SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.Arch
Title of the paper: Mathematics - I
Semester: I — Max.Marks: 80
Sub.Code: 521101 — Time: 3 Hours
Date: 14-05-2009 — Session: FN

PART - A (8 X 4 = 32)

Answer ALL the Questions

1. Find the sum and product of the eigen values of the matrix
$A = \begin{pmatrix} 1 & 2 & -2 \\ 1 & 0 & 3 \\ -2 & -1 & -3 \end{pmatrix}$.

2. Find the matrix form of the quadratic form
$3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$

3. Show that the spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ are orthogonal to each other.

4. Find the angle between the planes $2x - y + z = 6$ *and* $x + y + 2z = 3$.

5. Evaluate $\int_0^{\frac{\pi}{2}} \sin^{10} x dx$.

6. Evaluate $\int_1^b \int_1^a \frac{dxdy}{xy}$.

7. Prove that $2^5 \cos^6\theta = \cos 6\theta + 6\cos 4\theta + 15 \cos 2\theta + 10$.

8. Convert $x^2 \frac{d^2y}{dx^2} + 4y\frac{dy}{dx} + 2y = \sin(\log x)$ into a second order linear differential equation with constant coefficients using suitable transformation and hence find the C.F.

PART – B (4 x 12 = 48)

Answer All the Questions

9. Verify Cayley Hamilton theorem and hence find the inverse for

$$\begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$$

(or)

10. Deduce $\begin{pmatrix} 0 & 1 & -1 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{pmatrix}$ to the diagonal form.

11. Find the shortest distance and the equation of the line of shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$.

(or)

12. Find the centre and radius of the circle circle given by $x^2 + y^2 + z^2 - 2x - 4y - 6z - 2 = 0; x + 2y + 2z - 20 = 0.$

13. Change the order of integration in $\int_0^a \int_x^a (x^2 + y^2)\,dydx$. and hence evaluate it.

(or)

14. If, $I_n = \int x^n e^{-x} dx$, n being a positive integer, show that $I_n = -x^n e^{-x} + nI_{n-1}$. Hence show that $\int_0^\infty x^n e^{-x} dx = n!$

15. If x + iy = tan (A + iB), prove that

(a) $x^2 + y^2 + 2x \cot 2A = 1$

(b) $x^2 + y^2 + 1 - 2y \cot h2 B = 0$

(or)

16. Solve $\frac{d^2y}{dx^2} + 4y$ = 4 tan 2x using method of variation of parameters.