DATE : 10/04/2011

Aakash IIT-JEE

(Division of Aakash Educational Services Ltd.)

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Time : 3 hrs. **Max. Marks: 240**

Solutions

to

IIT-JEE 2011

PAPER - 2 (Code - 8)

Instructions :

1. The question paper consists of **3 parts** (Chemistry, Physics and Mathematics). Each part consists of **four sections.**

2. In **Section I** (Total Marks : 24), for each question you will be **awarded 3 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** if no bubble is darkened. In all other cases, **minus one (–1) mark** will be awarded.

3. In **Section II** (Total Marks : 16), for each question you will be **awarded 4 marks** if you darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** and **zero marks** otherwise. There are **no negative marks** in this section.

4. In **Section III** (Total Marks : 24), for each question you will be **awarded 4 marks** if you darken **ONLY** the bubble corresponding to the correct answer and **zero marks** otherwise. There are no **no negative mark** in this section.

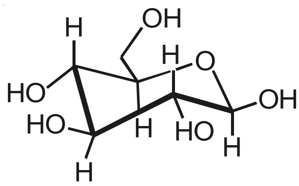
5. In **Section IV** (Total Marks : 16), for each question you will be **awarded 2 marks** for each row in which you have darken **ALL** the bubble(s) corresponding to the correct answer(s) **ONLY** And **zero marks** otherwise. Thus, each question in this section carries **a maximum of 8 marks**. There are **no negative marks** in this section.

PART–I : CHEMISTRY

SECTION - I (Total Marks : 21)
(Single Correct Answer Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

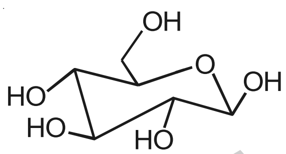
1. The following carbohydrate is



(A) A ketohexose
(B) An aldohexose
(C) An α-furanose
(D) An α-pyranose

Answer (B)

Hints :



β — D — Glucopyranose, which is cyclic form of an aldohexose

2. Oxidation states of the metal in the minerals haematite and magnetite, respectively, are

(A) II, III in haematite and III in magnetite
(B) II, III in haematite and II in magnetite
(C) II in haematite and II, III in magnetite
(D) III in haematite and II, III in magnetite

Answer (D)

Hints :

Haematite	Fe_2O_3	Fe^{+3}
Magnetite	Fe_3O_4	Fe^{+2} and Fe^{+3}

3. Among the following complexes (K – P),

$K_3[Fe(CN)_6]$(K), $[Co(NH_3)_6]Cl_3$(L), $Na_3[Co(oxalate)_3]^{-3}$(M), $[Ni(H_2O)_6]Cl_2$(N), $K_2[Pt(CN)_4]$(O) and $[Zn(H_2O)_6](NO_3)_2$(P)

the diamagnetic complexes are

(A) K, L, M, N
(B) K, M, O, P
(C) L, M, O, P
(D) L, M, N, O

Answer (C)

Hints :

$[Co(NH_3)_6]Cl_3$	d^2sp^3	diamagnetic
$Na_3[Co(OX)_3]$	d^2sp^3	diamagnetic
$K_2[Pt(CN)_4]$	dsp^2	diamagnetic
$[Zn(H_2O)_6](NO_3)_2$	sp^3d^2	diamagnetic

4. Passing H_2S gas into a mixture of Mn^{2+}, Ni^{2+}, Cu^{2+} and Hg^{2+} ions in an acidified aqueous solution precipitates

(A) CuS and HgS (B) MnS and CuS

(C) MnS and NiS (D) NiS and HgS

Answer (A)

Hints :

Cu^{+2} and Hg^{+2} belong with 2^{nd} group of basic radical.

5. Consider the following cell reaction: $2Fe_{(s)} + O_{2(g)} + 4H^{+}_{(aq)} \longrightarrow 2Fe^{2+}_{(aq)} + 2H_2O(l)$; E° = 1.67 V

At $[Fe^{2+}] = 10^{-3}$ M, $P(O_2) = 0.1$ atm and pH = 3, the cell potential at 25°C is

(A) 1.47 V (B) 1.77 V

(C) 1.87 V (D) 1.57 V

Answer (D)

Hints : $E_{cell} = 1.67 - \dfrac{0.0591}{4}\log\dfrac{[Fe^{2+}]^2}{pO_2 \times [H^+]^4}$

$$= 1.67 - \frac{0.0581}{4}\log\frac{(10^{-3})^2}{0.1\times(10^{-3})^4}$$

$$= 1.67 - \frac{0.0591}{4}\log\frac{10^{-6}}{10^{-13}}$$

$$= 1.67 - \frac{0.0591}{4}\log 10^7$$

$$= 1.67 - \frac{0.0591}{4}\times 7 = 1.57$$

6. The freezing point (in °C) of a solution containing 0.1 g of $K_3[Fe(CN)_6]$ (Mol. Wt. 329) in 100 g water (K_f = 1.86 K kg mol^{-1}) is

(A) -2.3×10^{-2} (B) -5.7×10^{-2}

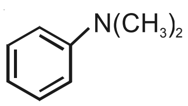
(C) -5.7×10^{-3} (D) -1.2×10^{-2}

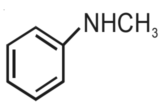
Answer (A)

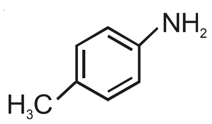
Hints : $T_f' - T_f' = 0 - T_f' = 4 \times 1.86 \times \dfrac{\frac{0.1}{329}}{\frac{100}{1000}}$

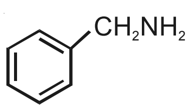
$$T_f' = -4 \times 1.86 \times \frac{0.1}{329} \times \frac{1000}{100} = -0.023$$

7. Amongst the compounds given, the one that would form a brilliant colored dye on treatment with $NaNO_2$ in dil. HCl followed by addition to an alkaline solution of β-naphthol is

(A) 

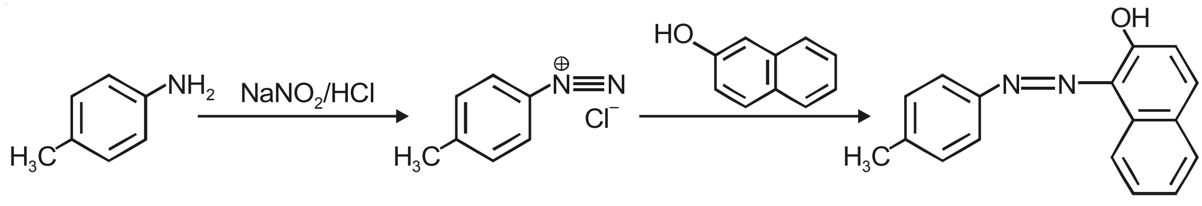
(B) 

(C) 

(D) 

Answer (C)

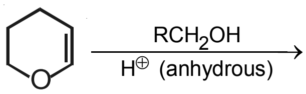
Hints :



Brilliant coloured dye

Note : Only primary aromatic amines will give benzenediazonium chloride chloride at 0°C, which will react with β-naphthol to give an azo dye.

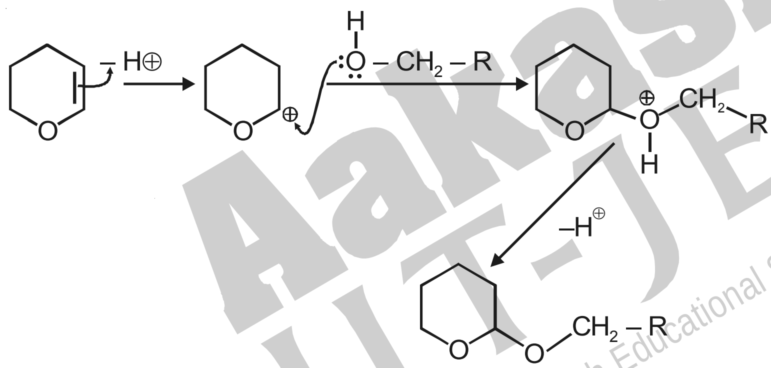
8. The major product of the following reaction is



(A) A hemiacetal
(B) An acetal
(C) An ether
(D) An ester

Answer (B)

Hints :



An acetal

SECTION - II (Total Marks : 16)

(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

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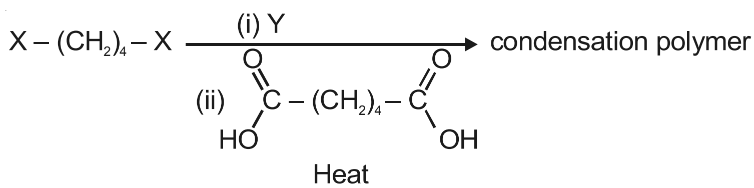
9. The correct functional group X and the reagent/reaction conditions Y in the following scheme are



(A) X = $COOCH_3$, Y = H_2/Ni/Heat
(B) X = $CONH_2$, Y = H_2/Ni/Heat
(C) X = $CONH_2$, Y = Br_2/NaOH
(D) X = CN, Y = H_2/Ni/Heat

Answer (A, B, C, D)

Hints : $X-(CH_2)_4-X$

when x = –COO Me and Y = H_2/Ni will give diol. Diol will form polyester with dicarboxylic acid.

In B, C and D diamine is obtained which will give polyamide with dicarboxylic acid.

10. For the first order reaction, $2N_2O_5(g) \longrightarrow 4NO_2(g) + O_2(g)$

(A) The concentration of the reactant decreases exponentially with time

(B) The half life of the reaction decreases with increasing temperature

(C) The half life of the reaction depends on the initial concentration of the reactant

(D) The reaction proceeds to 99.6% completion in eight half-life duration

Answer (A, B, D)

Hints : $t = \frac{2.303}{m}\log\frac{A}{0.4 \times 10^{-2} \times A}$

$\Rightarrow \frac{8 \times 0.693}{k} = \frac{2.303}{k}\log 250$

$= 2.303 \log\frac{10^3}{4}$

$= 2.303 \times (3 - 2 \times 0.3010)$

11. Reduction of the metal centre in aqueous permanganate ion involves

(A) 3 electrons in neutral medium

(B) 5 electrons in neutral medium

(C) 3 electrons in alkaline medium

(D) 5 electrons in acidic medium

Answer (A, D)

Hints : $MnO_4^- \xrightarrow{aq} MnO_2$

$MnO_4^- \xrightarrow{acidic} Mn^{2+}$

Therefore in aqueous and in acidic mediums 3 and 5 electrons will transfer respecitvely.

12. The equilibrium $2Cu^{I} \rightleftharpoons Cu^{O} + Cu^{II}$ in aqueous medium at 25°C shifts towards the left in the presence of

(A) NO_3^-

(B) Cl^-

(C) SCN^-

(D) CN^-

Answer (B, C, D)

SECTION - III (Total Marks : 24)

(Integer Answer Type)

This section contains **6 questions.** The answer to each of the questions is a **Single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

13. The volume (in mL) of 0.1 M $AgNO_3$ required for complete precipitation of chloride ions present in 30 mL of 0.01 M solution of $[Cr(H_2O)_5Cl]Cl_2$, as silver chloride is close to

Answer (6)

Hints :

Applying equation $N_1V_1 = N_2V_2$

$0.1 \times V = 2 \times 0.01 \times 30$

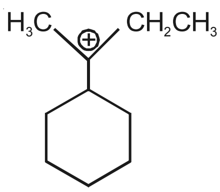
$V = \frac{2 \times 30 \times 10}{100 \times 1} = 6$

14. The number of hexagonal faces that are present in truncated octahedron is

Answer (8)

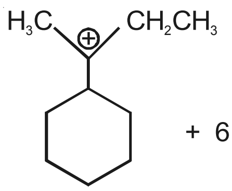
Hints : Truncated octahedron contain 14 faces out of that eight are hexagonal and six are square.

15. The total number of contributing structures showing hyperconjugation (involving C-H bonds) for the following carbocation is



Answer (7)

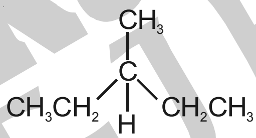
Hints :



+ 6 more hyperconjugative structures

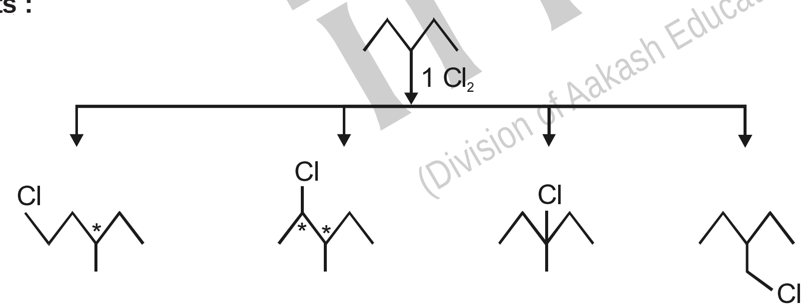
∴ Total number of hyperconjugative structures = 7

16. The maximum number of isomers (including stereoisomers) that are possible monochlorination of the following compound, is



Answer (8)

Hints :



Two Stereoisomers + Four Stereoisomers + One + One

∴ Total Number = 8

17. Among the following, the number of compounds than can react with PCl_5 to give $POCl_3$ is O_2, CO_2, SO_2, H_2O, H_2SO_4, P_4O_{10}.

Answer (5)

Hints :

O_2, SO_2, H_2O, H_2SO_4 and P_4O_{10} convert PCl_5 to $POCl_3$.

18. In 1 L saturated solution of AgCl[K_{SP}(AgCl) = 1.6 × 10^{-10}], 0.1 mol of CuCl [K_{SP}(CuCl) = 1.0 × 10^{-6}] is added. The resultant concentration of Ag^+ in the solution is 1.6 × 10^{-x}. The value of "x" is.

Answer (7)

Hints : $AgCl \longrightarrow \underset{x}{Ag^+} + \underset{x+y}{Cl^-}$

$K_{sp_1} = 1.6 \times 10^{-10} = x(x + y)$ ...(i)

$$\underset{}{CuCl} \longrightarrow \underset{y}{Cu^+} + \underset{y+x}{Cl^-}$$

$K_{sp_2} = 1 \times 10^{-6} = y(x + y)$ ...(ii)

From equation (i) and (ii)

$\dfrac{K_{sp_1}}{K_{sp_2}} = 1.6 \times 10^{-4} = \dfrac{x}{y}$ ...(iii)

$\Rightarrow \; x = 1.6 \times 10^{-4}\, y$

$\Rightarrow \; K_{sp_1} = 1.6 \times 10^{-10} = 1.6 \times 10^{-4} y(1.6 \times 10^{-4} y + y)$

$\Rightarrow \; 10^{-6} = y^2(1.6 \times 10^{-4} + 1)$

$\Rightarrow \; y = 10^{-3} \Rightarrow x = 1.6 \times 10^{-7}$

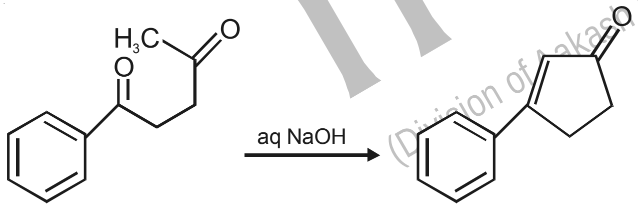
$\Rightarrow \; [Ag^+] = x = 1.6 \times 10^{-7}$

SECTION - IV (Total Marks : 16)
(Matrix-Match Type)

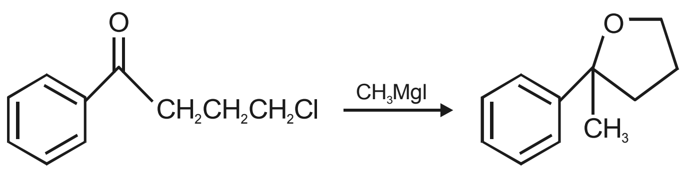
This section contains **2 questions**. Each Question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II**. Any given statement in **Column I** can have correct matching with **ONE** or **MORE** statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q are r, then for that particular question, against statement B, darken the bubble corresponding to q and r in the ORS.

19. Match the reactions in **Column-I** with appropriate types of steps/reactive intermediate involved in these reactions as given in **Column-II**

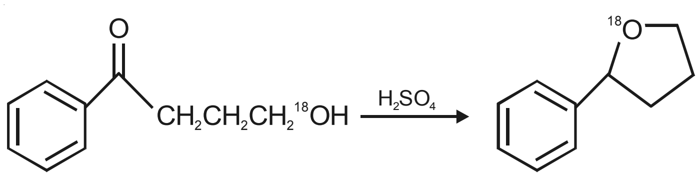
Column I | **Column II**

(A) 

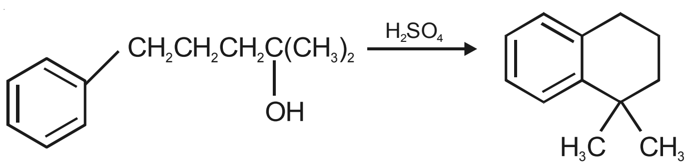
(p) Nucleophilic substitution

(B) 

(q) Electrophilic substitution

(C) 

(r) Dehydration

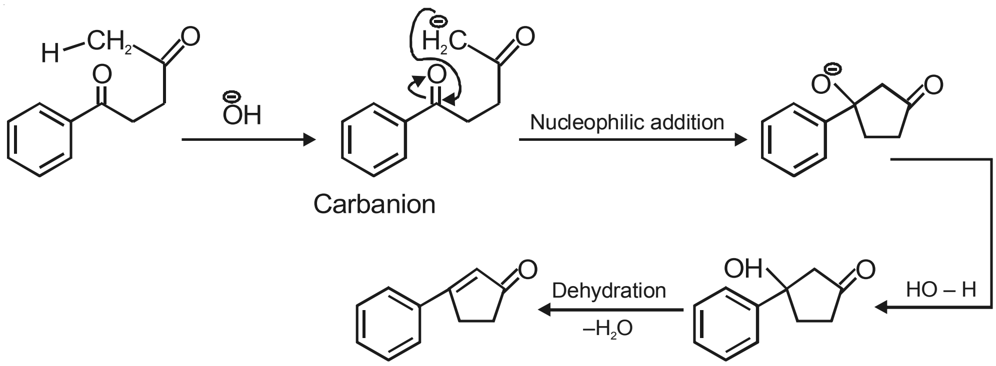
(D) 

(s) Nucleophilic addition

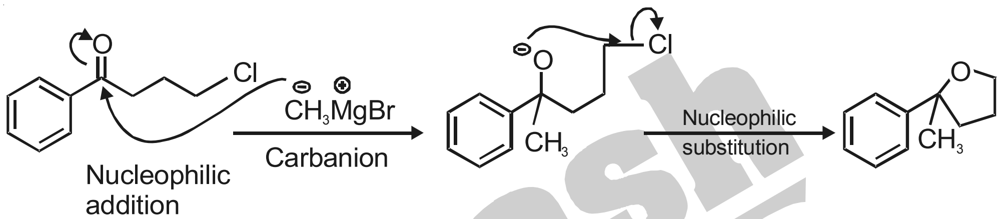
(t) Carbanion

Answer : A(r, s, t), B(p, s, t), C(r, s), D(q, r)

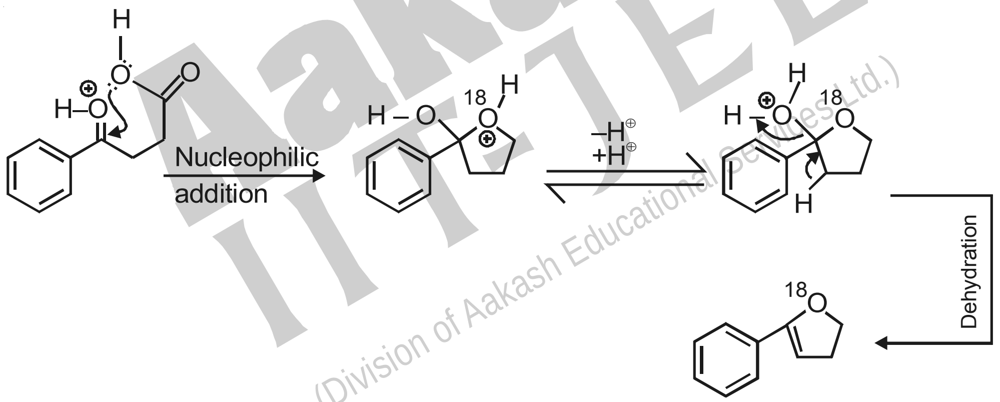
Hints : A.



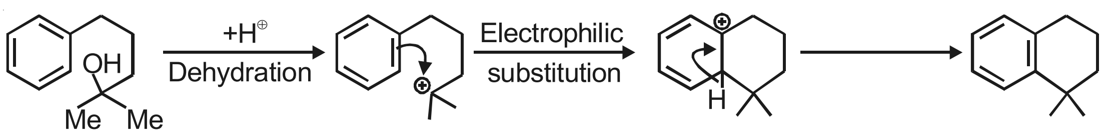
B.



C.



D.



20. Match the transformations in **Column-I** with appropriate options in **Column-II**.

Column I	**Column II**
(A) $CO_2(s) \longrightarrow CO_2(g)$	(p) Phase transition
(B) $CaCO_3(s) \longrightarrow CaO(s) + CO_2(g)$	(q) Allotropic change
(C) $2H \longrightarrow H_2(g)$	(r) ΔH is positive
(D) $P_{(\text{white, solid})} \longrightarrow P_{(\text{red, solid})}$	(s) ΔS is positive
	(t) ΔS is negative

Answer : A(p, r, s), B(p, r, s), C(p, t), D(p, q, t)

Hints : Phase is the part which is physically and chemically uniform throughout. During phase transition from solid to liquid or gas, ΔS = +ve.

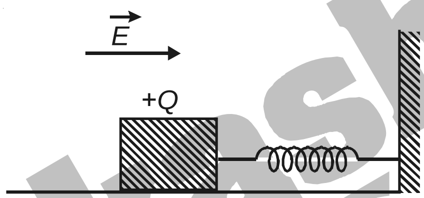
In $2H^{\bullet} \longrightarrow H_2$, ΔS is –ve because no. of entities decreases.

PART–II : PHYSICS

SECTION - I (Total Marks : 24)
(Single Correct Answer Type)

This section contains **7 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be



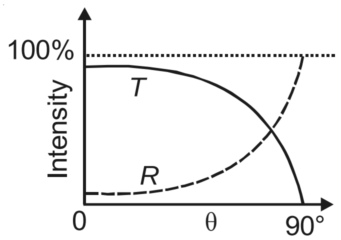
(A) Of the same frequency and with shifted mean position
(B) Of the same frequency and with the same mean position
(C) Of changed frequency and with shifted mean position
(D) Of changed frequency and with the same mean position

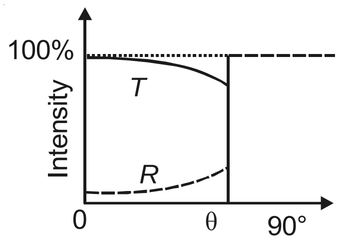
Answer (A)

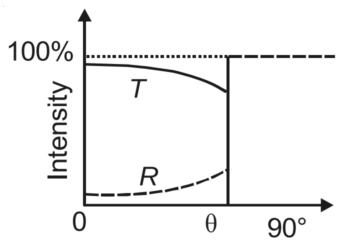
Hints : Frequency does not depend on constant external force.

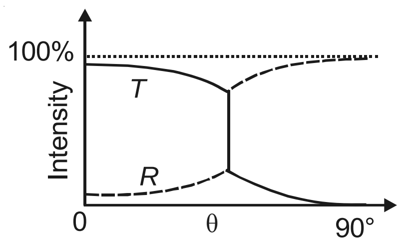
Mean position will shift to $x = \dfrac{qE}{k}$.

22. A light ray travelling in glass medium is incident on glass-air interface at an angle of indcidence θ. The reflected (R) and transmitted (T) intensities, both as function of θ, are plotted. The correct sketch is

(A)


(B)


(C)


(D)


Answer (C)

Hints : When $\theta < C$ partial transmission and reflection will occur. When $\theta > C$, only reflection takes place.

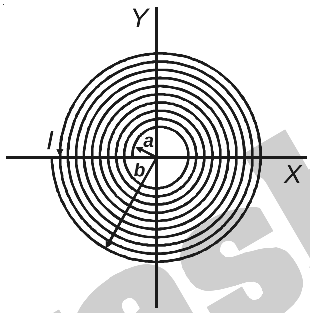
23. A satellite is moving with a constant speed V in a circular orbit about the earth. An object of mass m is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2\ mV^2$

Answer (B)

Hints : To escape speed $V_e = \sqrt{2}V_{orbital}$.

24. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the X-Y plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the center of the spiral is



(A) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 NI}{2(b-a)}\ln\left(\frac{b+a}{b-a}\right)$ (C) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 NI}{2b}\ln\left(\frac{b+a}{b-a}\right)$

Answer (A)

Hints : $dB = \frac{\mu_0 dNI}{2r} = \frac{\mu_0 NI}{(b-a)}\frac{dr}{r}$

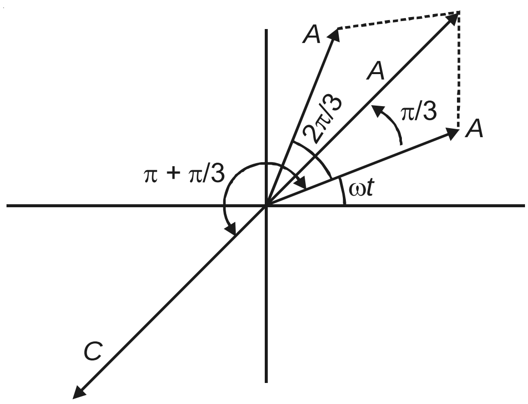
$B = \frac{\mu_0 NI}{(b-a)}\ln\left(\frac{b}{a}\right)$

25. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are

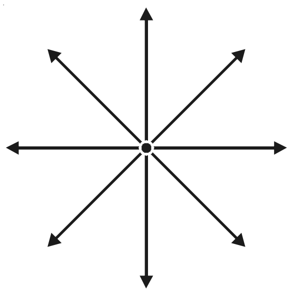
(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

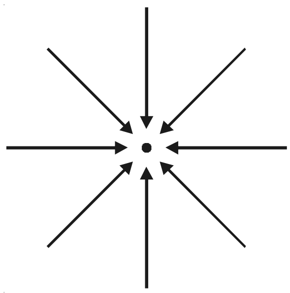
Answer (B)

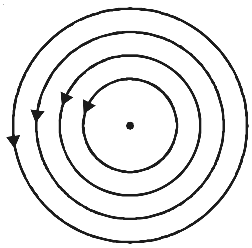
Hints : See the phasor

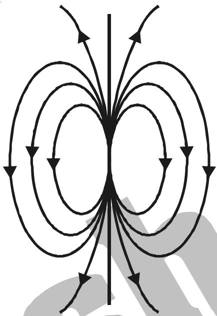


26. Which of the field patterns given below is valid for electric field as well as for magnetic field?

(A) 

(B) 

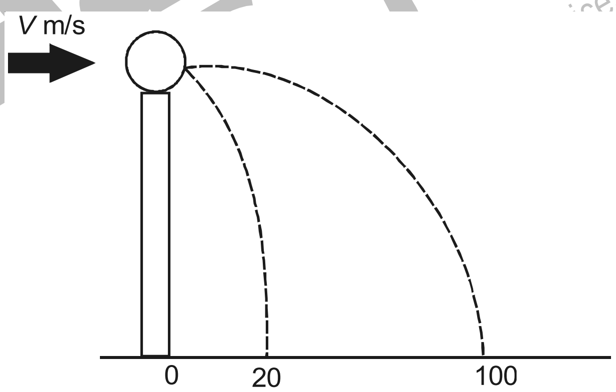
(C) 

(D) 

Answer (C)

Hints : Induced electric field and magnetic field can form closed loops.

27. A ball of mass 0.2 kg rests on a vertical post of height 5 m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is



(A) 250 m/s (B) $250\sqrt{2}$ m/s (C) 400 m/s (D) 500 m/s

Answer (D)

Hints : $t = \sqrt{\frac{2h}{g}} = 1\text{s}$

$\Rightarrow V_{ball}$ = 20 m/s and V_{bullet} = 100 m/s

By conservation of linear momentum, $0.01V = 0.2 \times 20 + 0.01 \times 100$

V = 500 m/s

28. The density of a solid ball is to be determined in an experiment. The diameter of the ball is measured with a screw gauge, whose pitch is 0.5 mm and there are 50 divisions on the circular scale. The reading on the main scale is 2.5 mm and that on the circular scale is 20 divisions. If the measured mass of the ball has a relative error of 2%, the relative percentage error in the density is

(A) 0.9% (B) 2.4% (C) 3.1% (D) 4.2%

Answer (C)

Hints : Δr = least count = $\dfrac{0.5}{50} = 0.01\,\text{mm}$

$$r = 2.5\,\text{mm} + 20 \times \frac{0.5}{50} = 2.5\,\text{mm} + 0.20\,\text{mm} = 2.70\,\text{mm}$$

$$\frac{\Delta r}{r} = \frac{0.01}{2.70}$$

$$d = \frac{m}{v}$$

$$\frac{\Delta d}{d} = \frac{\Delta m}{m} + \frac{3\Delta r}{r}$$

$$= 2\% + 3 \times \frac{1}{2.7}$$

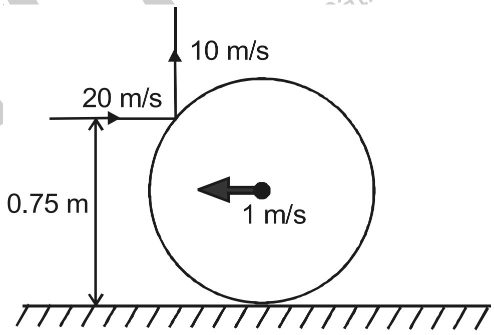
$= 2\% + 1.11\% = 3.11\%$

SECTION - II (Total Marks : 16)
(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE** may be correct.

---

29. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision



(A) The ring has pure rotation about its stationary CM

(B) The ring comes to a complete stop

(C) Friction between the ring and the ground is to the left

(D) There is no friction between the ring and the ground

Answer (A, C)

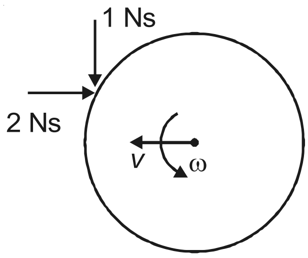
Hints : The data is incomplete, if we assume that the friction is not impulsive during impact then the solution is as follows

$2 = -2 \times v - (-2 \times 1)$

$\Rightarrow v = 0$

Thus centre of mass becomes stationary

Taking angular impulse about centre of mass of ring.



$$1\left(\frac{\sqrt{3}}{2}\times\frac{1}{2}\right)-2\times(0.5)\times\frac{1}{2}=2\times(0.5)^2\left[\omega-\frac{1}{0.5}\right]$$

$$\frac{1.732}{4}-0.5=0.5\omega-1$$

$0.5\omega = 0.5 + 0.433$

$\Rightarrow \omega > 0$ (*i.e.* anticlockwise)

30. Which of the following statement(s) is/are correct?

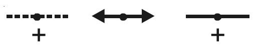
(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gausss law will still be valid

(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

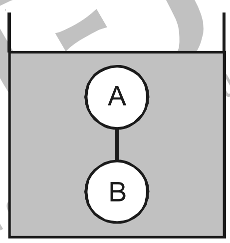
(C) If he electric field between two point charges is zero somewhere, then the sign of the two charges is the same

(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

Answer (C)

Hints : 

31. The solid spheres A and B of equals volume but of different densities d_A and d_B are connected by a string. The y are fully immersed in a fluid of density d_F. They get arranged into an equilibrium state as shown in the figure with a tension in the string. The arrangement is possible only if



(A) $d_A < d_F$

(B) $d_B > d_F$

(C) $d_A > d_F$

(D) $d_A + d_B = 2d_F$

Answer (A, B, D)

Hints : For string to be taught, $vd_Fg > vd_Ag$

$vd_Bg > vd_Fg$

Also, $vd_Fg + vd_Fg = vd_Ag + vd_Bg$

32. A series R-C circuit is connected to AC voltage source. Consider two cases; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true?

(A) $I_R^A > I_R^B$

(B) $I_R^A < I_R^B$

(C) $V_C^A > V_C^B$

(D) $V_C^A < V_C^B$

Answer (B, C)

Hints : $X_C = \frac{1}{\omega C}$

$X'_C = \frac{1}{\omega C'}$

$C' = KC$

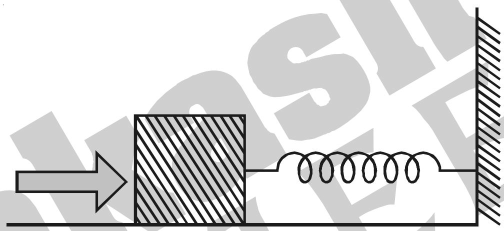
When X_C is more, current is less, but V_C is more.

SECTION - III (Total Marks : 24)

(Integer Answer Type)

This section contains **6 questions.** The answer to each of the questions is a **Single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

33. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is



Answer (4)

Hints : $\frac{1}{2}mv^2 = \mu mgx + \frac{1}{2}kx^2$

$\frac{1}{2} \times 0.18 \times v^2 = 0.1 \times 0.18 \times 10 \times 0.06 + \frac{1}{2} \times 2 \times (0.06)^2$

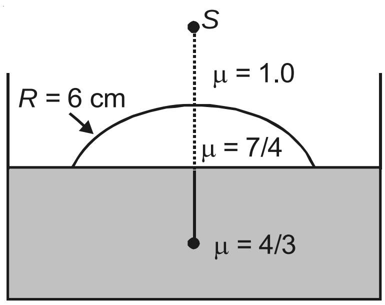
$= 108 \times 10^{-4} + 36 \times 10^{-4}$

$0.9v^2 = 144 \times 10^{-4}$

$0.3v = 12 \times 10^{-2}$

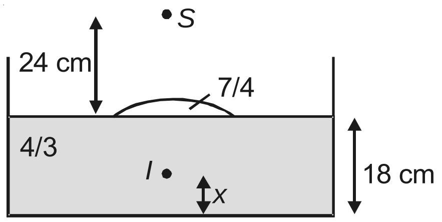
$v = 40 \times 10^{-2} = 0.4$ m/s

34. Water (with refractive index = $\frac{4}{3}$) in a tank is 18 cm deep. Oil of refractive index $\frac{7}{4}$ lies on water making a convex surface of radius of curvature 'R = 6 cm' as shown. Consider oil to act as a thin lens. An object 'S' is placed 24 cm above water surface. The location of its image is at 'x' cm above the bottom of the tank. Then 'x' is



Answer (2)

Hints :



$$\frac{4/3}{(18-x)} = -\frac{1}{24} + \frac{\left(\frac{7}{4}-1\right)}{6}$$

$$\frac{4}{3(18-x)} = -\frac{1}{24} + \frac{3}{24} = \frac{1}{12}$$

$$48 = 54 - 3x$$

$3x = 6$

$x = 2$ cm

35. A silver sphere of radius 1 cm and work function 4.7 eV is suspended from an insulting thread in free-space. It is under continuous illumination of 200 nm wavelength light. As photoelectrons are emitted, the sphere gets charged and acquires a potential. The maximum number of photoelectrons emitted from the sphere is A × 10^Z (where 1 < A < 10). The value of Z is

Answer (7)

Hints : $\frac{hc}{\lambda} - \phi = ev_0$

$$v_0 = \frac{ne}{4\pi\varepsilon_0 r}$$

$$\frac{1240}{200}eV - 4.7eV = \left(\frac{xne}{4\pi\varepsilon_0 r}\right)eV$$

$$6.2 - 4.7 = \frac{9\times 19^9 \times n \times 1.6\times 10^{-19}}{10^{-2}}$$

$$n = \frac{1.5\times 10^{-2}}{9\times 1.6\times 10^{-10}} \approx 1.04\times 10^7 = 7$$

36. A series R-C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is

Answer (4)

Hints : $Z = \sqrt{R^2 + X_C^2} = \sqrt{1.25}R$

$\Rightarrow R^2 + X_C{}^2 = 1.25\ R^2$

$$X_C{}^2 = \frac{R^2}{4}$$

$$X_C = \frac{R}{2}$$

$$\frac{1}{\omega c} = \frac{R}{2}$$

$$R_C = \frac{2}{\omega} = \frac{2}{500}\text{s} = 4 \text{ ms.}$$

37. A train is moving along a straight line with a constant acceleration *a*. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is

Answer (5)

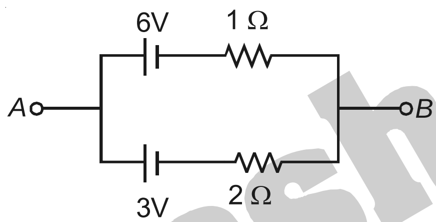
Hints : $T = \frac{2u_y}{g} = \frac{2 \times 10 \sin 60}{g} = \sqrt{3}\text{ s}$

$R = 1.15 \text{ m} = u_x t - \frac{1}{2}at^2$

$1.15 = 10\cos 60 \times \sqrt{3} - \frac{1}{2}a\left(\sqrt{3}\right)^2$

$\Rightarrow a = 5\text{m/s}^2$

38. Two batteries of different emfs and different internal resistance are connected as shown. The voltage across *AB* in volts is



Answer (5)

Hints : $V = \frac{\frac{6}{1}+\frac{3}{2}}{\frac{1}{1}+\frac{1}{2}} = \frac{7.5}{\frac{3}{2}} = 5$

SECTION - IV (Total Marks : 16)
(Matrix-Match Type)

This section contains **2 questions**. Each Question has **four statements** (A, B, C and D) given in **Column I** and **five statements** (p, q, r, s and t) in **Column II.** Any given statement in **Column I** can have correct matching with **ONE** or **MORE** statement(s) given in **Column II**. For example, if for a given question, statement B matches with the statements given in q are r, then for that particular question, against statement B, darken the bubble corresponding to q and r in the ORS.

39. **Column I** shows four systems, each of the same length *L*, for producing standing waves. The lowest possible natural frequency of a system is called its fundamental frequency, whose wavelength is denoted as λ_f. Match each system with statements given in **Column II** describing the nature and wavelength of the standing waves.

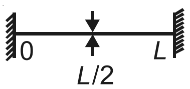
Column I

(A) Pipe closed at one end

(B) Pipe open at both ends

(C) Stretched wire clamped at both ends

(D) Stretched wire clamped at both ends and at mid-point



Column II

(p) Longitudinal waves

(q) Transverse waves

(r) $\lambda_f = L$

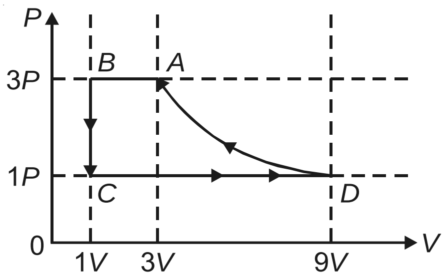
(s) $\lambda_f = 2L$

(t) $\lambda_f = 4L$

Answer : A(p, t), B(p, s), C(q, s), D(q, r)

Hints : In organ pipes, longitudinal wares exist. In strings, transverse waves exist. Open end is antinode, fixed end is antinode. Least distance between node and antinode is $\lambda/4$ and between two nodes is $\lambda/2$

40. One mole of a monatomic ideal gas is taken through a cycle *ABCDA* as shown in the *P*-*V* diagram. **Column II** gives the characteristics involved in the cycle. Match them with each of the processes given in **Column I**.



Column I	**Column II**
(A) Process $A \to B$	(p) Internal energy decreases
(B) Process $B \to C$	(q) Internal energy increases
(C) Process $C \to D$	(r) Heat is lost
(D) Process $D \to A$	(s) Heat is gained
	(t) Work is done on the gas

Answer : A(p, r, t), B(p, r), C(q, s), D(r, t)

Hints : In *AB* temperature and volume are decreasing.
In *BC* temperature decreases, volume does not change
In *CD* temperature and volume increase
In *DA* final temperature equals initial temperature. Also, volume decreases
For all processes use $\Delta U = nC_V\Delta T$, $W = \int pdV$, $Q = \Delta U + W$

PART–III : MATHEMATICS

SECTION - I (Total Marks : 24)
(Single Correct Answer Type)

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

41. Let (x, y) be any point on the parabola $y^2 = 4x$. Let P be the point that divides the line segment from (0, 0) to (x, y) in the ratio 1 : 3. Then the locus of P is

(A) $x^2 = y$ (B) $y^2 = 2x$ (C) $y^2 = x$ (D) $x^2 = 2y$

Answer (C)

Hints : Let $P(\alpha, \beta)$ be the point intersecting the line-segment joining $O(0, 0)$ and $Q(t^2, 2t)$ in the ratio 1 : 3.

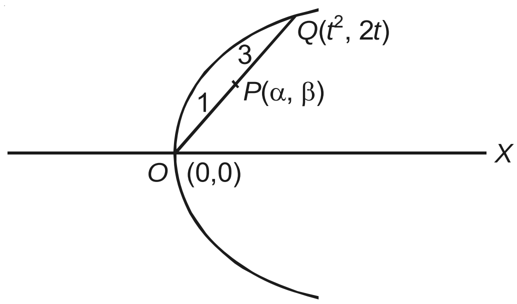
Then $\alpha = \dfrac{t^2+0}{4}$, $\beta = \dfrac{2t+0}{4}$

$\Rightarrow \alpha = \dfrac{t^2}{4}$, $\beta = \dfrac{t}{2}$

$\Rightarrow 4\alpha = (2\beta)^2$

$\Rightarrow \beta^2 = \alpha$

Locus of (α, β) is $y^2 = x$.



42. Let $P(6, 3)$ be a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. If the normal at the point P intersects the x-axis at $(9, 0)$, then the eccentricity of the hyperbola is

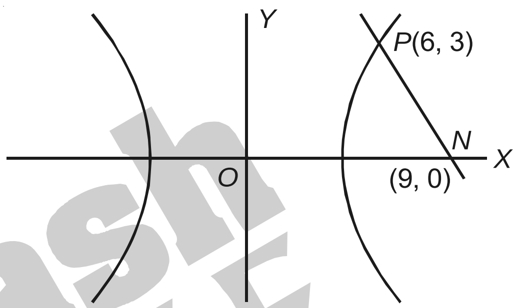
(A) $\sqrt{\frac{5}{2}}$ (B) $\sqrt{\frac{3}{2}}$ (C) $\sqrt{2}$ (D) $\sqrt{3}$

Answer (B)

Hints : The equation of the normal at $P(6, 3)$ to the given hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$ is

$$\frac{a^2x}{6}+\frac{b^2y}{3}=a^2+b^2$$

which meets x-axis at $(9, 0)$, hence



$$\frac{9a^2}{6}=a^2+b^2$$

$$\Rightarrow \frac{3}{2}=1+\frac{b^2}{a^2}=e^2$$

$$\Rightarrow e=\frac{\sqrt{3}}{2}$$

43. A value of b for which the equations

$x^2 + bx - 1 = 0$

$x^2 + x + b = 0,$

have one root in common is

(A) $-\sqrt{2}$ (B) $-i\sqrt{3}$ (C) $i\sqrt{5}$ (D) $\sqrt{2}$

Answer (B)

Hints : Let α be a common root between given equations $x^2 + bx - 1 = 0$ and $x^2 + x + b = 0$

$$\Rightarrow \frac{\alpha^2}{b^2+1}=\frac{\alpha}{-1-b}=\frac{1}{1-b}$$

$$\Rightarrow \alpha^2=\frac{b^2+1}{1-b} \text{ and } \alpha=-\left(\frac{1+b}{1-b}\right)$$

$$\Rightarrow \frac{b^2+1}{1-b}=\left(\frac{1+b}{1-b}\right)^2$$

$$\Rightarrow b^2+1=\frac{(1+b)^2}{1-b}$$

$$\Rightarrow b^2 - b^3 + 1 - b = 1 + 2b + b^2$$

$$\Rightarrow b^3 + 3b = 0$$

$$\Rightarrow b=0, b=\pm\sqrt{3}i$$

$$\Rightarrow b=-\sqrt{3}i$$

44. Let $\omega \neq 1$ be a cube root of unity and S be the set of all non-singular matrices of the form

$$\begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$$

where each of a, b and c is either ω or ω^2. Then the number of distinct matrices in the set S is

(A) 2 (B) 6 (C) 4 (D) 8

Answer (C)

Hints : We have,

$$M = \begin{bmatrix} 1 & a & b \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{bmatrix}$$

$$\Rightarrow \quad |M| = \begin{vmatrix} 0 & a-\omega^2 & b+c+1 \\ \omega & 1 & c \\ \omega^2 & \omega & 1 \end{vmatrix}$$

$= -\omega(a - \omega^2 - b\omega - c\omega - \omega) + \omega^2(ac - \omega^2 c - b - c - 1)$

$= -(a + c)\omega + ac\omega^2 + 1$

$a + c \neq 1$, $ac \neq 1$

Since a, b, c are ω or ω^2

$\Rightarrow \quad a = c$

If $a = \omega \quad \Rightarrow \quad c = \omega$

$\therefore$ Number of ways of selecting a, b, c = $1 \times 1 \times 2 = 2$

If $a = \omega^2$, then number of ways = $1 \times 1 \times 1 = 2$

Total number of distinct matrices in the given set S = 4.

45. The circle passing through the point (–1, 0) and touching the y-axis at (0, 2) also passes through the point

(A) $\left(-\frac{3}{2}, 0\right)$ (B) $\left(-\frac{5}{2}, 2\right)$ (C) $\left(-\frac{3}{2}, \frac{5}{2}\right)$ (D) (–4, 0)

Answer (D)

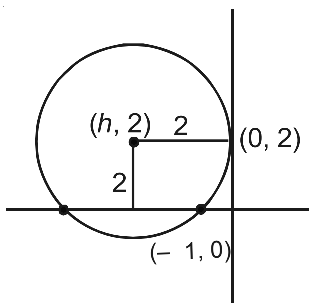
Hints : The equation of the circle directing y-axis at (0, 2) can be put in the form

$(x - h)^2 + (y - 2)^2 = h^2$

which will pass through (– 1, 0) if

$(-1 - h)^2 + 4 = h^2$

$\Rightarrow \quad h = -\frac{5}{2}$



Thus the equation of the circle is

$$\left(x+\frac{5}{2}\right)^2 + (y-2)^2 = \left(\frac{5}{2}\right)^2$$

which passes through (– 4, 0)

46. If $\lim\limits_{x\to 0}[1+x\ln(1+b^2)]^{1/x} = 2b\sin^2\theta$, $b > 0$ and $\theta \in (-\pi, \pi]$, then the value of θ is

(A) $\pm\frac{\pi}{4}$ (B) $\pm\frac{\pi}{3}$ (C) $\pm\frac{\pi}{6}$ (D) $\pm\frac{\pi}{2}$

Answer (D)

Hints : We have,

$$\lim_{x\to 0}\left[1+x\ln(1+b^2)\right]^{\frac{1}{x}} = 2b\sin^2\theta, \qquad b > 0 \text{ and } \theta \in (-\pi, \pi)$$

$$\Rightarrow \quad e^{\lim\limits_{x\to 0}\frac{x\ln(1+b^2)}{x}} = 2b\sin^2\theta$$

$\Rightarrow$ $1 + b^2 = 2b\sin^2\theta \le 2b$

But $1 + b^2 \ge 2b$, by A.M.-G.M. inequality.

Hence $\sin\theta = \pm 1$

$$\Rightarrow \quad \theta = \pm\frac{\pi}{2}$$

47. Let $f : [-1, 2] \to [0, \infty)$ be a continuous function such that $f(x) = f(1 - x)$ for all $x \in [-1, 2]$. Let $R_1 = \int_{-1}^{2} x\, f(x)\, dx$, and R_2 be the area of the region bounded by $y = f(x)$, $x = -1$, $x = 2$, and the x-axis. Then

(A) $R_1 = 2R_2$ (B) $R_1 = 3R_2$ (C) $2R_1 = R_2$ (D) $3R_1 = R_2$

Answer (C)

Hints : We have,

$$R_1 = \int_{-1}^{2} x f(x)dx = \int_{-1}^{2}(1-x)f(1-x)dx$$

$$= \int_{-1}^{2} f(1-x)dx - \int_{-1}^{2} xf(1-x)dx$$

$= R_2 - R_1$

$\Rightarrow$ $2R_1 = R_2$

48. Let $f(x) = x^2$ and $g(x) = \sin x$ for all $x \in \mathbb{R}$. Then the set of all x satisfying $(fogogof)(x) = (gogof)(x)$, where $(fog)(x) = f(g(x))$, is

(A) $\pm\sqrt{n\pi}, n \in \{0, 1, 2, ...\}$ (B) $\pm\sqrt{n\pi}, n \in \{1, 2, ...\}$

(C) $\frac{\pi}{2} + 2n\pi, n \in \{..., -2, -1, 0, 1, 2, ...\}$ (D) $2n\pi$, $n \in \{..., -2, -1, 0, 1, 2, ...\}$

Answer (A)

Hints : We have,

$f(x) = x^2$ and $g(x) = \sin x$, $\forall x$

$\Rightarrow$ $f(g(g(f(x)))) = g(g(f(x)))$

$\Rightarrow$ $g(f(x)) = g(x^2) = \sin x^2$

$\Rightarrow$ $g(g(f(x))) = g(\sin x^2) = \sin(\sin x^2)$

$\Rightarrow$ $f(g(g(f(x)))) = (\sin(\sin x^2))^2$

$\Rightarrow$ $(\sin \sin x^2)^2 = \sin(\sin x^2)$

$\Rightarrow$ $\sin(\sin x^2) = 0$ or $\sin(\sin x^2) = 1$

But $\sin(\sin x^2) = 1$ is not possible hence $\sin x^2 = 0$

$\Rightarrow$ $x^2 = n\pi$

$\Rightarrow$ $x = \pm\sqrt{n\pi}, n \in \{0, 1, 2, 3......\}$

SECTION - II (Total Marks : 16)

(Multiple Correct Answer(s) Type)

This section contains **4 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE** may be correct.

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49. Let $f : (0, 1) \to \mathbb{R}$ be defined by

$$f(x) = \frac{b-x}{1-bx}$$

where b is a constant such that $0 < b < 1$. Then

(A) f is not invertible on (0, 1)

(B) $f \neq f^{-1}$ on (0, 1) and $f'(b) = \dfrac{1}{f'(0)}$

(C) $f = f^{-1}$ on (0, 1) and $f'(b) = \dfrac{1}{f'(0)}$

(D) f^{-1} is differentiable on (0, 1)

Answer (A)

Hints : Let $f : (0, 1) \to \mathbb{R}$ defined by

$$f(x) = \frac{b-x}{1-bx}, \text{ where } 0 < b < 1$$

We observe that

$$f'(x) = \frac{1+b^2}{(1-bx)^2} > 0$$

$\Rightarrow$ $f(x)$ is strictly increasing $\forall x \in (0, 1)$

It is obvious that $f(x)$ does not take all real values for $0 < b < 1$

$\Rightarrow$ $f : (0, 1) \to \mathbb{R}$ is into function, and hence its increase does not exist.

50. Let L be a normal to the parabola $y^2 = 4x$. If L passes through the point (9, 6), then L is given by

(A) $y - x + 3 = 0$

(B) $y + 3x - 33 = 0$

(C) $y + x - 15 = 0$

(D) $y - 2x + 12 = 0$

Answer (A, B, D)

Hints : The equation of the normal to the given parabola $y^2 = 4x$ in slope form is

$y = mx - 2m - m^3$

which will pass through (9, 6) if

$6 = 9m - 2m - m^3$

$\Rightarrow$ $m^3 - 7m + 6 = 0$

$\Rightarrow$ $m = 1, 2, -3$

Consequently the equation of the normal L is

$y = x - 3 \quad \Rightarrow \quad y - x + 3 = 0$

or $y = 2x - 12 \quad \Rightarrow \quad y - 2x + 12 = 0$

or $y = -3x + 33 \quad \Rightarrow \quad y + 3x - 33 = 0$

51. If

$$f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le -\frac{\pi}{2}, \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1, \end{cases}$$

then

(A) $f(x)$ is continuous at $x = -\frac{\pi}{2}$

(B) $f(x)$ is not differentiable at $x = 0$

(C) $f(x)$ is differentiable at $x = 1$

(D) $f(x)$ is differentiable at $x = -\frac{3}{2}$

Answer (A, B, C, D)

Hints : The given function f is defined as

$$f(x) = \begin{cases} -x - \frac{\pi}{2}, & x \le -\frac{\pi}{2} \\ -\cos x, & -\frac{\pi}{2} < x \le 0 \\ x - 1, & 0 < x \le 1 \\ \ln x, & x > 1 \end{cases}$$

We have,

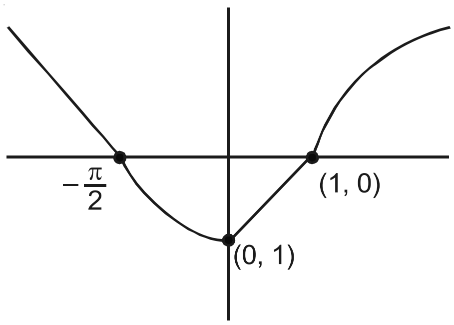
$$\lim_{h \to 0} f\left(-\frac{\pi}{2} - h\right) = \lim_{h \to 0} + \frac{\pi}{2} + h - \frac{\pi}{2} = 0$$

$$\lim_{h \to 0} f\left(-\frac{\pi}{2} - h\right) = \lim_{h \to 0} -\cos\left(-\frac{\pi}{2} + h\right) = 0$$

$$f\left(-\frac{\pi}{2}\right) = \frac{\pi}{2} - \frac{\pi}{2} = 0$$

$\Rightarrow$ $f(x)$ is continuous at $x = -\frac{\pi}{2}$

Let us draw the graph of the given function



From graph we observe that all the options are correct.

52. Let E and F be two independent events. The probability that exactly one of them occurs is $\frac{11}{25}$ and the probability of none of them occurring is $\frac{2}{25}$. If $P(T)$ denotes the probability of occurrence of the event T, then

(A) $P(E)=\frac{4}{5}, P(F)=\frac{3}{5}$

(B) $P(E)=\frac{1}{5}, P(F)=\frac{2}{5}$

(C) $P(E)=\frac{2}{5}, P(F)=\frac{1}{5}$

(D) $P(E)=\frac{3}{5}, P(F)=\frac{4}{5}$

Answer (A, D)

Hints : We have,

$$P(E)+P(F)-2P(E\cap F)=\frac{11}{25}$$

$$P(E^c\cap F^c)=(1-P(E))(1-P(F))=\frac{2}{25}$$

Solving these equations, we shall get

$$P(E)=\frac{4}{5},\ P(F)=\frac{3}{5}$$

or $$P(E)=\frac{3}{5},\ P(F)=\frac{4}{5}$$

SECTION - III (Total Marks : 24)

(Integer Answer Type)

This section contains **6 questions.** The answer to each of the questions is a **single-digit integer,** ranging from 0 to 9. The bubble corresponding to the correct answer is to be darkened in the ORS.

53. The number of distinct real roots of $x^4 - 4x^3 + 12x^2 + x - 1 = 0$ is

Answer (2)

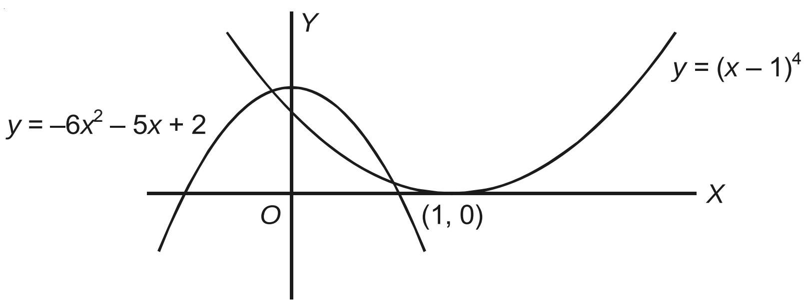
Hints : The given equation is

$$x^4 - 4x^3 + 12x^2 + x - 1 = 0$$

$$\Rightarrow x^4 - 4x^3 + 6x^2 - 4x + 1 + 6x^2 + 5x - 2 = 0$$

$$\Rightarrow (x-1)^4 = -6x^2 - 5x + 2$$

In order to find the number of solutions of the given equations, it is sufficient to find the number of point of intersections of the given curve $y = (x-1)^4$ and $y = -6x^2 - 5x + 2$.



Clearly, there are two solutions of the given equation.

54. Let M be a 3 × 3 matrix satisfying

$$M\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix},\ M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix},\text{ and }M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$$

Then the sum of the diagonal entries of M is

Answer (9)

Hints : Let $M=\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}$ be the given matrix.

Using the given conditions, we have

$$\begin{bmatrix}a_{11} & a_{12} & a_{13}\\ a_{21} & a_{22} & a_{23}\\ a_{31} & a_{32} & a_{33}\end{bmatrix}\begin{bmatrix}0\\1\\0\end{bmatrix}=\begin{bmatrix}-1\\2\\3\end{bmatrix}$$

$\Rightarrow$ $a_{12}=-1$

$a_{22}=2$

$a_{32}=3$

Also, $M\begin{bmatrix}1\\-1\\0\end{bmatrix}=\begin{bmatrix}1\\1\\-1\end{bmatrix}$

$\Rightarrow$ $a_{11}-a_{12}=1$

$a_{21}-a_{22}=1$

$a_{31}-a_{32}=-1$

Using above equations, we shall get

$a_{11}=0$

Moreover, $M\begin{bmatrix}1\\1\\1\end{bmatrix}=\begin{bmatrix}0\\0\\12\end{bmatrix}$

$\Rightarrow$ $a_{11}+a_{12}+a_{13}=0$

$a_{21}+a_{22}+a_{23}=0$

$a_{31}+a_{32}+a_{33}=12$

Using above results, we get

$a_{33}=7$

Finally, the sum of elements of leading diagonals

$=a_{11}+a_{22}+a_{33}$

$=0+2+7$

$=9$

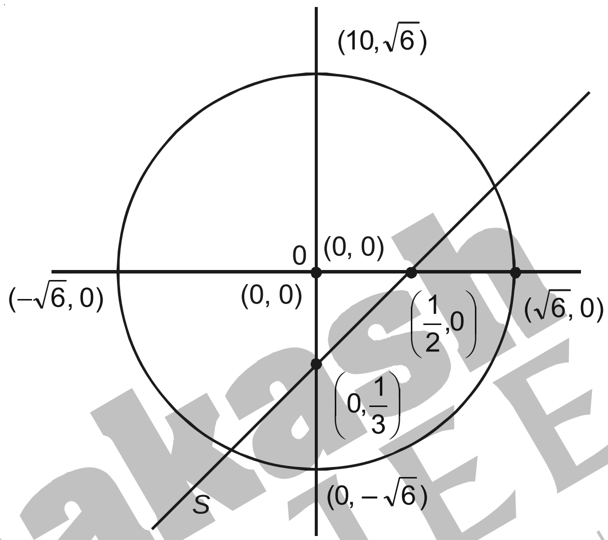
55. The straight line $2x - 3y = 1$ divides the circular region $x^2 + y^2 \le 6$ into two parts. If

$$S = \left\{\left(2, \frac{3}{4}\right), \left(\frac{5}{2}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right), \left(\frac{1}{8}, \frac{1}{4}\right)\right\},$$

then the number of point(s) in S lying inside the smaller part is

Answer (3)

Hints : We observe that the points $\left(2, \frac{3}{4}\right), \left(\frac{5}{3}, \frac{3}{4}\right), \left(\frac{1}{4}, -\frac{1}{4}\right)$



lie on the opposite side of origin with respect to the given line. Hence there are exactly three points of the set lie in the smaller part

56. Let $\vec{a} = -\hat{i} - \hat{k}$, $\vec{b} = -\hat{i} + \hat{j}$ and $\vec{c} = \hat{i} + 2\hat{j} + 3\hat{k}$ be three given vectors. If $\vec{r}$ is a vector such that $\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$ and $\vec{r}.\vec{a} = 0$, then the value of $\vec{r}.\vec{b}$ is

Answer (9)

Hints : We have

$$\vec{r} \times \vec{b} = \vec{c} \times \vec{b}$$

$$\Rightarrow \quad (\vec{r} - \vec{c}) \times \vec{b} = \vec{0}$$

$$\Rightarrow \quad \vec{r} - \vec{c} = \lambda\vec{b}, \lambda \ne 0$$

$$\Rightarrow \quad \vec{r} = \vec{c} + \lambda\vec{b}$$

Since $\vec{r} \cdot \vec{a} = 0$

$$\Rightarrow \quad (\vec{c} + \lambda\vec{b}) \cdot \vec{a} = 0$$

$$\Rightarrow \quad \lambda = 4$$

$$\Rightarrow \quad \vec{r}.\vec{b} = (\vec{c} + 4\vec{b}) \cdot \vec{b}$$

$$= (-3\hat{i} + 6\hat{j} + 3\hat{k}) \cdot (-\hat{i} + \hat{j})$$

$$= 3 + 6 = 9$$

57. Let $\omega = e^{i\pi/3}$, and a, b, c, x, y, z be non-zero complex number such that

$$a + b + c = x$$

$$a + b\omega + c\omega^2 = y$$

$$a + b\omega^2 = c\omega$$

Then the value of $\dfrac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2}$ is

Answer (3)

Hints : We have

$$|x|^2 = x\bar{x} = (a+b+c)\cdot(\vec{a}+\vec{b}+\vec{c})$$

$$= |\vec{a}|^2 + |\vec{b}|^2 + |c|^2 + a(\vec{b}+\vec{c})$$

$$+ b(\vec{c}+\vec{a}) + c(\vec{a}+\vec{b})$$

$$|y|^2 = y\bar{y} = (a + b\omega + c\omega^2)(\bar{a} + \bar{b}\omega^2 + \bar{c}\omega)$$

Similarly

$$|z|^2 = z\bar{z} = (a + b\omega^2 + c\omega)(\bar{a} + \bar{b}\bar{\omega}^2 + \bar{c}\bar{\omega})$$

On adding them, we get

$$|x|^2 + |y|^2 + |z|^2 = 3(|a|^2 + |b|^2 + |c|^2)$$

$$\Rightarrow \frac{|x|^2 + |y|^2 + |z|^2}{|a|^2 + |b|^2 + |c|^2} = 3$$

58. Let $y'(x) + y(x)g'(x) = g(x)g'(x)$, $y(0) = 0$, $x \in \mathbb{R}$, where $f'(x)$ denotes $\dfrac{df(x)}{dx}$ and g(x) is a given non-constant differentiable function on $\mathbb{R}$ with g(0) = g(2) = 0. Then the value of y (2) is

Answer (0)

Hints : $y'(x) + y(x)g'(x) = g(x)\cdot g'(x)$

$\Rightarrow$ which is linear differential equation

$$\text{I.F.} = e^{\int g'(x)dx} = e^{g(x)}$$

Solution is

$$y(x)e^{g(x)} = \int e^{g(x)} \cdot g(x)g'(x)dx$$

$$y(x)e^{g(x)} = e^{g(x)}(g(x) - 1) + k$$

where k is a constant of integration

For $x = 0$, $k = 1$

For $x = 2$, $y(2) = 0$

SECTION - IV (Total Marks : 16)

(Matrix-Match Type)

This section contains **2 questions.** Each question has **four statements** (A, B, C and D) given in **Column I** and five **statements** (p, q, r, s and t) in **Column II.** Any given statement in Column I can have correct matching with **ONE** or **MORE** statement (s) given in Column II. For example, if for a given question, statement B matches with the statements given in q and r, then for the particular question, against statement B, darken the bubbles corresponding to q and r in the ORS.

59. Match the statements given in **Column I** with the intervals/union of intervals given in **Column II**

Column I	**Column II**
(A) The set $\left\{\mathrm{Re}\left(\frac{2iz}{1-z^2}\right): z \text{ is a complex number }, f\,\lvert z\rvert=1, z\neq\pm1\right\}$ is	(p) $(-\infty,-1)\cup(1,\infty)$
(B) The domain of the function $f(x)=\sin^{-1}\left(\frac{8(3)^{x-2}}{1-3^{2(x-1)}}\right)$ is	(q) $(-\infty,-0)\cup(0,\infty)$
(C) If $f(\theta)=\begin{vmatrix} 1 & \tan\theta & 1 \\ -\tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$, then the set $\left\{f(\theta): 0\le\theta<\frac{\pi}{2}\right\}$ is	(r) $[2,\infty)$
(D) If $f(x)=x^{3/2}(3x-10),\ x\ge 0,$ then $f(x)$ is increasing in	(s) $(-\infty,-1]\cup[1,\infty)$
	(t) $(-\infty,0]\cup[2,\infty)$

Answer : A(p, s), B(r, t), C(r), D(r)

Hints : (A) We have $|z| = 1$ and $z \neq \pm 1$

$z = \cos\theta + i\sin\theta$

$$\frac{2z}{1=z^2}=\frac{2(\cos\theta+i\sin\theta)}{1-(\cos\theta+i\sin\theta)^2}$$

$$=\frac{2(\cos\theta+i\sin\theta)}{1-\cos 2\theta-i\sin 2\theta}$$

$$=\frac{1}{-i\sin\theta}\left(\frac{\cos\theta+i\sin\theta}{\cos\theta+i\sin\theta}\right)$$

$$= = \frac{1}{i\sin\theta}=\frac{i}{\sin\theta}$$

$$\Rightarrow\quad R_e\left(\frac{i2z}{1-z^2}\right)=-\frac{1}{\sin\theta}=-\operatorname{cosec}\theta$$

$\Rightarrow\quad D_f = (-\infty, 1] \cup [1, \infty)$

(B) For the domain of the given function

$$-1 \le \frac{8.3^{x-2}}{1-3^{2(x-1)}} \le 1$$

$\Rightarrow$ $|8.3^{x-2}| \le |1 - 3^{2(x-1}|$

$\Rightarrow$ $\dfrac{8.3^x}{9} \le 1 - \dfrac{3^{2x}}{9}, \dfrac{3^{2x}}{9} < 1$

$\Rightarrow$ $8a \le 9 - a^2$, $a = 3^x$

$\Rightarrow$ $-9 \le a \le 1$

$\Rightarrow$ $3^x \le 3^0$

$\Rightarrow$ $x \le 0$

(C) We have

$$f(\theta) = \begin{vmatrix} 1 & \tan\theta & 1 \\ \tan\theta & 1 & \tan\theta \\ -1 & -\tan\theta & 1 \end{vmatrix}$$

$= (1 + \tan^2\theta) - \tan\theta(-\tan\theta + \tan\theta) + 1(\tan^2\theta + 1)$

$= 2(1 + \tan^2\theta) = 2\sec^2\theta$

$D_f = [2, \infty)$

(D) We have

$$f(x) = x^{\frac{3}{2}}(3x-10)$$

$$f'(x) = \frac{3}{2}x^{\frac{1}{2}}(3x-10) + 3 \cdot x^{\frac{3}{2}}$$

$$= \frac{3x^{\frac{1}{2}}}{2}(3x-10+2x)$$

$$= \frac{15}{2}x^{\frac{1}{2}}(x-2)$$

Since $f(x)$ is increasing, hence

$f'(x) \ge 0$

$\Rightarrow$ $x \ge 2$ as $x \ge 0$

D_f is $[2, \infty)$

60. Match the statements given in **Column I** with the values given in **Column II**

Column I	**Column II**
(A) If $\vec{a} = \hat{j} + \sqrt{3}\,\hat{k}, \vec{b} = -\hat{j} + \sqrt{3}\hat{k}$ and $\vec{c} = 2\sqrt{3}\hat{k}$ form a triangle, then internal angle of the triangle between $\vec{a}$ and $\vec{b}$ is	(p) $\dfrac{\pi}{6}$
(B) If $\int_a^b (f(x) - 3x)dx = a^2 - b^2$, then the value of $f\left(\dfrac{\pi}{6}\right)$ is	(q) $\dfrac{2\pi}{3}$

(C) The value of $\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec(\pi x)dx$ is | (r) $\frac{\pi}{3}$

(D) The maximum value of $\left|Arg\left(\frac{1}{1-z}\right)\right|$ | (s) π

$|z|=1, z \neq 1$ is given by

(t) $\frac{\pi}{2}$

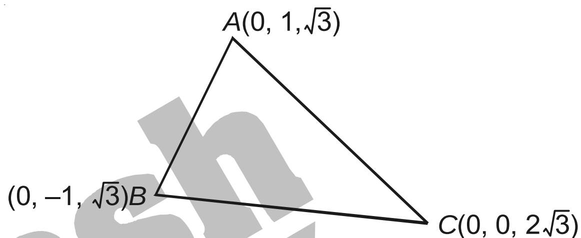
Answer : A(r), B(p), C(s), D(s)

Hints : (A) We have,

$\vec{a} = \hat{j} + \sqrt{3}\hat{k}$

$\vec{b} = -\hat{j} + \sqrt{3}\hat{k}$

$\vec{c} = 2\sqrt{3}\hat{k}$



We observe that

$\vec{a} + \vec{b} = \vec{c}$

$\Rightarrow \quad |\vec{a}|^2 + |\vec{b}|^2 + 2\vec{a}\cdot\vec{b} = |\vec{c}|^2$

$\Rightarrow \quad 4 + 4 + 8\cos\theta = 12$

$\cos\theta = \frac{1}{2}$

$\Rightarrow \quad \theta = \frac{\pi}{3}$

(B) We have,

$$\int_a^b (f(x) - 3x)\,dx = a^2 - b^2$$

Keeping a constant and differentiating both sides w.r.t. b, we get

$f(b) - 3b = -2b$

$\Rightarrow \quad f(b) = b$

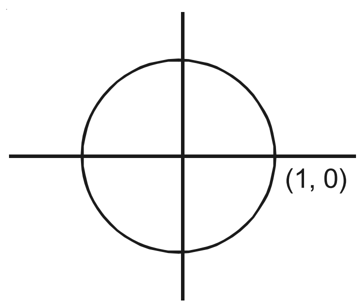
$\Rightarrow \quad f\left(\frac{\pi}{6}\right) = \frac{\pi}{6}$

(C) Using $\int \sec \pi x\, dx = \frac{1}{\pi}\ln|\sec \pi x + \tan \pi x|$, we get

$$\frac{\pi^2}{\ln 3}\int_{7/6}^{5/6} \sec \pi x\, dx = \pi$$

(D) $\left|\arg\left(\frac{1}{1-z}\right)\right| = |\arg(1) - \arg(1-z)|$

$= |\arg(1-z)|$



But z lies on $|z| = 1$

Hence, max$|\arg(1-z)| = \pi$

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