SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.Arch
Title of the paper: Mathematics - I
Semester: I — Max.Marks: 80
Sub.Code: 621101(2006-07-08) — Time: 3 Hours
Date: 14-05-2009 — Session: FN

PART - A (8 X 4 = 32)

Answer ALL the Questions

1. If $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 3 \end{pmatrix}$, write down the sum of the eigen values of A and product of the eigen values of A.

2. Use Cayley-Hamilton theorem to find the inverse of $A = \begin{pmatrix} 7 & 3 \\ 2 & 6 \end{pmatrix}$.

3. Evaluate $\int_0^{\frac{\pi}{2}} \sin^6 x \, dx$.

4. Change the order of integration in $\int_0^1 \int_0^{2\sqrt{x}} f(x, y) dy dx$

5. Solve the P.I. of $(D^2 - 2D + 5)\, y = e^x \sin 2x$.

6. Solve $(D^2 - 4D + 4)\, y = 0$.

7. Find the equation of the plane which passes through the points A (0, 1, 3) B (1, 0, -4) C(1, 1, -1)

8. Find the equation of the sphere whose centre is (2, -3, 4) and radius 5.

PART – B (5 x 12 = 60)

Answer All the Questions

9. Reduce the quadratic form $2x_1^2 + x_2^2 + x_3^2 + 2x_1x_2 - 2x_1x_3 - 4x_2x_3$ to canonical form by orthogonal reduction. Find the nature of the quadratic form.

(or)

10. (a) Using Cayley-Hamilton theorem to find the inverse of the

matrix $A = \begin{pmatrix} 1 & 3 & 7 \\ 4 & 2 & 3 \\ 1 & 2 & 1 \end{pmatrix}$

(b) Diagonalise the matrix $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$ by means of orthogonal transformation.

11. (a) Change the order of integration and hence evaluate it $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dydx$

(b) Evaluate $\int_0^1 \int_0^{1-x} \int_0^{1-x-y} \frac{dzdydx}{(x+y+z+1)^3}$

(or)

12. (a) Drive the Reduction formula $\int \sin^n x dx$.

(b) Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sin^n x}{(\sin^n x + \cos^n x)} dx$

13. (a) Solve $(D^2 + 5D + 4)\, y = e^{-x} \sin 2x$.

(b) Solve $(x^2D^2 - 2xD - 4)\, y = 32\,(\log x)^2$.

(or)

14. (a) Solve by the method of variation of parameter $y'' + 7y' - 8y = e^{2x}$.

(b) Solve $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$

15. (a) Show that the lines $\frac{x-1}{2} = \frac{y-2}{3} = \frac{z-3}{4}$ and $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$ are coplanar and the equation of the plane containing them.

(b) Prove that the two spheres $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$, $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$ touch each other and find the coordinates of the point of contact.

(or)

16. (a) Find the shortest distance between the lines $\frac{x-3}{3} = \frac{y-8}{-1} = \frac{z-3}{1}$ and $\frac{x+3}{-3} = \frac{y+7}{2} = \frac{z-6}{4}$. Also find the equation of the line of the shortest distance.

(b) Find the equation of the plane passing through the points A (2, 5, -3), B (-2, -3, 5) C (5, 3, -3).