(3) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$

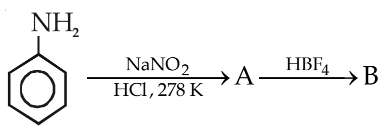
(4) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$

Ans. (1)

Sol. $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

Decreasing ionic radii with increasing effective nuclear charge for isoelectronic species.

34. In the chemical reactions,



the compounds 'A' and 'B' respectively are

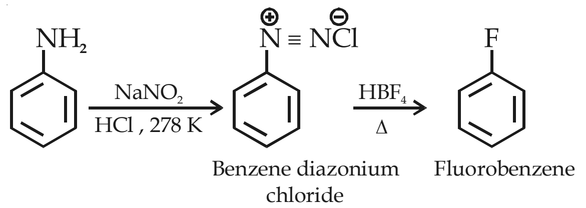
(1) Nitrobenzene and chlorobenzene

(2) Nitrobenzene and fluorobenzene

(3) Phenol and benzene

(4) Benzene diazonium chloride and fluorobenzene

Ans. (4)

Sol. 

35. If 10^{-4} dm^3 of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established?

(Given : Vapour pressure of H_2O at 300 K is 3170 Pa; R = 8.314 J K^{-1} mol^{-1})

(1) 1.27×10^{-3} mol (2) 5.56×10^{-3} mol

(3) 1.53×10^{-2} mol (4) 4.46×10^{-2} mol

Ans. (1)

Sol. PV = nRT

$3170 \times 10^{-3} = n \times 8.314 \times 300$

$$n = \frac{31.7 \times 10^{-3}}{8.314 \times 3} = 1.27 \times 10^{-3}$$

36. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is

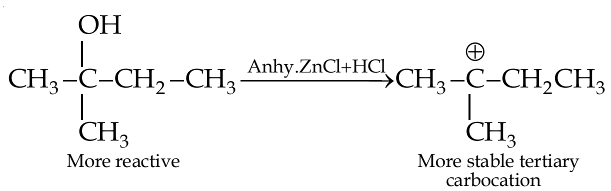
(1) 1-Butanol (2) 2-Butanol

(3) 2-Methylpropan-2-ol (4) 2-Methylpropanol

Ans. (3)

Sol. Alcohols which give more stable carbocation is more reactive with Lucas reagent

(Anhy. $ZnCl_2$ + conc. HCl)



37. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1})

(1) 0.0186 K (2) 0.0372 K

(3) 0.0558 K (4) 0.0744 K

Ans. (3)

Sol. $\Delta T_f = i\,K_f m$

i for Na_2SO_4 is 3(100% ionisation)

$$\Delta T_f = 3 \times 1.86 \times \frac{0.01}{1}$$

$\Delta T_f = 0.0558$ K

38. Three reactions involving $H_2PO_4^-$ are given below

(i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$

(ii) $H_2PO_4^- + H_2O \rightarrow H_2PO_4^{2-} + H_3O^+$

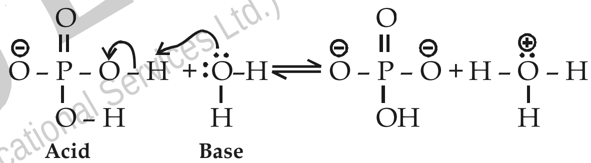
(iii) $H_2PO_4^- + OH^- \rightarrow H_3PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid?

(1) (i) only (2) (ii) only

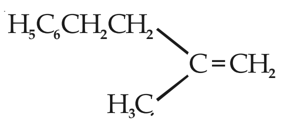
(3) (i) and (ii) (4) (iii) only

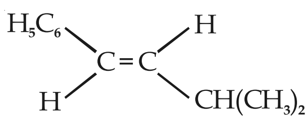
Ans. (2)

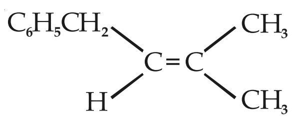
Sol. 

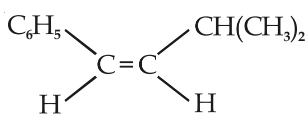
39. The main product of the following reaction is

$$C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{conc. } H_2SO_4} ?$$

(1) 

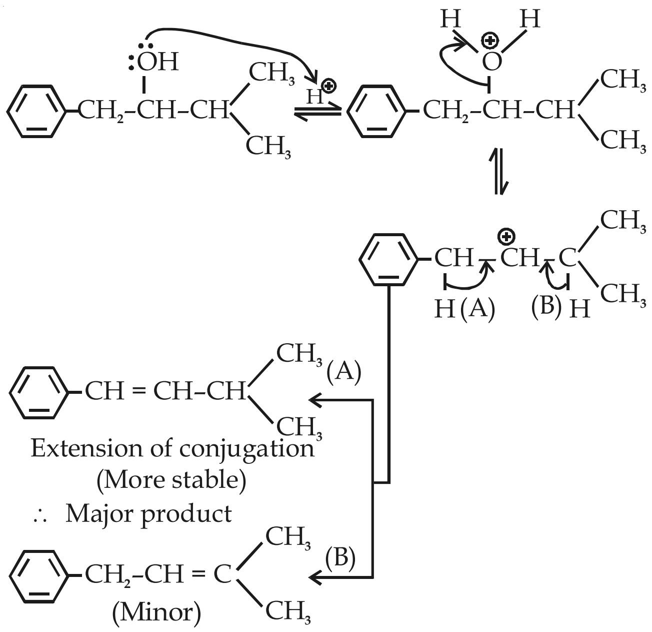
(2) 

(3) 

(4) 

Ans. (2)

Sol.



40. The energy required to break one mole of Cl – Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl – Cl bond is

($c = 3 \times 10^8$ ms^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1})

(1) 494 nm (2) 594 nm

(3) 640 nm (4) 700 nm

Ans. (1)

Sol. $E = \dfrac{hc}{\lambda}$

$$E = \frac{242 \times 10^3}{6.023 \times 10^{23}} \text{ J / atom}$$

$$\therefore \quad \frac{242 \times 10^3}{6.023 \times 10^{23}} = \frac{6.6 \times 10^{-34} \times 3 \times 10^8}{\lambda}$$

$$\lambda = \frac{19.8 \times 10^{-26} \times 6.023 \times 10^{23}}{242 \times 10^3} = 0.494 \times 10^{-6}$$

= 494 nm

41. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralisation. The percentage of nitrogen in the compound is

(1) 29.5 (2) 59.0

(3) 47.4 (4) 23.7

Ans. (4)

Sol. $\%N = \dfrac{1.4\,NV}{W}$

$$= \frac{1.4 \times 0.1 \times (20-15)}{29.5 \times 10^{-3}} = \frac{700}{29.5} = 23.7$$

42. Ionisation energy of He^+ is 19.6×10^{-18} atom^{-1}. The energy of the first stationary state (n = 1) of Li^{2+} is

(1) 8.82×10^{-17} J atom^{-1}

(2) 4.41×10^{-16} J atom^{-1}

(3) -4.41×10^{-17} J atom^{-1}

(4) -2.2×10^{-15} J atom^{-1}

Ans. (3)

Sol. $\dfrac{E_{He^+}}{E_{Li^{+2}}} = \dfrac{Z^2_{He^+}}{Z^2_{Li^{+2}}}$

$$\frac{19.6 \times 10^{-18}}{E_{Li^{+2}}} = \frac{4}{9}$$

$$E_{Li^{+2}} = \frac{9}{4} \times 19.6 \times 10^{-18}$$

$= 4.41 \times 10^{-17}$ J/atom

∴ Energy of orbit of Li^{+2} is -4.41×10^{-17} J/atom

43. On mixing, heptane and octane from an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})

(1) 144.5 kPa (2) 72.0 kPa

(3) 36.1 kPa (4) 96.2 kPa

Ans. (2)

Sol. $P = X_A P_A^0 + X_B P_B^0$

$$n_{heptane} = \frac{25}{100} = 0.25$$

$$n_{octane} = \frac{35}{114} = 0.307$$

$$P = \frac{0.25}{0.25 + 0.307} \times 105 + \frac{0.307}{0.25 + 0.307} \times 45$$

= 47.127 + 24.84 = 71.96

≈ 72 kPa

44. Which one of the following has an optical isomer?

(1) $[Zn(en)_2]^{2+}$

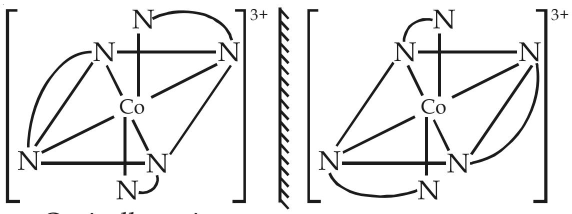
(2) $[Zn(en)(NH_3)_2]^{2+}$

(3) $[Co(en)_3]^{3+}$

(4) $[Co(H_2O)_4(en)]^{3+}$

Ans. (3)

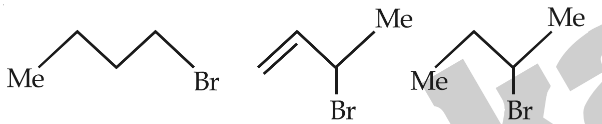
Sol.



Optically active

Exist as enantiomeric pair $\left[Co(en)_3\right]^{3+}$

45. Consider the following bromides



(A) (B) (C)

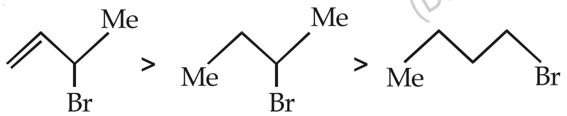
The correct order of S_N1 reactivity is

(1) A > B > C (2) B > C > A

(3) B > A > C (4) C > B > A

Ans. (2)

Sol. Formation of carbocation is rate determining step in S_N1 reaction. Hence alkyl halide which gives more stable carbocation is more reactive towards S_N1 reaction



decreasing order of S_N1 reactivity

46. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene

(1) Ethane

(2) Propene

(3) 1-butene

(4) 2-butene

Ans. (4)

Sol. $CH_3-CH=CH-CH_3 \xrightarrow[\text{(ii) Zn-}H_2O]{\text{(i) }O_3} CH_3-CHO$

44 amu

47. Consider the reaction

$Cl_2\,(aq) + H_2S\,(aq) \rightarrow S(s) + 2H^+(aq) + 2Cl^-(aq)$

The rate equation for this reaction is

rate = k $[Cl_2]\,[H_2S]$

Which of these mechanisms is/are consistent with this rate equation?

A. $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)

$Cl^+ + HS^- \rightarrow H^+ + Cl^- + S$ (fast)

B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)

$Cl_2 + HS^- \rightarrow 2Cl^- + H^+ + S$ (slow)

(1) A only (2) B only

(3) Both (A) & (B) (4) Neither (A) nor (B)

Ans. (1)

Sol. Rate depends only on slow step

48. The Gibbs energy for the decomposition Al_2O_3 at 500°C is as follows

$$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2,\ \Delta_rG = +966\,kJ\,mol^{-1}$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least

(1) 5.0 V (2) 4.5 V

(3) 3.0 V (4) 2.5 V

Ans. (4)

Sol. $\frac{2}{3}Al_2O_3 \longrightarrow \frac{4}{3}Al + O_2$

ΔG = 966 kJ/mol

$\Rightarrow 4e^-$ are involved

$\therefore\ \Delta G = -\,nFE$

$966 \times 10^3 = -4 \times 96500 \times E$

$$E = -\frac{966}{4\times 965}V = -2.5\,V$$

$\therefore$ 2.5 V potential difference is required

49. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is

(1) $RCO\bar{O} < HC\equiv\bar{C} < \bar{N}H_2 < \bar{R}$

(2) $RCO\bar{O} < HC\equiv\bar{C} < \bar{R} < \bar{N}H_2$

(3) $\bar{R} < HC\equiv\bar{C} < RCO\bar{O} < \bar{N}H_2$

(4) $RCO\bar{O} < \bar{N}H_2 < HC\equiv\bar{C} < \bar{R}$

Ans. (1)

Sol. $\underset{\text{increasing basic strength}}{\xrightarrow{R-CO\bar{O} < HC\equiv C^{\ominus} < \overset{\ominus}{N}H_2 < \bar{R}}}$

As sp^3 C is less electronegative than sp^3 N alkyl carbanion $(\bar{R})$ is more basic than $\overset{\ominus}{N}H_2$. However sp hybridized carbon is more electronegative than sp^3 N. Hence $\overset{\ominus}{N}H_2$ is more basic than $HC\equiv\overset{\ominus}{C}$

50. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

(1) 144 pm

(2) 288 pm

(3) 398 pm

(4) 618 pm

Ans. (1)

Sol. In fcc crystal, $R + r = \frac{a}{2}$

$\therefore\ 110 + r = \frac{508}{2} = 254$

r = 144 pm

51. Out of the following, the alkene that exhibits optical isomerism is

(1) 2-methyl-2-pentene

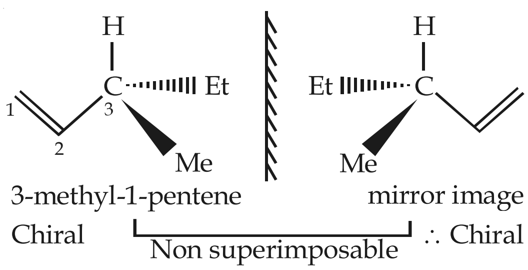
(2) 3-methyl-2-pentene

(3) 4-methyl-1-pentene

(4) 3-methyl-1-pentene

Ans. (4)

Sol.



52. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(1) $T = T_e$ (2) $T_e > T$

(3) $T > T_e$ (4) T_e is 5 times T

Ans. (3)

Sol. $\Delta G = \Delta H - T\Delta S$

For equilibrium $\Delta G = 0$

For spontaneous reaction, $\Delta G < 0$

$\therefore\ T > T_e$

53. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively

(1) 48% and 26%

(2) 30% and 26%

(3) 26% and 32%

(4) 32% and 48%

Ans. (3)

Sol. Packing fractions of fcc and bcc lattices are 74% and 68%

∴ Vacancies are 26% and 32%

54. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is

(1) Natural rubber

(2) Teflon

(3) Nylon 6, 6

(4) Polystyrene

Ans. (3)

Sol. Nylon 6, 6 involves amide linkage therefore, it will also have very strong intermolecular hydrogen bonding between $>N-H\cdots O=C<$ group of two polyamide chains

55. At 25°C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions?

(1) 8 (2) 9

(3) 10 (4) 11

Ans. (3)

Sol. $K_{sp} = [Mg^{+2}]\,[OH^-]^2$

$1 \times 10^{-11} = 0.001 \times [OH^-]^2$

$\therefore [OH^-] = 10^{-4}$ M

pOH = 4, pH = 10

56. The correct order of $E^o_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(1) Cr > Mn > Fe > Co (2) Mn > Cr > Fe > Co

(3) Cr > Fe > Mn > Co (4) Fe > Mn > Cr > Co

Ans. (2)

Sol. Mn > Cr > Fe > Co

$E^o_{Mn^{2+}/Mn} = -1.18$

$E^o_{Cr^{2+}/Cr} = -0.91$

$E^o_{Fe^{2+}/Fe} = -0.44$

$E^o_{CO^{2+}/CO} = -0.28$

57. Biuret test is not given by

(1) Proteins (2) Carbohydrates

(3) Polypeptides (4) Urea

Ans. (2)

Sol. Biuret test is only given by amides. Carbohydrates are not amides and hence it does not give biuret test.

58. The time for half life period of a certain reaction $A \longrightarrow$ Products is 1 h. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction?

(1) 1 h (2) 4 h

(3) 0.5 h (4) 0.25 h

Ans. (4)

Sol. $K = \dfrac{a}{2t_{\frac{1}{2}}} = \dfrac{2}{2 \times 1} = 1$

$t = \dfrac{C_0 - C_t}{K} = \dfrac{0.5 - 0.25}{1} = 0.25$ h

59. A solution containing 2.675 g of $CoCl_3.6NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is

(At. mass of Ag = 108 u)

(1) $[CoCl(NH_3)_5]Cl_2$

(2) $[Co(NH_3)_6]Cl_3$

(3) $[CoCl_2(NH_3)_4]Cl$

(4) $[CoCl_3(NH_3)_3]$

Ans. (2)

Sol. Moles of complex = $\dfrac{2.675}{267.5} = 0.01$

Moles of AgCl precipitated = $\dfrac{4-78}{143.5} = 0.033$

It means $3Cl^-$ are released by one molecule of complex

$\therefore [Co(NH_3)_6]Cl_3$

60. The standard enthalpy of formation of NH_3 is –46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is –436 kJ mol^{-1} and that of N_2 is – 712 kJ mol^{-1}, the average bond enthalpy of N – H bond is NH_3 is

(1) –1102 kJ mol^{-1}

(2) –964 kJ mol^{-1}

(3) +352 kJ mol^{-1}

(4) +1056 kJ mol^{-1}

Ans. (3)

Sol. $\dfrac{1}{2}N_2 + \dfrac{3}{2}H_2 \longrightarrow NH_3$

$\Delta H_{NH_3} = -46$ kJ

$NH_3 \longrightarrow \dfrac{1}{2}N_2 + \dfrac{3}{2}H_2$

$46 = 3\Delta H_{N-H} - \dfrac{1}{2} \times (712) - \dfrac{3}{2} \times 436$

$\Delta H_{N-H} = 352$ kJ / mol

PART–C : MATHEMATICS

61. Consider the following relations :

R = {(x, y) | x, y are real numbers and $x = wy$ for some rational number w};

$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle| m, n, p \text{ and } q \text{ are integers such that } n,\right.$ $q \neq 0$ and $qm = pn\}$. Then

(1) R is an equivalence relation but S is not an equivalence relation

(2) Neither R nor S is an equivalence relation

(3) S is an equivalence relation but R is not an equivalence relation

(4) R and S both are equivalence relations

Ans. (3)

Sol. R is not an equivalence relation because $0\,R\,1$ but $1 \not R\, 0$, S is an equivalence relation.

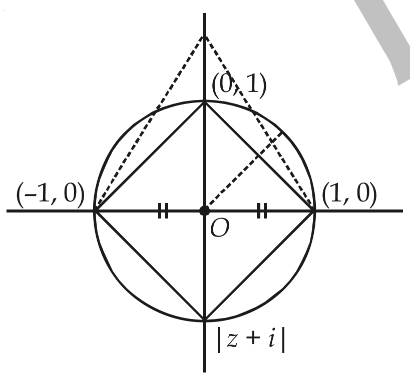
62. The number of complex numbers z such that $|z - 1| = |z + 1| = |z - i|$ equals

(1) 0 (2) 1

(3) 2 (4) ∞

Ans. (2)

Sol.



We have,

$|z - 1| = |z + 1| = |z - i|$

Clearly z is the circumcentre of the triangle formed by the vertices (1, 0) and (0, 1) and (–1, 0), which is unique.

63. If α and β are the roots of the equation $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) –2 (2) –1

(3) 1 (4) 2

Ans. (3)

Sol. α and β are roots of the equation $x^2 - x + 1 = 0$.

$\Rightarrow \alpha + \beta = 1, \alpha\beta = 1$

$\Rightarrow x = \frac{1 \pm \sqrt{3}i}{2}, \frac{1+\sqrt{3}i}{2}, \frac{1-\sqrt{3}i}{2}$

$\Rightarrow x = -\omega$ or ω^2

Thus, $\alpha = -\omega^2$, then $\beta = -\omega$

$\alpha = -\omega$, then $\beta = -\omega^2$ where $\omega^3 = 1$

$\alpha^{2009} + \beta^{2009} = (-\omega)^{2009} + (-\omega^2)^{2009}$

$= -[(\omega^3)^{669}.\omega^2 + (\omega^3)^{1337}.\omega]$

$= -[\omega^2 + \omega] = -(-1) = 1$

64. Consider the system of linear equations:

$x_1 + 2x_2 + x_3 = 3$

$2x_1 + 3x_2 + x_3 = 3$

$3x_1 + 5x_2 + 2x_3 = 1$

The system has

(1) Infinite number of solutions

(2) Exactly 3 solutions

(3) A unique solutions

(4) No solution

Ans. (4)

Sol. The given system of linear equations can be put in the matrix form as

$$\begin{bmatrix} 1 & 2 & 1 \\ 2 & 3 & 1 \\ 3 & 5 & 2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 1 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & -1 & -1 \\ 0 & -1 & -1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -3 \\ -8 \end{bmatrix} \quad \text{by } \begin{matrix} R_2 \to R_2 - 2R_1 \\ R_3 \to R_3 - 3R_1 \end{matrix}$$

$$\sim \begin{bmatrix} 1 & 2 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ 3 \\ 5 \end{bmatrix} \quad R_3 \to R_3 - R_2$$

Clearly the given system of equations has no solution.

Alter

Subtracting the addition of first two equations from third equation, we get,

$0 = -5$, which is an absurd result.

Hence the given system of equation has no solution.

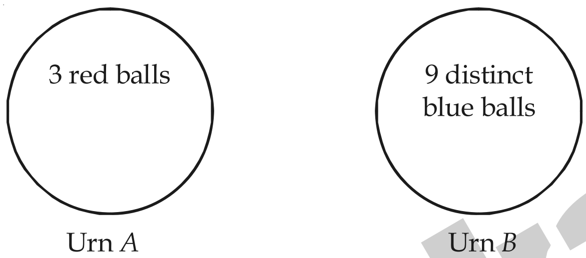
65. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(1) 3 (2) 36

(3) 66 (4) 108

Ans. (4)

Sol.



Two balls from urn A and two balls from urn B can be selected in ${}^3C_2 \times {}^9C_2$ ways

$= 3 \times 36 = 108$

66. Let $f : (-1, 1) \to R$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

(1) 4 (2) –4

(3) 0 (4) –2

Ans. (2)

Sol. We have,

$$f : (-1,\ 1) \longrightarrow R$$

$$f(0) = -1 \qquad f'(0) = 1$$

$$g(x) = [f(2f(x) + 2)]^2$$

$$g'(x) = 2[f(2f(x) + 2)] \times f'(2f(x) + 2) \times 2f'(x)$$

$$\Rightarrow g'(0) = 2[f(2f(0) + 2)] \times f'(2f(0) + 2) \times 2f'(0)$$

$$= 2[f(0)] \times f'(0) \times 2f'(0)$$

$$= 2 \times -1 \times 1 \times 2 \times 1 = -4$$

67. Let $f : R \to R$ be a positive increasing function with $\lim\limits_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then $\lim\limits_{x\to\infty} \frac{f(2x)}{f(x)} =$

(1) 1 (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) 3

Ans. (1)

Sol. We have,

$$f : R \to R$$

$$\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$$

$$\frac{f(2x)}{f(x)} = \frac{f(2x)}{f\left(\frac{2}{3}x\right)} \cdot \frac{f\left(\frac{2}{3}x\right)}{f(x)}$$

$$= \frac{f(2x)}{f\left(\frac{2}{3}x\right)} \cdot \frac{1}{\frac{f(x)}{f\left(\frac{x}{3}\right)}} \cdot \frac{f\left(\frac{x}{3}\right)}{f\left(\frac{2x}{3}\right)}$$

Taking limit $x \to \infty$ and $\lim\limits_{x\to\infty} \frac{f(2x)}{f(x)} = l$

We find that,

$$l = 1 \times \frac{1}{1} \times \frac{1}{l}$$

$$\Rightarrow l^2 = 1 \Rightarrow l = 1.$$

68. Let $p(x)$ be a function defined on R such that $\lim\limits_{x\to\infty} \frac{f(3x)}{f(x)} = 1$ $p'(x) = p'(1 - x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then $\int_0^1 p(x)dx$ equals

(1) $\sqrt{41}$ (2) 21

(3) 41 (4) 42

Ans. (2)

Sol. We have,

$p'(x) = p'(1 - x)$, $\forall\ x \in [0, 1]$, $p(0) = 1$, $p(1) = 41$

$$p(x) = -p(1 - x) + C$$

$$\Rightarrow 1 = -41 + C$$

$$\Rightarrow C = 42$$

$$\Rightarrow p(x) + p(1 - x) = 42$$

$$I = \int_0^1 p(x)dx = \int_0^1 p(1-x)dx$$

$$\Rightarrow 2I = \int_0^1 (p(x) + p(1-x))dx = \int_0^1 42.dx = 42$$

$$\Rightarrow I = 21$$

69. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = ... = a_{10} = 150$ and $a_{10}, a_{11}, ...$ are in an AP with common difference –2, then the time taken by him to count all notes is

(1) 24 minutes (2) 34 minutes

(3) 125 minutes (4) 135 minutes

Ans. (2)

Sol. Number of notes person counts in 10 minutes.

$= 10 \times 150 = 1500$

Since, $a_{10}, a_{11}, a_{12}, ...$ are in A.P. with common difference = –2

$\Rightarrow$ Let n be the time taken to count remaining 3000 notes, then

$$\frac{n}{2}[2\times148+(n-1)\times-2]=3000$$

$\Rightarrow n^2 - 149n + 3000 = 0$

$\Rightarrow (n - 24)(n - 125) = 0$

$\Rightarrow n = 24, 125$

Time taken by the person to count all notes

$= 10 + 24 = 34$ minutes

70. The equation of the tangent to the curve $y = x+\frac{4}{x^2}$, that is parallel to the x-axis, is

(1) $y = 0$ (2) $y = 1$

(3) $y = 2$ (4) $y = 3$

Ans. (4)

Sol. We have,

$$y = x+\frac{4}{x^2}$$

$$\Rightarrow \frac{dy}{dx}=1-\frac{8}{x^3}$$

The tangent is parallel to x-axis, hence

$$\frac{dy}{dx}=0$$

$\Rightarrow x^3 = 8$

$\Rightarrow x = 2$

and $y = 3$

The equation of the tangent to the given curve at (2, 3) is

$$y-3=\left(\frac{dy}{dx}\right)_{(2,\,3)}(x-2)=0$$

$\Rightarrow y = 3$

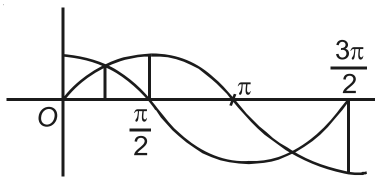
71. The area bounded by the curves $y = \cos x$ and $y = \sin x$ between the ordinates $x = 0$ and $x = \frac{3\pi}{2}$ is

(1) $4\sqrt{2}-2$ (2) $4\sqrt{2}+2$

(3) $4\sqrt{2}-1$ (4) $4\sqrt{2}+1$

Ans. (1)

Sol.



Required area

$$= \int_0^{\pi/4}(\cos x-\sin x)dx+\int_{\pi/4}^{5\pi/4}(\sin x-\cos x)dx$$

$$+\int_{5\pi/4}^{3\pi/2}(\cos x-\sin x)dx$$

$= (4\sqrt{2}-2)$ sq. units

72. Solution of the differential equation

$$\cos x dy = y(\sin x - y)dx,\ 0<x<\frac{\pi}{2}$$ is

(1) $\sec x = (\tan x + c)y$ (2) $y\sec x = \tan x + c$

(3) $y\tan x = \sec x + c$ (4) $\tan x = (\sec x + c)y$

Ans. (1)

Sol. The given differential equation can be put in the form

$$\frac{1}{y^2}\frac{dy}{dx}-\frac{1}{y}\tan x=-\sec x$$

$$\Rightarrow \frac{dz}{dx}+(\tan x)z=+\sec x,\ z=\frac{1}{y}$$

which is linear is z

$$\text{I.F} = e^{\int \tan x\,dx}=e^{\ln \sec x}=\sec x$$

The solution is

$$z.\sec x=\int \sec^2 x dx=\tan x+c$$

where c is a constant of integration

$$\Rightarrow \sec x = y(\tan x + c)$$

73. Let $a = \hat{j} - \hat{k}$ and $c = \hat{i} - \hat{j} - \hat{k}$. Then the vector b satisfying $a \times b + c = 0$ and $a.b = 3$ is

(1) $-\hat{i} + \hat{j} - 2\hat{k}$ (2) $2\hat{i} - \hat{j} + 2\hat{k}$

(3) $\hat{i} - \hat{j} - 2\hat{k}$ (4) $\hat{i} + \hat{j} - 2\hat{k}$

Ans. (1)

Sol. We have

$$a \times b + c = 0$$

$$\Rightarrow a \times (a \times b) + a \times c = 0$$

$$\Rightarrow (a.b)a - (a.a)b + a \times c = 0$$

$$\Rightarrow 3a - 2b + a \times c = 0$$

$$\Rightarrow 2b = 3a + a \times c \; ; \; a \times c = -2i - j - k$$

$$= 3j - 3k - 2i - j - k$$

$$= -2i + 2j - 4k$$

$$\Rightarrow b = -i + j - 2k$$

74. If the vectors $a = \hat{i} - \hat{j} + 2\hat{k}$, $b = 2\hat{i} + 4\hat{j} + \hat{k}$ and $c = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then (λ, μ) =

(1) (–3, 2) (2) (2, –3)

(3) (–2, 3) (4) (3, –2)

Ans. (1)

Sol. We have

$$a.b = 2 - 4 + 2 = 0$$

$$a.c = \lambda - 1 + 2\mu = 0$$

$$b.c = 2\lambda + 4 + \mu = 0$$

Thus $\lambda = 1 - 2\mu$

and $2 - 4\mu + 4 + \mu = 0$

$\Rightarrow 3\mu = 6, \Rightarrow \mu = 2$

$\lambda = -3$

$(\lambda, \mu) = (-3, 2)$

75. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of p is

(1) $x = 1$ (2) $2x + 1 = 0$

(3) $x = -1$ (4) $2x - 1 = 0$

Ans. (3)

Sol. Locus of P from which two perpendicular tangents are drawn to the parabola is the directrix of the parabola

Hence locus is, $x = -1$

76. The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is

(1) $\frac{23}{\sqrt{15}}$ (2) $\sqrt{17}$

(3) $\frac{17}{\sqrt{15}}$ (4) $\frac{23}{\sqrt{17}}$

Ans. (4)

Sol. $\frac{13}{5} + \frac{32}{b} = 1 \Rightarrow \frac{32}{b} = -\frac{8}{5} \therefore b = -20$

The line K must have equation

$$\frac{x}{5} - \frac{y}{20} = a \text{ or } \frac{x}{5a} - \frac{y}{20a} = 1$$

Comparing with $\frac{x}{c} + \frac{y}{3} = 1$

$\left(\text{Given } 20a = -3, \; c = 5a = -\frac{3}{4}\right)$

Distance between lines is

$$= \frac{|a-1|}{\sqrt{\frac{1}{25} + \frac{1}{400}}} = \frac{\left|\frac{-3}{20} - 1\right|}{\sqrt{\frac{17}{400}}} = \frac{23}{\sqrt{17}}$$

77. A line AB in three-dimensional space makes angles 45° and 120° with the positive x-axis and the positive y-axis respectively. If AB makes an acute angle θ with the positive z-axis, then θ equals

(1) 30° (2) 45°

(3) 60° (4) 75°

Ans. (3)

Sol. $\cos^2 45° + \cos^2 120° + \cos^2\theta = 1$

$$\frac{1}{2}+\frac{1}{4}+\cos^2\theta=1$$

$$\therefore \cos^2\theta=\frac{1}{4}$$

$$\cos\theta=\pm\frac{1}{2}\Rightarrow\theta=60^\circ \text{ or } 120^\circ$$

78. Let S be a non-empty subset of R Consider the following statement :

P: There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P?

(1) There is a rational number $x \in S$ such that $x \le 0$

(2) There is no rational number $x \in S$ such that $x \le 0$

(3) Every rational number $x \in S$ satisfies $x \le 0$

(4) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational

Ans. (3)

79. Let $\cos(\alpha+\beta)=\frac{4}{5}$ and let $\sin(\alpha-\beta)=\frac{5}{13}$, where $0\le\alpha,\beta\le\frac{\pi}{4}$. Then $\tan 2\alpha$ =

(1) $\frac{25}{16}$ (2) $\frac{56}{33}$

(3) $\frac{19}{12}$ (4) $\frac{20}{7}$

Ans. (2)

Sol. $\cos(\alpha+\beta)=\frac{4}{5}$ $\Rightarrow \alpha + \beta \in 1^{st}$ quadrant

$\sin(\alpha-\beta)=\frac{5}{13}$ $\Rightarrow \alpha - \beta \in 1^{st}$ quadrant

$2\alpha = (\alpha + \beta) + (\alpha - \beta)$

$$\therefore \quad \tan 2\alpha=\frac{\tan(\alpha+\beta)+\tan(\alpha-\beta)}{1-\tan(\alpha+\beta)\tan(\alpha-\beta)}$$

$$=\frac{\frac{3}{4}+\frac{5}{12}}{1-\frac{3}{4}\cdot\frac{5}{12}}=\frac{56}{33}$$

80. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $-85 < m < -35$ (2) $-35 < m < 15$

(3) $15 < m < 65$ (4) $35 < m < 85$

Ans. (2)

Sol. Centre ≡ (2, 4) $r^2 = 4 + 16 + 5 = 25$

Distance of (2, 4) from $3x - 4y = m$ must be less than radius

$$\therefore \quad \frac{|6-16-m|}{5}<5$$

$\Rightarrow$ $-25 < 10 + m < 25$

$\therefore$ $-35 < m < 15$

81. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$

(3) 6 (4) $\frac{13}{2}$

Ans. (2)

Sol. $E(X^2) - (E(X))^2 = 4$

$\therefore$ $E(X^2) = 4 + 4 = 8$

$\sum X_i^2 = 40$

$E(Y^2) - (E(Y))^2 = 5$

$E(Y^2) = 5 + 16 = 21$

$\therefore$ $\sum Y_i^2 = 105$

$\sum X_i = 10, \sum Y_i = 20$

$\therefore$ $\sum (X_i + Y_i) = 30$

$\sum (X_i^2 + Y_i^2) = 145$

$\therefore$ Variance(combined data) = $\frac{145}{10}-9=\frac{55}{10}=\frac{11}{2}$

82. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(1) $\frac{1}{3}$ (2) $\frac{2}{7}$

(3) $\frac{1}{21}$ (4) $\frac{2}{23}$

Ans. (2)

Sol. Total number of cases = ${}^9C_3 = 84$

Favourable cases = ${}^3C_1 . {}^4C_1 . {}^2C_1 = 24$

$$p = \frac{24}{84} = \frac{2}{7}$$

83. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A ***false*** statement among the following is

(1) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

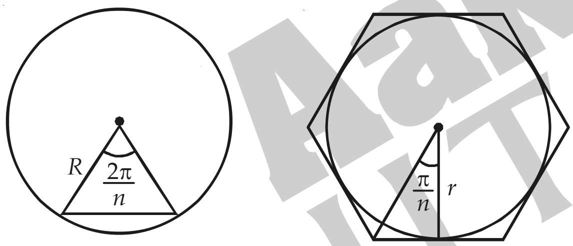
(2) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

(4) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$

Ans. **(3)**

Sol.



$$\frac{a}{2R} = \sin\frac{\pi}{n}$$

$$\frac{a}{2r} = \tan\frac{\pi}{n} \qquad \therefore \qquad \frac{r}{R} = \cos\frac{\pi}{n}$$

$n = 3$ gives $\frac{r}{R} = \frac{1}{2}$

$n = 4$ gives $\frac{r}{R} = \frac{1}{\sqrt{2}}$

$n = 6$ gives $\frac{r}{R} = \frac{\sqrt{3}}{2}$

84. The number of 3 × 3 non-singular matrices, with four entries as 1 and all other entries as 0, is

(1) Less than 4 (2) 5

(3) 6 (4) At least 7

Ans. **(4)**

Sol. Consider $\begin{pmatrix} 1 & * & * \\ * & 1 & * \\ * & * & 1 \end{pmatrix}$. By placing a1 in any one of the 6 * position and 0 elsewhere. We get 6 nonsingular matrices.

Similarly $\begin{pmatrix} * & * & 1 \\ * & 1 & * \\ 1 & * & * \end{pmatrix}$ gives at least one nonsingular

85. Let $f : \mathbf{R} \to \mathbf{R}$ be defined by

$$f(x) = \begin{cases} k - 2x, & \text{if} \quad x \le -1 \\ 2x + 3, & \text{if} \quad x > -1 \end{cases}$$

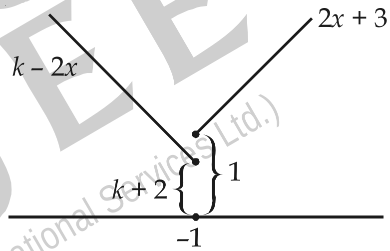
If f has a local minimum at $x = -1$, then a possible value of k is

(1) 1 (2) 0

(3) $-\frac{1}{2}$ (4) –1

Ans. **(4)**

Sol.



$k + 2 \le 1$

$\therefore \quad k \le -1$

Directions : *Questions number 86 to 90 are Assertion - Reason type questions. Each of these questions contains two statements.*

Statement-1 : (Assertion) and

Statement-2 : (Reason).

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

86. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, ..., 20}.

Statement-1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement-2 : If the four chosen numbers from an AP, then the set of all possible values of common difference is {+1, +2, +3, +4, +5}.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans. (3)

Sol. Statement-2 is false.

The outcomes 2, 8, 14, 20 is an AP with common difference 6.

87. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,^{10}C_j$ and

$S_3 = \sum_{j=1}^{10} j^2\ ^{10}C_j$

Statement-1 : $S_3 = 55 \times 2^9$

Statement-2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans. (3)

Sol. $S_2 = \sum_{j=1}^{10} j\ ^{10}C_j = 10.2^9$

$\therefore$ Statement-2 is false.

Only choice is (3).

88. **Statement-1 :** The point $A(3, 1, 6)$ is the mirror image of the point $B(1, 3, 4)$ in the plane $x - y + z = 5$.

Statement-2 : The plane $x - y + z = 5$ bisects the line segment joining $A(3, 1, 6)$ and $B(1, 3, 4)$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans. (1)

Sol. The image of the point (3, 1, 6) w.r.t. the plane $x - y + z = 5$ is

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-6}{1} = \frac{-2(3-1+6-5)}{1+1+1}$$

$$\Rightarrow \frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-6}{1} = -2$$

$\Rightarrow$ $x = 3 - 2 = 1$

$y = 1 + 2 = 3$

$z = 6 - 2 = 4$

which shows that statement-1 is true.

We observe that the line segment joining the points $A(3, 1, 6)$ and $B(1, 3, 4)$ has direction ratios 2, –2, 2 which one proportional to 1, –1, 1 the direction ratios of the normal to the plane. Hence statement-2 is true.

89. Let $f : \mathbf{R} \to \mathbf{R}$ be a continuous function defined by

$$f(x) = \frac{1}{e^x + 2e^{-x}}$$

Statement-1 : $f(c) = \frac{1}{3}$, for some $c \in \mathbf{R}$.

Statement-2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in \mathbf{R}$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is *not* a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans. (1)

Sol. $f(0) = \frac{1}{3}$

$\therefore$ Statement-1 is true.

$$f(x) = \frac{1}{\frac{e^x}{2} + \frac{e^x}{2} + e^{-x} + e^{-x}}$$

By AM – GM

$$\frac{e^x}{2} + \frac{e^x}{2} + e^{-x} + e^{-x} \ge 4\sqrt[4]{\frac{1}{4}} = 4^{3/4}$$

$\therefore \quad 0 < f(x) \le \frac{1}{4^{3/4}} = \frac{1}{2\sqrt{2}}$

Equality holds if $e^x = 2e^{-x} \Rightarrow e^{2x} = 2$.

Since $\frac{1}{3} \le \frac{1}{2\sqrt{2}}$ by intermediate value theorem

$f(c) = \frac{1}{3}$ same $c \in R$.

90. Let A be a 2×2 matrix with non-zero entries and let $A^2 = I$, where I is 2×2 identity matrix. Define

Tr(A) = sum of diagonal elements of A and $|A|$ = determinant of matrix A.

Statement-1 : Tr(A) = 0.

Statement-2 : $|A| = 1$.

(1) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(2) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is false, Statement-2 is true

Ans. (3)

Sol. A satisfies A^2 – Tr(A). A + (det A)I = 0 comparing with $A^2 - I = 0$, it follows Tr A = 0, $|A| = -1$.

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