PAPER – 1 : MATHEMATICS, PHYSICS & CHEMISTRY

Test Booklet Code

C

Do not open this Test Booklet until you are asked to do so.

Read carefully the Instructions on the Back Cover of this Test Booklet.

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. Use of pencil is strictly prohibited.
2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.
3. The test is of **3 hours** duration.
4. The Test Booklet consists of **90** questions. The maximum marks are **432.**
5. There are ***three*** parts in the question paper.
   The distribution of marks subjectwise in each part is as under for each correct response.
   Part A – MATHEMATICS (144 marks) - Questions No. 1, 5 to 12 and 16 to 30 consist of **FOUR (4)** marks each and Questions No. 2 to 4 and 13 to 15 consist of **EIGHT (8)** marks each for each correct response.
   Part B – PHYSICS (144 marks) – Questions No. 31 to 38, 43 to 49 and 52 to 60 consist of **FOUR (4)** marks each and Questions No. 39 to 42 and 50 to 51 consist of **EIGHT (8)** marks each for each correct response.
   Part C – CHEMISTRY (144 marks) – Questions No. 64 to 78 and 82 to 90 consist of **FOUR (4)** marks each and Questions No. 61 to 63 and 79 to 81 consist of **EIGHT (8)** marks each for each correct response.
6. *Candidates will be awarded marks as stated above in Instruction No. 5 for correct response of each question. ¼ (one fourth) marks will be deducted for indicating incorrect response of each question.* ***No deduction*** *from the total score will be made* ***if no response*** *is indicated for an item in the Answer Sheet.*
7. Use ***Blue/Black Ball Point Pen only*** for writing particulars/marking responses on ***Side-1*** and ***Side-2*** of the Answer Sheet. ***Use of pencil is strictly prohibited.***
8. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc., except the Admit Card inside the examination hall/room.
9. Rough work is to be done on the space provided for this purpose in the Test Booklet only. This space is given at the bottom of each page and in 4 pages (Pages 20 – 23) at the end of the booklet.
10. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. ***However, the candidates are allowed to take away this Test Booklet with them.***
11. The CODE for this Booklet is **C.** Make sure that the CODE printed on **Side-2** of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet.
12. **Do not fold or make any stray marks on the Answer Sheet.**

Name of the Candidate (in Capital letters) : ______________________________

Roll Number : in figures | | | | | | | | |

: in words ______________________________

Examination Centre Number : | | | | | | |

Name of Examination Centre (in Capital letters) : ______________________________

Candidate's Signature : ______________ Invigilator's Signature : ______________

READ THE FOLLOWING INSTRUCTIONS CAREFULLY :

1. The candidates should fill in the required particulars on the Test Booklet and Answer Sheet (***Side-1***) with ***Blue/Black Ball Point Pen.***

2. For writing/marketing particulars on ***Side-2*** of the Answer Sheet, use ***Blue/Black Ball Point Pen only.***

3. The candidates should not write their Roll Numbers anywhere else (except in the specified space) on the Test Booklet/Answer Sheet.

4. Out of the four options given for each question, only one option is the correct answer.

5. For each ***incorrect response, one-fourth (¼)*** of the total marks allotted to the question would be deducted from the total score. ***No deduction*** from the total score, however, will be made ***if no response*** is indicated for an item in the Answer Sheet.

6. Handle the Test Booklet and Answer Sheet with care, as under no circumstances (except for discrepancy in Test Booklet Code and Answer Sheet Code), will another set be provided.

7. The candidates are not allowed to do any rough work or writing work on the Answer Sheet. All calculations/writing work are to be done in the space provided for this purpose in the Test Booklet itself, marked 'Space for Rough Work'. This space is given at the bottom of each page and in 4 pages (Pages 20 – 23) at the end of the booklet.

8. On completion of the test, the candidates must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. **However, the candidates are allowed to take away this Test Booklet with them.**

9. Each candidate must show on demand his/her Admit Card to the Invigilator.

10. No candidate, without special permission of the Superintendent or Invigilator, should leave his/her seat.

11. The candidates should not leave the Examination Hall without handing over their Answer Sheet to the Invigilator on duty and sign the Attendance Sheet again. Cases where a candidate has not signed the Attendance Sheet a second time will be deemed not to have handed over the Answer Sheet and dealt with as an unfair means case. **The candidates are also required to put their left hand THUMB impression in the space provided in the Attendance Sheet.**

12. Use of Electronic/Manual Calculator and any Electronic Item like mobile phone, pager etc. is prohibited.

13. The candidates are governed by all Rules and Regulations of the Board with regard to their conduct in the Examination Hall. All cases of unfair means will be dealt with as per Rules and Regulations of the Board.

14. No part of the Test Booklet and Answer Sheet shall be detached under any circumstances.

15. **Candidates are not allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, electronic device or any other material except the Admit Card inside the examination hall/room.**

AIEEE 2010

Test Booklet Code

C

Questions and Solutions

PAPER - 1 : MATHEMATICS, PHYSICS & CHEMISTRY

PART- A : MATHEMATICS

1. The line L given by $\frac{x}{5}+\frac{y}{b}=1$ passes through the point (13, 32). The line K is parallel to L and has the equation $\frac{x}{c}+\frac{y}{3}=1$. Then the distance between L and K is

(1) $\frac{23}{\sqrt{17}}$ (2) $\frac{23}{\sqrt{15}}$ (3) $\sqrt{17}$ (4) $\frac{17}{\sqrt{15}}$

Solution : (1)

$\frac{x}{5}+\frac{y}{b}=1$ line passes through (13, 32)

$\therefore$, b = –20

$\therefore$, Eq. of line L is 4x – y = 20

Since line K is parallel to L, $c=-\frac{3}{4}$

$\therefore$, Eq. of K = 4 x – y = –3

Hence, distance between K and L is $\frac{23}{\sqrt{17}}$.

2. The number of 3 × 3 non–singular matrices, with four entries as 1 and all other entries as 0, is

(1) at least 7 (2) less than 4 (3) 5 (4) 6

Solution : (1)

More than 7 examples are given below,

$$\begin{bmatrix}0&1&1\\1&0&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&1&0\\1&0&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&1&0\\0&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&0&0\\1&1&0\\0&0&1\end{bmatrix},\begin{bmatrix}1&0&1\\0&1&0\\0&0&1\end{bmatrix},$$

$$\begin{bmatrix}1&0&0\\0&1&0\\1&0&1\end{bmatrix},\begin{bmatrix}1&0&1\\1&0&0\\0&1&0\end{bmatrix},\begin{bmatrix}0&0&1\\1&0&1\\0&1&0\end{bmatrix}$$

3. Let f : R → R be defined by, $f(x)=\begin{cases}k-2x & \text{if } x\le -1\\ 2x+3 & \text{if } x>-1\end{cases}$

If f has a local minimum at x = –1, then a possible value of k is

(1) –1 (2) 1 (3) 0 (4) $-\frac{1}{2}$

Solution : (1)

f(x) has a local minimum at x = –1,

if K – 2 (–1) ≤ 2 (–1) + 3

⇒ K + 2 ≤ 1

⇒ K ≤ –1

Directions : Questions number **4** to **8** are Assertion – Reason type questions. Each of these questions contains two statements.

Statement - 1 (Assertion) and **Statement – 2 (Reason).**

Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select the correct choice.

4. Four numbers are chosen at random (without replacement) from the set {1, 2, 3, …, 20}.

Statement – 1 : The probability that the chosen numbers when arranged in some order will form an AP is $\frac{1}{85}$.

Statement – 2 : If the four chosen numbers form an AP, then the set of all possible values of common difference is $\{\pm1, \pm2, \pm3, \pm4, \pm5\}$

(1) Statement - 1 is false, Statement - 2 is true.
(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.
(3) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(4) Statement -1 is true, Statement - 2 is false.

Solution : (4)

Required probability $\Rightarrow$ $P = \dfrac{17+14+11+8+5+2}{^{20}C_4}$

$$P = \frac{1}{85}$$

Statement (2) is false as the common difference can be ± 6.
(Ex. 1, 7, 13, 19)

5. Let A be a 2 × 2 matrix with non–zero entries and let $A^2 = I$, where I is 2 × 2 identity matrix. Define Tr(A) = sum of diagonal elements of A and | A | = determinant of matrix A.

Statement – 1 : Tr (A) = 0
Statement – 2 : | A | = 1

(1) Statement - 1 is false, Statement - 2 is true.
(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.
(3) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(4) Statement -1 is true, Statement - 2 is false.

Solution : (4)

Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$A^2 = \begin{bmatrix} a^2+bc & (a+d)b \\ (a+d)c & d^2+bc \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$\Rightarrow$ $a + d = 0$ $(\because b, c \neq 0)$

$\therefore$ Statement (1) is true.

Also $|A|^2 = |I| = 1 \Rightarrow |A| = \pm 1$

$\therefore$, Statement 1 is true and statement 2 is false.

6. Let f : R → R be a continuous function defined by $f(x) = \dfrac{1}{e^x + 2e^{-x}}$.

Statement – 1 : $f(c) = \frac{1}{3}$, for some $c \in R$.

Statement – 2 : $0 < f(x) \le \frac{1}{2\sqrt{2}}$, for all $x \in R$.

(1) Statement - 1 is false, Statement - 2 is true.
(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.
(3) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(4) Statement -1 is true, Statement - 2 is false.

Solution : (2)

$f : \mathbb{R} \to \mathbb{R}$,

$f(x) = \dfrac{1}{e^x + 2e^{-x}}$

$y = f(x) = \dfrac{e^x}{e^{2x} + 2}$

$e^x + 2e^{-x} \geq 2\sqrt{e^x \cdot 2e^{-x}} = 2\sqrt{2}$ (using AM–GM)

$\therefore \dfrac{1}{e^x + 2e^{-x}} \leq \dfrac{1}{2\sqrt{2}}$

and $\dfrac{1}{e^x + 2e^{-x}} > 0 \quad \forall\, x \in \mathbb{R}$

$\therefore \quad 0 < \dfrac{1}{e^x + 2e^{-x}} \leq \dfrac{1}{2\sqrt{2}}$

Note that, $\dfrac{1}{e^x + 2e^{-x}}$ is continuous and $\dfrac{1}{3} < \dfrac{1}{2\sqrt{2}}$.

$\exists\, c \in \mathbb{R}$ s.t., $f(c) = \dfrac{1}{3}$

7. **Statement – 1 :** The point A (3, 1, 6) is the mirror image of the point B(1, 3, 4) in the plane $x - y + z = 5$

Statement – 2 : The plane $x - y + z = 5$ bisects the line segment joining A (3, 1, 6) and B (1, 3, 4).

(1) Statement - 1 is false, Statement - 2 is true.
(2) Statement - 1 is true, Statement-2 is true; Statement - 2 is a correct explanation for Statement - 1.
(3) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(4) Statement -1 is true, Statement - 2 is false.

Solution : (3)

$x - y + z = 5$;

Image of point (3, 1, 6) is given by

$$\frac{x-3}{1} = \frac{y-1}{-1} = \frac{z-6}{1} = -2\left(\frac{3-1+6-5}{1^2+1^2+1^2}\right) = -2\left(\frac{3}{3}\right) = -2$$

$(3 - 2, 2 + 1, -2 + 6) = (1, 3, 4)$

$\therefore$ Statement (1) is correct, Statement (2) is correct but statement (2) is not the correct explanation for Statement (1).

8. Let $S_1 = \sum_{j=1}^{10} j(j-1)\,{}^{10}C_j$, $S_2 = \sum_{j=1}^{10} j\,{}^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^2\,{}^{10}C_j$

Statement – 1 : $S_3 = 55 \times 2^9$

Statement – 2 : $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.

(1) Statement - 1 is false, Statement - 2 is true.
(2) Statement - 1 is true, Statement - 2 is true; Statement - 2 is a correct explanation for Statement - 1.
(3) Statement - 1 is true, Statement - 2 is true; Statement - 2 is not a correct explanation for Statement - 1.
(4) Statement -1 is true, Statement - 2 is false.

Solution : (4)

$(1+x)^{10} = {}^{10}C_0 + {}^{10}C_1 x + {}^{10}C_2 x^2 + \ldots.. + {}^{10}C_{10} x^{10}$

S_2 is obtained by differentiating the given expansion and putting x = 1.

S_1 is obtained by differentiating the given expansion twice and putting x = 1.

S_3 is obtained by differentiating, multiplying by x, differentiating again and putting x = 1.

$\therefore$, $S_1 = 90 \times 2^8$

$S_2 = 10 \times 2^9$

$S_3 = 55 \times 2^9$

9. A line AB in three–dimensional space makes angles 45° and 120° with the positive x − axis and the positive y–axis respectively. If AB makes an acute angle θ with the positive z–axis, then θ equals

(1) 75° (2) 30° (3) 45° (4) 60°

Solution : (4)

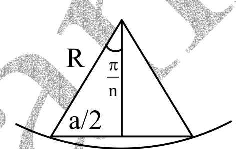
$$\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$$
$$\cos^2 45 + \cos^2 120 + \cos^2\gamma = 1$$
$$\frac{1}{2} + \frac{1}{4} + \cos^2\gamma = 1$$
$$\cos^2\gamma = 1 - \frac{3}{4}$$
$$\cos^2\gamma = \frac{1}{4}$$
$$\cos\gamma = \pm\frac{1}{2}$$
$$\gamma = 120° \quad \text{or} \quad 60°.$$

10. For a regular polygon, let r and R be the radii of the inscribed and the circumscribed circles. A false statement among the following is

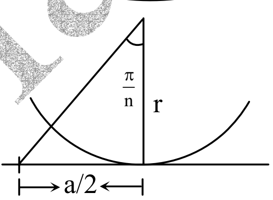
(1) There is a regular polygon with $\frac{r}{R} = \frac{\sqrt{3}}{2}$ (2) There is a regular polygon with $\frac{r}{R} = \frac{1}{2}$

(3) There is a regular polygon with $\frac{r}{R} = \frac{1}{\sqrt{2}}$ (4) There is a regular polygon with $\frac{r}{R} = \frac{2}{3}$

Solution :(4)



$$\frac{a}{2R} = \sin\left(\frac{\pi}{n}\right)$$
$$\frac{a}{2r} = \tan\left(\frac{\pi}{n}\right)$$
$$\therefore \frac{r}{R} = \cos\left(\frac{\pi}{n}\right)$$



$\therefore \frac{r}{R}$ can be $\frac{\sqrt{3}}{2}, \frac{1}{2}, \frac{1}{\sqrt{2}}$ natural values of n.

11. Let $\cos(\alpha + \beta) = \frac{4}{5}$ and let $\sin(\alpha - \beta) = \frac{5}{13}$, where $0 \le \alpha, \beta \le \frac{\pi}{4}$. Then $\tan 2\alpha =$

(1) $\frac{20}{7}$ (2) $\frac{25}{16}$ (3) $\frac{56}{33}$ (4) $\frac{19}{12}$

Solution : (3)

$$\tan(2\alpha) = \tan(\alpha + \beta + \alpha - \beta)$$
$$= \frac{\sin(\alpha + \beta + \alpha - \beta)}{\cos(\alpha + \beta + \alpha - \beta)} = \frac{\sin(\alpha+\beta)\cos(\alpha-\beta) + \cos(\alpha+\beta)\sin(\alpha-\beta)}{\cos(\alpha+\beta)\cos(\alpha-\beta) - \sin(\alpha+\beta)\sin(\alpha-\beta)}$$
$$= \frac{\frac{3}{5}\times\frac{12}{13} + \frac{4}{5}\times\frac{5}{13}}{\frac{4}{5}\times\frac{12}{13} - \frac{5}{13}\times\frac{3}{5}} = \frac{56}{33}$$

12. Let S be a non–empty subset of R. Consider the following statement :

P : There is a rational number $x \in S$ such that $x > 0$.

Which of the following statements is the negation of the statement P ?

(1) $x \in S$ and $x \le 0 \Rightarrow x$ is not rational.

(2) There is a rational number $x \in S$ such that $x \le 0$.

(3) There is no rational number $x \in S$ such that $x \le 0$.

(4) Every rational number $x \in S$ satisfies $x \le 0$.

Solution : (4)

13. Let p (x) be a function defined on R such that $p'(x) = p'(1-x)$, for all $x \in [0, 1]$, $p(0) = 1$ and $p(1) = 41$. Then, $\int_0^1 p(x)\,dx$ equals

(1) 42 (2) $\sqrt{41}$ (3) 21 (4) 41

Solution : (3)

$p'(x) = p'(1-x) \qquad \forall\, x \in [0, 1];\ p(0) = 1,\ p(1) = 41$

$$\int_0^x p'(x)dx = \int_0^x p'(1-x)\,dx \qquad \text{put } (1-x) = t$$

$$p(x) - 1 = -\int_1^{1-x} p'(t)\,dt$$

$$1 - p(x) = p(1-x) - p(1)$$

$$1 - p(x) = p(1-x) - 41$$

$$p(x) + p(1-x) = 42$$

$$\int_0^1 p(x)dx + \int_0^1 p(1-x)\,dx = \int_0^1 42\,dx$$

$$\int_0^1 p(x)dx - \int_1^0 p(t)\,dt = 42$$

$$2\int_0^1 p(x)\,dx = 42 \quad \Rightarrow \quad \int_0^1 p(x)\,dx = 21$$

14. A person is to count 4500 currency notes. Let a_n denote the number of notes he counts in the n^{th} minute. If $a_1 = a_2 = \ldots = a_{10} = 150$ and $a_{10}, a_{11}, \ldots$ are in an AP with common difference –2, then the time taken by him to count all notes is

(1) 135 minutes (2) 24 minutes (3) 34 minutes (4) 125 minutes

Solution : (3)

In first 10 minutes , he will count 1500 notes

Then $a_{11} = 148$, $a_{12} = 146$, ….

For an A.P., 148, 146, ….

$$\frac{n}{2}\left[2\times 148 + (n-1)(-2)\right] = 3000$$

$\Rightarrow\ n^2 - 149n + 3000 = 0$

$\Rightarrow\ n = 125, 24$

$n \neq 125$ as terms will become negative for n = 125

$\therefore\ n = 24$

Hence, total time taken = 10 + 24 = 34 minutes.

15. Let $f : R \to R$ be a positive increasing function with $\lim_{x\to\infty} \frac{f(3x)}{f(x)} = 1$. Then , $\lim_{x\to\infty} \frac{f(2x)}{f(x)} =$

(1) 3 (2) 1 (3) $\frac{2}{3}$ (4) $\frac{3}{2}$

Solution :(2)

f is increasing

$f(x) < f(2x) < f(3x)$

Divide by f(x),

$$\frac{f(x)}{f(x)} < \frac{f(2x)}{f(x)} < \frac{f(3x)}{f(x)}$$

$$\lim_{x\to\infty}\frac{f(x)}{f(x)} \le \lim_{x\to\infty}\frac{f(2x)}{f(x)} \le \lim_{x\to\infty}\frac{f(3x)}{f(x)}$$

$$1 \le \lim_{x\to\infty} \frac{f(2x)}{f(x)} \le 1$$

By sandwich theorem, $\lim_{x\to\infty} \frac{f(2x)}{f(x)} = 1$

16. Solution of the differential equation cos x dy = y (sin x − y) dx, $0 < x < \frac{\pi}{2}$ is

(1) tan x = (sec x + c) y
(2) sec x = (tan x + c) y
(3) y sec x = tan x + c
(4) y tan x = sec x + c

Solution : (2)

$$\frac{dy}{dx} = y\tan x - y^2 \sec x$$

$$\frac{dy}{dx} - y\tan x = -y^2 \sec x$$

$$\Rightarrow -\frac{1}{y^2}\frac{dy}{dx} + \frac{1}{y}\tan x = \sec x$$

$$\frac{1}{y} = t \quad \Rightarrow \quad -\frac{1}{y^2}\frac{dy}{dx} = \frac{dt}{dx}$$

$$\Rightarrow \frac{dt}{dx} + t\tan x = \sec x$$

$$\text{I.F.} = e^{\int \frac{\sin x}{\cos x}dx} = e^{\log|\sec x|} = \sec x$$

$$\text{So, } t.\sec x = c + \int \sec^2 x\, dx$$

$$t \,.\, \sec x = c + \tan x$$

$$\Rightarrow \frac{1}{y}\sec x = c + \tan x \quad \Rightarrow \quad \sec x = (c + \tan x)\, y$$

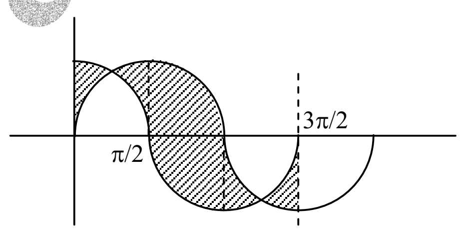
17. The area bounded by the curves y = cos x and y = sin x between the ordinates x = 0 and $x = \frac{3\pi}{2}$ is

(1) $4\sqrt{2}+1$
(2) $4\sqrt{2}-2$
(3) $4\sqrt{2}+2$
(4) $4\sqrt{2}-1$

Solution : (2)

$$A = 4\int_0^{\pi/4} (\cos x - \sin x)\, dx + 2\int_{\pi/2}^{\pi} \sin x\, dx$$

$$= 4\sqrt{2} - 2$$



18. The equation of the tangent to the curve $y = x + \frac{4}{x^2}$, that is parallel to the x−axis, is

(1) y = 3
(2) y = 0
(3) y = 1
(4) y = 2

Solution : (1)

$$y = x + \frac{4}{x^2}$$

$$y_1 = 1 - \frac{8}{x^3}$$

Critical points, $1 - \frac{8}{x^3} = 0$

$\Rightarrow x = 2$

$\therefore$ Tangent is at x = 2.

i.e., $y = 2 + \frac{4}{4} = 3 \qquad \therefore \quad y = 3$

19. Let f : $(-1, 1) \to \mathbf{R}$ be a differentiable function with $f(0) = -1$ and $f'(0) = 1$. Let $g(x) = [f(2f(x) + 2)]^2$. Then $g'(0) =$

(1) -2 (2) 4 (3) -4 (4) 0

Solution : (3)

$$
\begin{aligned}
g(x) &= f^2(2.f(x) + 2) \\
g'(x) &= 2.f(2f(x) + 2)\, f'(2f(x) + 2)\,.\,\{2f'(x)\} \\
g'(0) &= 2f(2f(0) + 2)\,.\,f'(2f(0) + 2)\,.\,2.f'(0) \\
&= 2f(2 \times (-1) + 2)\,.\,f'(-2 + 2)\,.\,2(1) \\
&= 2f(0).\,f'(0)\,.\,2 \\
&= 4.(-1)\,.\,(1) = -4
\end{aligned}
$$

20. There are two urns. Urn A has 3 distinct red balls and urn B has 9 distinct blue balls. From each urn two balls are taken out at random and then transferred to the other. The number of ways in which this can be done is

(1) 108 (2) 3 (3) 36 (4) 66

Solution : (1)

$${}^9C_2 \times {}^3C_2 = 9 \times 4 \times 3 = 108.$$

21. Consider the system of linear equations :

$$x_1 + 2x_2 + x_3 = 3$$
$$2x_1 + 3x_2 + x_3 = 3$$
$$3x_1 + 5x_2 + 2x_3 = 1$$

The system has

(1) no solution (2) infinite number of solutions
(3) exactly 3 solutions (4) a unique solution

Solution : (1)

$$x_1 + 2x_2 + x_3 = 3 \quad \ldots (1)$$
$$2x_1 + 3x_2 + x_3 = 3 \quad \ldots (2)$$
$$3x_1 + 5x_2 + 2x_3 = 1 \quad \ldots (3)$$

Adding (1) and (2), we get,

$$3x_1 + 5x_2 + 2x_3 = 6 \quad \ldots (4)$$
$$3x_1 + 5x_2 + 2x_3 = 1 \quad \ldots (3)$$

$\Rightarrow$ no solution.

22. Let $\vec{a} = \hat{j} - \hat{k}$ and $\vec{c} = \hat{i} - \hat{j} - \hat{k}$. Then the vector $\vec{b}$ satisfying $\vec{a} \times \vec{b} + \vec{c} = \vec{0}$ and $\vec{a} \cdot \vec{b} = 3$ is

(1) $\hat{i} + \hat{j} - 2\hat{k}$ (2) $-\hat{i} + \hat{j} - 2\hat{k}$ (3) $2\hat{i} - \hat{j} + 2\hat{k}$ (4) $\hat{i} - \hat{j} - 2\hat{k}$

Solution : (2)

$$\vec{b} = b_1\hat{i} + b_2\hat{j} + b_3\hat{k}$$

$$\vec{a} \times \vec{b} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 0 & 1 & -1 \\ b_1 & b_2 & b_3 \end{vmatrix} = \hat{i}(b_3 + b_2) - \hat{j}(b_1) + \hat{k}(-b_1)$$

$$= (b_2 + b_3)\hat{i} - b_1\hat{j} - b_1\hat{k}$$

$$\vec{a} \times \vec{b} + \vec{c} = \vec{0}$$

$$\Rightarrow (b_2 + b_3 + 1)\hat{i} - (b_1 + 1)\hat{j} - (b_1 + 1)\hat{k} = \vec{0}$$

$$\Rightarrow b_2 + b_3 + 1 = 0 \quad \ldots (1)$$
$$b_1 = -1 \quad \ldots (2)$$

So, $\vec{a}.\vec{b} = 3$

$$\Rightarrow b_2 - b_3 = 3 \quad \ldots (3)$$

Addition (1) and (3), we get

$$2b_2 = 2$$
$$b_2 = 1$$

$\therefore$ So, from (3),

$b_2 - 3 = b_3$

$\Rightarrow$ $b_3 = -2$

So, $\vec{b} = -\hat{i} + \hat{j} - 2\hat{k}$

23. For two data sets, each of size 5, the variances are given to be 4 and 5 and the corresponding means are given to be 2 and 4, respectively. The variance of the combined data set is

(1) $\frac{13}{2}$ (2) $\frac{5}{2}$ (3) $\frac{11}{2}$ (4) 6

Solution : (3)

$$\bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{5\times4 + 5\times2}{5+5} = 3$$

Variance $= \frac{1}{n_1+n_2}\left[n_1(\sigma_1{}^2 + d_1{}^2) + n_2(\sigma_2{}^2 + d_2{}^2)\right]$

Where $d_1{}^2 = (\bar{x}_1 - \bar{x})^2$, $d_2{}^2 = (\bar{x}_2 - \bar{x})^2$, $\sigma_1{}^2 = V_1$, $\sigma_2{}^2 = V_2$

$$= \frac{1}{10}\left[5(4+1^2) + 5(5+1^2)\right] = \frac{11}{2}$$

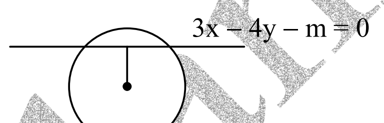
24. The circle $x^2 + y^2 = 4x + 8y + 5$ intersects the line $3x - 4y = m$ at two distinct points if

(1) $35 < m < 85$ (2) $-85 < m < -35$ (3) $-35 < m < 15$ (4) $15 < m < 65$

Solution : (3)

$x^2 + y^2 - 4x - 8y - 5 = 0$

$\therefore$ centre (2, 4), $r = \sqrt{2^2 + 4^2 + 5} = 5$



$$\left|\frac{3\times2 - 4\times4 - m}{5}\right| < 5$$

$\Rightarrow$ $|6 - 16 - m| < 25$ $\Rightarrow$ $|m + 10| < 25$

$\Rightarrow$ $-25 < m + 10 < 25$ $\Rightarrow$ $-35 < m < 15$

25. An urn contains nine balls of which three are red, four are blue and two are green. Three balls are drawn at random without replacement from the urn. The probability that the three balls have different colours is

(1) $\frac{2}{23}$ (2) $\frac{1}{3}$ (3) $\frac{2}{7}$ (4) $\frac{1}{21}$

Solution :(3)

So, required probability $= \frac{{}^3C_1 \times {}^4C_1 \times {}^2C_1}{{}^9C_3} = \frac{3\times4\times2}{\frac{9\times8\times7}{3\times2}} = \frac{2}{7}$

26. If two tangents drawn from a point P to the parabola $y^2 = 4x$ are at right angles, then the locus of P is

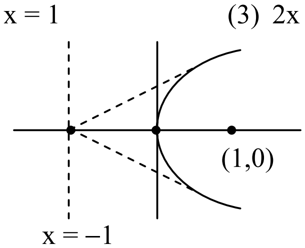
(1) $2x - 1 = 0$ (2) $x = 1$ (3) $2x + 1 = 0$ (4) $x = -1$

Solution : (4)

As right angles then,

Locus of ‘p’ is directrix.

So equation is $x = -1$.



27. If the vectors $\vec{a} = \hat{i} - \hat{j} + 2\hat{k}$, $\vec{b} = 2\hat{i} + 4\hat{j} + \hat{k}$ and $\vec{c} = \lambda\hat{i} + \hat{j} + \mu\hat{k}$ are mutually orthogonal, then $(\lambda, \mu) =$

(1) $(3, -2)$ (2) $(-3, 2)$ (3) $(2, -3)$ (4) $(-2, 3)$

Solution : (2)

As orthogonal $\Rightarrow$ $\vec{a}.\vec{c} = 0 \Rightarrow \lambda + 2\mu = 1$ ……(1)

& $\vec{b}.\vec{c} = 0 \Rightarrow 2\lambda + \mu = -4$ ……(2)

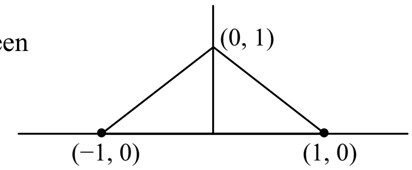
$\therefore$ $(\lambda, \mu) \equiv (-3, 2)$

28. The number of complex numbers z such that, $|z-1| = |z+1| = |z-i|$ equals

(1) ∞ (2) 0 (3) 1 (4) 2

Solution : (3)

Only from (0, 0), distance between (1, 0), (0, 1) and (–1, 0) are same.
Hence only one solution.



29. If α and β are the roots of the equation, $x^2 - x + 1 = 0$, then $\alpha^{2009} + \beta^{2009} =$

(1) 2 (2) −2 (3) −1 (4) 1

Solution : (4)

α, β root of $x^2 - x + 1 = 0$

$\alpha, \beta \equiv -\omega$ and $-\omega^2$

$$\alpha^{2009} + \beta^{2009} = -\omega^{2009} - (\omega^2)^{2009}$$
$$= -\omega^2 - (\omega^2)^2 \qquad [\because \omega^{2009} = \omega^{3\times 669+2} = \omega^2]$$
$$= -\omega^2 - \omega = 1$$

30. Consider the following relations :

R = {(x, y) | x, y are real numbers and x = wy for some rational number w};

$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle|\right.$ m, n, p and q are integers such that n, q $\neq$ 0 and qm = pn }

Then,

(1) R and S both are equivalence relations
(2) R is an equivalence relation but S is not an equivalence relation
(3) neither R nor S is an equivalence relation
(4) S is an equivalence relation but R is not an equivalence relation

Solution : (1)

$R = \{(x, y) \mid x, y \in \mathbb{R}\}$

x = wy for some rational number w.

$(x, x) \in R$ as, $x = 1 . x \Rightarrow$ reflexive as 1 is rational number.

$(x, y) \in R$ as, $x = wy$

$\Rightarrow y = \frac{1}{w}z \Rightarrow (y, x) \in R \Rightarrow$ symmetric as $\frac{1}{w}$ is also rational number.

Now, (x, y) & $(y, z) \in R$

$\Rightarrow x = w_1y$ and $y = w_2z$, where $w_1, w_2 \in Q$

$\Rightarrow$ so, $x = w_1y = w_1(w_2z)$

$\Rightarrow x = (w_1 w_2)z$

$\Rightarrow (x, z) \in R$ as $w_1 w_2 \in Q$. So, R is equivalence relation.

$$S = \left\{\left(\frac{m}{n}, \frac{p}{q}\right) \middle/ m, n, p \,\&\, q \text{ integers s.t. } n, q \neq 0 \,\&\, qm = pn\right\}$$

Let $\left(\frac{m}{n}, \frac{m}{n}\right) \in S \Rightarrow mn = mn \Rightarrow$ Reflexive

Now, let $\left(\frac{m}{n}, \frac{p}{q}\right) \in S \Rightarrow mq = np \Rightarrow np = mq$

$\Rightarrow \left(\frac{p}{q}, \frac{m}{n}\right) \in S \Rightarrow$ symmetric

Now, let $\left(\frac{m}{n}, \frac{p}{q}\right) \in S$ and $\left(\frac{p}{q}, \frac{x}{y}\right) \in S \Rightarrow mq = pn$ & $py = qx$

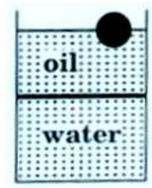
Now, $mq = \left(\frac{qx}{y}\right)n \Rightarrow my = nx \Rightarrow \left(\frac{m}{n}, \frac{x}{y}\right) \in S$

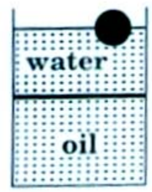
$\Rightarrow$ transitive. So, S is also equivalence relation.

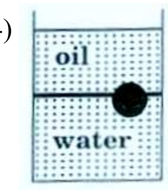
PART B : PHYSICS

31. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ?

(1) 

(2) 

(3) 

(4) 

Solution : (4)

Density of water is high so water will be at bottom. (2) is not possible as oil is less dense than ball.

Directions : Questions number 32 – 33 are based on the following paragraph.

A nucleus of mass M + Δm is at rest and decays into two daughter nuclei of equal mass M/2 each. Speed of light is c.

32. The speed of daughter nuclei is

(1) $c\sqrt{\frac{\Delta m}{M}}$ (2) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (3) $c\frac{\Delta m}{M+\Delta m}$ (4) $c\sqrt{\frac{2\Delta m}{M}}$

Solution : (1)

Total nuclear energy released = $\Delta\, mc^2$

Both daughter nuclei will have same speed (conservation of momentum)

By conservation of energy :

$$\frac{1}{2}Mv^2 + \frac{1}{2}Mv^2 = \Delta\, mc^2$$

$$\therefore \quad v = c\sqrt{\frac{\Delta m}{M}}$$

33. The binding energy per nucleon for the parent nucleus is E_1 and that for the daughter nuclei is E_2. Then

(1) $E_2 > E_1$ (2) $E_1 = 2E_2$ (3) $E_2 = 2E_1$ (4) $E_1 > E_2$

Solution : (1)

Mass number decreases and daughter nuclei are more stable :

$$\therefore E_2 > E_1$$

34. In a series LCR circuit R = 200 Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lags behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is

(1) Zero W (2) 242 W (3) 305 W (4) 210 W

Solution : (2)

Without inductance : phase difference ϕ is 30°.

$$\tan 30° = \frac{1}{\omega CR} \Rightarrow \frac{1}{\omega C} = R\tan 30°$$

without capacitance : phase difference ϕ is 30°

$$\tan 30° = \frac{\omega L}{R} \Rightarrow \omega L = R\tan 30°$$

$$\therefore \omega L = \frac{1}{\omega C} \Rightarrow \omega^2 = \frac{1}{LC}$$

Hence RLC circuit is driven at resonant frequency.

$\therefore$ Impedance Z = R and $\phi = 0$

$$P = E_{rms} \times I_{rms} \times \cos\theta = 220 \times \frac{220}{200} = 242 \text{ W.}$$

35. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_o\left(\frac{5}{4}-\frac{r}{R}\right)$ upto r = R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origin is given by

(1) $\frac{4\rho_o r}{3\varepsilon_o}\left(\frac{5}{4}-\frac{r}{R}\right)$ (2) $\frac{\rho_o r}{3\varepsilon_o}\left(\frac{5}{4}-\frac{r}{R}\right)$ (3) $\frac{4\pi\rho_o r}{3\varepsilon_o}\left(\frac{5}{3}-\frac{r}{R}\right)$ (4) $\frac{\rho_o r}{4\varepsilon_o}\left(\frac{5}{3}-\frac{r}{R}\right)$

Solution : (4)

Charge contained in sphere of radius (r < R)

$$q(r) = \int_0^r (4\pi r^2\, dr)\,\rho = 4\pi\rho_0\left[\frac{5}{4}\int_0^r r^2.dr - \frac{1}{R}\int_0^r r^3\,.dr\right]$$

$$\Rightarrow q(r) = \pi\rho_0\left[4\times\frac{5}{4}\times\frac{r^3}{3}-\frac{4}{R}.\frac{r^4}{4}\right]$$

$$\Rightarrow q(r) = \pi r^3\rho_0\left[\frac{5}{3}-\frac{r}{R}\right]$$

$$\therefore E(r) = \frac{q(r)}{4\pi\varepsilon_0 r^2} = \frac{\rho_0 r}{4\varepsilon_0}\left[\frac{5}{3}-\frac{r}{R}\right]$$

Directions : Questions number 36 – 38 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

36. The speed of light in the medium is

(1) directly proportional to the intensity I
(2) maximum on the axis of the beam
(3) minimum on the axis of the beam
(4) the same everywhere in the beam

Solution : (3)

$$\mu = \mu_0 + \mu_2 I = \frac{C_0}{C}$$

C_0 = speed of light in vacuum
C = speed of light in medium

$$\Rightarrow C = \frac{C_0}{(\mu_0+\mu_2 I)}.$$

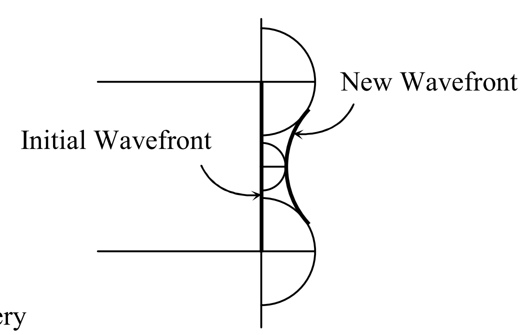
I is maximum at axis. Therefore, C is minimum at axis.

37. As the beam enters the medium, it will

(1) diverge near the axis and converge near the periphery
(2) travel as a cylindrical beam
(3) diverge
(4) converge

Solution : (4)

The axis moves with minimum speed and the speed increases as one moves out towards the periphery. Thus the beam converges.

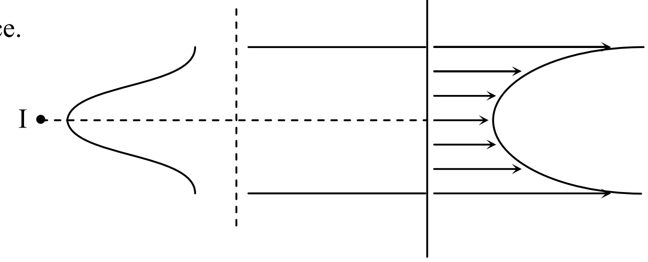


38. The initial shape of the wavefront of the beam is

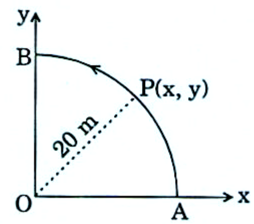
(1) convex near the axis and concave near the periphery
(2) planar
(3) convex
(4) concave

Solution : (4)

Assuming normal incidence.



39. A point P moves in counter-clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that is sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2 s is nearly



(1) 7.2 m/s^2 (2) 14 m/s^2
(3) 13 m/s^2 (4) 12 m/s^2

Solution : (2)

$s = t^3 + 5$

$v = \frac{ds}{dt} = 3t^2$ (By definition)

$a_T = \frac{dv}{dt} = 6t$ (tangential acceleration)

$a_c = \frac{v^2}{R} = \frac{9t^4}{R}$ (centripetal)

at 2 seconds :

$a_T = 12 \text{ ms}^{-2}$

$a_c = \frac{9\times16}{20} = \frac{36}{5} \approx 7\text{ms}^{-2}$

$\therefore\ a = \sqrt{\left[a_T^2 + a_c^2\right]} = \sqrt{[144+49]} = \sqrt{193} \approx 14 \text{ ms}^{-2}$

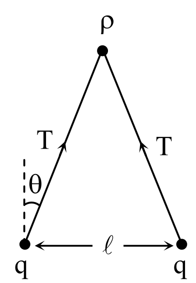
40. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is

(1) 2 (2) 1 (3) 4 (4) 3

Solution : (1)

$T\cos\theta = mg$

$T\sin\theta = \frac{Kq^2}{\ell^2} \Rightarrow \tan\theta = \frac{Kq^2}{\ell^2(mg)}$



Inside liquid ; buoyant force will act

$m = \rho_S V$ (ρ_S = density of solid)

Inside liquid ;

$m_{eff} = m - m_w$ (m_L = mass of liquid displaced)

$\therefore\ m_{eff} = (\rho_D - \rho_L)\,V$

For dielectric ; $\tan\theta = \frac{Kq^2}{\ell^2 m_{eff} g\varepsilon}$ ε = dielectric constant

$\Rightarrow \varepsilon = \left(\frac{m}{m_{eff}}\right) = \frac{1.6\times V}{(1.6-0.8)V} = 2$ [V = Volume of sphere]

41. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly

(1) $\alpha_1 + \alpha_2, \frac{\alpha_1\,\alpha_2}{\alpha_1+\alpha_2}$ (2) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$

(3) $\frac{\alpha_1+\alpha_2}{2}, \quad \alpha_1+\alpha_2$ (4) $\alpha_1+\alpha_2, \quad \frac{\alpha_1+\alpha_2}{2}$

Solution : (2)

$R_S = R_1 + R_2$

Original resistance $R_S = R_0 + R_0 = 2R_0$

$dR_S = dR_1 + dR_2 = R_0\,\alpha_1\,dT + R_0\,\alpha_2\,dT$

$= R_0\,(\alpha_1 + \alpha_2)\,dT$

$$\alpha_S = \frac{dR_S}{R_S\,dT} = \frac{\alpha_1 + \alpha_2}{2}$$

$$\frac{1}{R_P} = \frac{1}{R_1} + \frac{1}{R_2}$$

Original resistance $= \frac{R_0}{2}$

$$\frac{-dR_P}{R_P^2} = -\frac{dR_1}{R_1^2} - \frac{dR_2}{R_2^2}$$

$$\frac{dR_P}{R_P} = R_P\left[\frac{dR_1}{R_1^2} + \frac{dR_2}{R_2^2}\right] = \frac{R_0}{2}\left[\frac{dR_1/R_1}{R_1} + \frac{dR_2/R_2}{R_2}\right]$$

$$\frac{dR_1}{R_1} = \alpha_1 dT, \quad \frac{dR_2}{R_2} = \alpha_2 dT$$

$$\frac{dR_P}{R_P} = \frac{R_0}{2}\left[\frac{\alpha_1 dT}{R_0} + \frac{\alpha_2 dT}{R_0}\right]$$

$$\frac{dR_P}{R_P} = \frac{\alpha_1 + \alpha_2}{2} dT$$

$$\frac{d\,R_P}{R_P\,dT} = \alpha_P = \frac{\alpha_1 + \alpha_2}{2}$$

42. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^6}$, where a and b are constants and x is the distance between the atoms. If the dissociation energy of the molecule is $D = [U(x = \infty) - U_{\text{at equilibrium}}]$, D is

(1) $\frac{b^2}{4a}$ (2) $\frac{b^2}{6a}$ (3) $\frac{b^2}{2a}$ (4) $\frac{b^2}{12a}$

Solution : (1)

At equilibrium ; $\frac{dU}{dx} = 0$ (U is minimum)

$$\Rightarrow -\frac{12_a}{x^{13}} + \frac{6b}{x^7} = 0$$

$$\Rightarrow x_0^6 = \frac{2a}{b}$$ [x_0 is equilibrium distance].

$\therefore$ Dissociation energy $= U_\infty - U_{x_0}$

$$= 0 - \left[\frac{a}{\left(\frac{2a}{b}\right)^2} - \frac{b}{\frac{2a}{b}}\right] = \frac{b^2}{2a} - \frac{b^2}{4a} = \frac{b^2}{4a}$$

Directions : Questions number 43 – 44 contain statement – 1 and statement – 2. Of the four choices given after the statements, choose the one that best describes the two statements.

43. Statement 1 : When ultraviolet light is incident on a photocell, its stopping potential is V_0 and the maximum kinetic energy of the photoelectrons is K_{max}. When the ultraviolet light is replaced by X-rays, both V_0 and K_{max} increase.

Statement 2 : Photoelectrons are emitted with speeds ranging from zero to a maximum value because of the range of frequencies present in the incident light.

(1) Statement-1 is false, Statement-2 is true
(2) Statement-1 is true, Statement-2 is false.
(3) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** the correct explanation of Statement-1.

Solution : (2)

Statement 1 is true as stopping potential is related to the maximum kinetic energy of the ejected electrons. The kinetic energy of the ejected electron depends on the number of collisions it undergoes in the metal.

44. Statement-1 : Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

(1) Statement-1 is false, Statement-2 is true.
(2) Statement-1 is true, Statement-2 is false.
(3) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1.
(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** the correct explanation of Statement-1.

Solution : (3)

$m_1u_1 + m_2u_2 = (m_1 + m_2)v$ [By conservation of momentum].

$\therefore$ Final K.E. is not zero ($\because$ u_1 & u_2 are in same direction).

Hence statement (2) is correct explanation for statement (1).

45. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be

(1) $\frac{A-Z-12}{Z-4}$ (2) $\frac{A-Z-4}{Z-2}$ (3) $\frac{A-Z-8}{Z-4}$ (4) $\frac{A-Z-4}{Z-8}$

Solution : (4)

Mass no. of He = 4

$_{Z}X^{A} \xrightarrow{\alpha} {}_{Z-2}X^{A-4} \xrightarrow{\alpha} {}_{Z-4}H^{A-8} \longrightarrow {}_{Z-6}H^{A-12}$

To emit a positron, following conversion happens.

$p = n + e^{+}$

Mass no. does not change ; no. of protons decreases.

$_{Z-6}H^{A-12} \xrightarrow{2e^{+}} {}_{Z-8}H^{A-12}$

no. of protons = Z – 8.

no. of neutrons = A – 12 – Z + 8 = A – Z – 4

$\therefore$ Required ratio $= \frac{A-Z-4}{Z-8}$

46. If a source of power 4 kW produces 10^{20} photons/second, the radiation belongs to a part of the spectrum called

(1) microwaves (2) γ-rays (3) X-rays (4) ultraviolet rays

Solution : (4)

E = Energy per photon

$10^{20} \times E = 4 \times 10^{3}$

$E = 4 \times 10^{-17}$

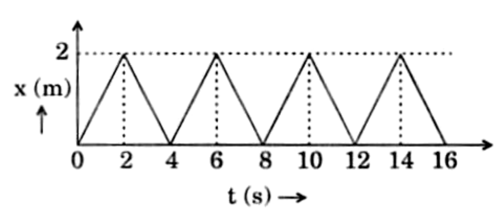
$\frac{hc}{\lambda} = 4 \times 10^{-17}$

$\lambda = \frac{6.64\times10^{-34}\times3\times10^{8}}{4\times10^{-17}} = \frac{3\times6.64}{4}\times10^{-9} = 3\times1.66\times10^{-9}$

$\lambda = 5 \times 10^{-9}$ m = 5 nm

$\therefore$ Ultraviolet

47. The figure shows the position – time (x – t) graph of one-dimensional motion of a body of mass 0.4 kg. The magnitude of each impulse is



(1) 1.6 Ns (2) 0.2 Ns
(3) 0.4 Ns (4) 0.8 Ns

Solution : (4)

The velocity undergoes abrupt change at t = 2, 4, 6 …sec

Velocity during t = 0 to t = 2 sec = $\frac{2\text{m}}{2\text{s}}$ = 1 m/s

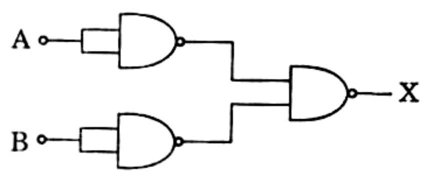
Velocity during t = 2 sec to t = 4 sec = $\frac{-2\text{m}}{2\text{s}}$ = −1m/s

Impulse = $P_f - P_i$

$= m\,[V_f - V_i]$

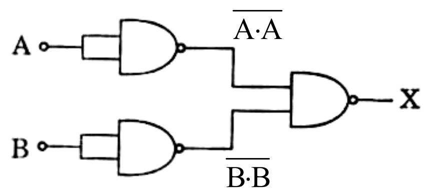
$= 0.4\,[-1 -1] = -\,0.8$

48. The combination of gates shown below yields



(1) XOR gate (2) NAND gate (3) OR gate (4) NOT gate

Solution : (3)

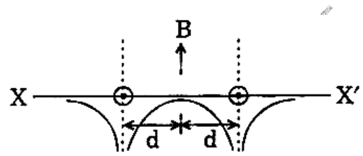


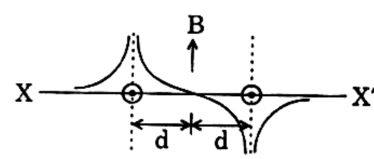
$X = \overline{\overline{A\cdot A}\cdot\overline{B\cdot B}} = \overline{\overline{A}\cdot\overline{B}}$

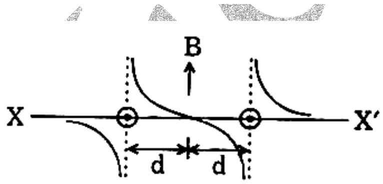
$= \overline{\overline{A}} + \overline{\overline{B}} = A + B$ [De–Morgan's Law]

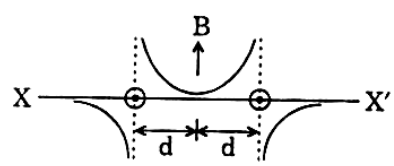
Hence OR – GATE.

49. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX′ is given by

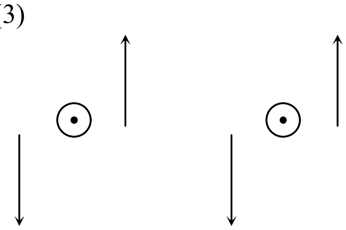
(1) 

(2) 

(3) 

(4) 

Solution : (3)



Magnetic field due to individual wires is as shown.

The influence of a wire is greatest near the wire and decreases as one moves away from it.

50. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/ t_2 will be

(1) $\frac{1}{4}$ (2) 2 (3) 1 (4) $\frac{1}{2}$

Solution : (1)

$$q = q_0 e^{-t/RC}$$

$$E = \frac{q^2}{2C}$$

$$\frac{E}{E_0} = \frac{1}{2} \quad \Rightarrow \quad \frac{q}{q_0} = \frac{1}{\sqrt{2}}$$

$$\therefore \quad \frac{1}{\sqrt{2}} = e^{-(t_1/RC)}$$

$$q = \frac{1}{4} q_0$$

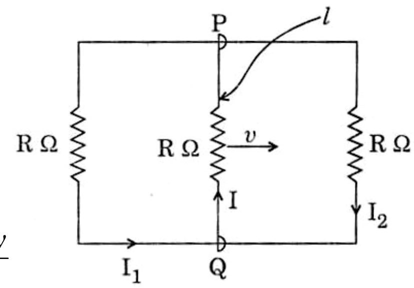
$$\frac{1}{4} = e^{-(t_2/RC)}$$

$$\left(\frac{1}{\sqrt{2}}\right)^4 = e^{(-t_2/RC)}$$

$$\left[e^{-(t_1/RC)}\right]^4 = \left[e^{-t_2/RC}\right]$$

$$t_2 = 4t_1 \quad \Rightarrow \quad \frac{t_1}{t_2} = \frac{1}{4}$$

51. A rectangular loop has a sliding connector PQ of length ℓ and resistance R Ω and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are

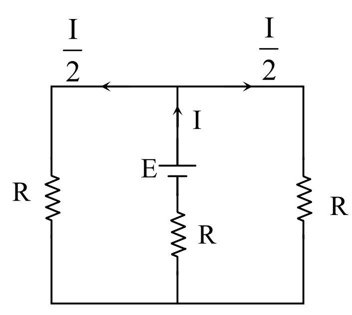


(1) $I_1 = I_2 = I = \frac{B\ell v}{R}$ (2) $I_1 = I_2 = \frac{B\ell v}{6R}$, $I = \frac{B\ell v}{3R}$

(3) $I_1 = -I_2 = \frac{B\ell v}{R}$, $I = \frac{2B\ell v}{R}$ (4) $I_1 = I_2 = \frac{B\ell v}{3R}$, $I = \frac{2B\ell v}{3R}$

Solution : (4)

An emf $f = vB\ell$ is induced in the sliding connector.
The circuit can be drawn as,



$$E - \frac{I}{2}R - IR = 0$$

$$E = \frac{3}{2}IR \quad \Rightarrow \quad I = \frac{E}{\frac{3}{2}R} = \frac{2vB\ell}{3R}$$

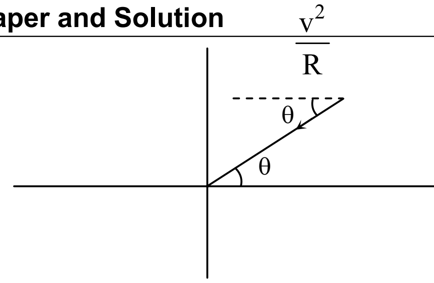
$$\therefore \quad I_1 = I_2 = \frac{I}{2} = \frac{vB\ell}{3R}$$

52. For a particle in uniform circular motion, the acceleration $\vec{a}$ at a point P(R, θ) on the circle of radius R is (Here θ is measured from the x-axis)

(1) $-\frac{v^2}{R}\cos\theta\,\hat{i} - \frac{v^2}{R}\sin\theta\,\hat{j}$ (2) $\frac{v^2}{R}\hat{i} + \frac{v^2}{R}\hat{j}$

(3) $-\frac{v^2}{R}\cos\theta\,\hat{i} + \frac{v^2}{R}\sin\theta\,\hat{j}$ (4) $-\frac{v^2}{R}\sin\theta\,\hat{i} + \frac{v^2}{R}\cos\theta\,\hat{j}$

Solution : (1)

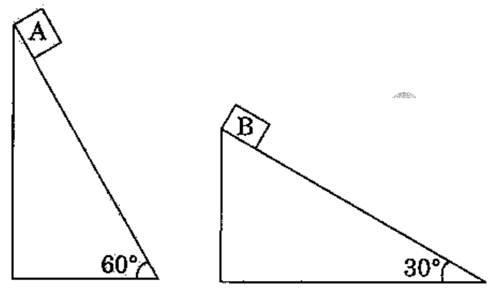


x–component : $-\dfrac{v^2}{R}\cos\theta\,\hat{i}$

y–component : $-\dfrac{v^2}{R}\sin\theta\,\hat{j}$

$\vec{v} = -\dfrac{v^2}{R}\cos\theta\,\hat{i} - \dfrac{v^2}{R}\sin\theta\,\hat{j}$

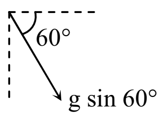
53. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?



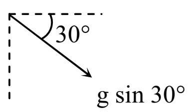
(1) Zero
(2) 4.9 ms^{-2} in vertical direction
(3) 4.9 ms^{-2} in horizontal direction
(4) 9.8 ms^{-2} in vertical direction

Solution : (2)

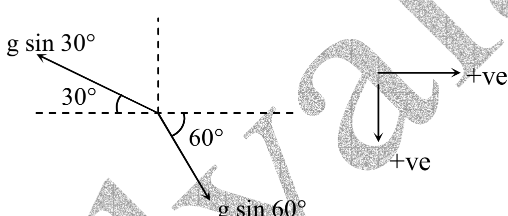
a_A =



a_B =



$a_{A,\,B} = a_A - a_B$



<u>x–component</u> :

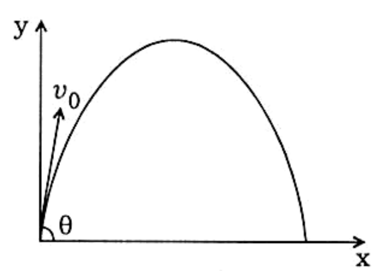
$g\sin 60^\circ\cos 60^\circ - g\sin 30^\circ\cos 30^\circ = 0$

<u>y–component</u> :

$g\sin 60^\circ\sin 60^\circ - g\sin 30^\circ\sin 30^\circ$

$= g\left[\left(\dfrac{\sqrt{3}}{2}\right)^2 - \left(\dfrac{1}{2}\right)^2\right] = g\left(\dfrac{3}{4}-\dfrac{1}{4}\right) = \dfrac{9.8}{2} = 4.9\ m/s^2$

54. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \dfrac{v_0\sin\theta}{g}$, the angular momentum of the particle is



(1) $-\dfrac{1}{2}mg v_0 t^2\cos\theta\,\hat{k}$ (2) $\dfrac{1}{2}mg v_0 t^2\cos\theta\,\hat{i}$
(3) $-mg v_0 t^2\cos\theta\,\hat{j}$ (4) $mg v_0 t\cos\theta\,\hat{k}$

where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively.

Solution : (1)

At time t,

$\vec{v} = v_x\,\hat{i} + v_y\,\hat{j} = v_0\cos\theta\,\hat{i} + (v_0\sin\theta - g\,t)\,\hat{j}$

$$\vec{r} = x\hat{i} + y\hat{j} = v_0 \cos\theta\, t\,\hat{i} + (v_0 \sin\theta\, t - \frac{1}{2} g t^2)\,\hat{j}$$

$$\vec{L} = \vec{r} \times \vec{p} = m\,\vec{r} \times \vec{v} = m\left[v_0 \cos\theta\, t \times \left(v_0 \sin\theta - g\,t\right)\hat{k} - \left(v_0 \sin\theta\, t - \frac{1}{2} g\,t^2\right) v_0 \cos\theta\,\hat{k}\right]$$

$$= mt\left[v_0^2 \sin\theta\cos\theta - v_0 \cos\theta\, g\, t - v_0^2 \sin\theta\cos\theta + \frac{1}{2} gt\, v_0 \cos\theta\right]\hat{k}$$

$$= mt\left[-\frac{1}{2} g\, t\, v_0 \cos\theta\right]\hat{k} = -\frac{1}{2} mg\, t^2\, v_0 \cos\theta\,\hat{k}$$

55. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by,

$y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is

(1) 0.5 N (2) 6.25 N (3) 4.0 N (4) 12.5 N

Solution : (2)

$$y = 0.02\,(m)\sin\left[2\pi\left[\frac{t}{0.04\,(s)} - \frac{x}{0.50\,(m)}\right]\right]$$

$$v = \sqrt{\frac{T}{\mu}} = \frac{\lambda}{T} = \frac{0.5}{0.04} = \frac{25}{2}$$

$$\frac{T}{\mu} = \frac{625}{4} \Rightarrow T = \frac{625}{4} \times 0.04 = 6.25\text{ N}$$

56. A diatomic ideal gas is used in a Carnot engine as the working substance. If during the adiabatic expansion part of the cycle the volume of the gas increases from V to 32 V, the efficiency of the engine is

(1) 0.99 (2) 0.25 (3) 0.5 (4) 0.75

Solution : (4)

$$TV^{\gamma-1} = \text{constant}$$

$$T_1 V^{\gamma-1} = T_2 (32V)^{\gamma-1},\ \gamma = \frac{7}{5}$$

$$T_1 = T_2\left(\frac{32V}{V}\right)^{\gamma-1} = T_2\,(32)^{(7/5-1)}$$

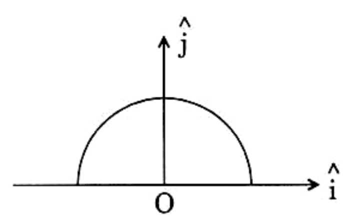
$$T_1 = T_2\ (2^5)^{2/5}$$

$$T_1 = T_2 \times 4$$

$$\frac{T_2}{T_1} = \frac{1}{4}$$

$$\therefore\ \eta = 1 - \frac{1}{4} = 0.75$$

57. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is



(1) $-\frac{q}{2\pi^2\varepsilon_o r^2}\hat{j}$ (2) $\frac{q}{2\pi^2\varepsilon_o r^2}\hat{j}$

(3) $\frac{q}{4\pi^2\varepsilon_o r^2}\hat{j}$ (4) $-\frac{q}{4\pi^2\varepsilon_o r^2}\hat{j}$

57. (1)

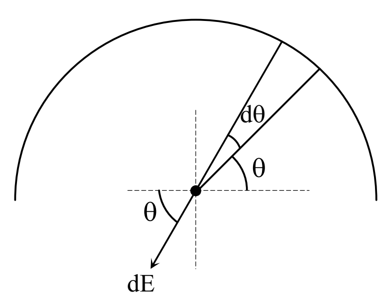
Consider a small element dθ at angle θ.

The length of the arc $d\ell = r\,d\theta$

The charge on the element $dq = \lambda\,d\ell = \frac{q}{\pi r} \times d\ell = \frac{q}{\pi} d\theta$

The electric field due to this element

$$dE = \frac{1}{4\pi\varepsilon_0}\frac{dq}{r^2} = \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi r^2}d\theta$$



The horizontal component gets cancelled.

$\therefore$ The vertical component, $dE_y = dE\sin\theta$

$$E_y = \int dE_y = \int_0^{\pi} \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi r^2}\sin\theta\, d\theta$$

$$= \frac{1}{4\pi\varepsilon_0}\frac{q}{\pi r^2}\left[-\cos\theta\right]_0^{\pi} = \frac{q}{2\pi^2\varepsilon_0 r^2} \text{ in the downward direction.}$$

58. The respective number of significant figures for the numbers 23.023, 0.0003 and 2.1×10^{-3} are

(1) 5, 5, 2 (2) 4, 4, 2 (3) 5, 1, 2 (4) 5, 1, 5

Solution : (3)

59. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is

(1) xy = constant (2) $y^2 = x^2$ + constant (3) $y = x^2$ + constant (4) $y^2 = x$ + constant

Solution : (2)

$\vec{V} = Ky\,\hat{i} + Kx\,\hat{j}$

$V_x = Ky$, $V_y = Kx$

$\frac{dx}{dt} = Ky$ ……(1)

$\frac{dy}{dt} = Kx$ ……(2)

Multiply (1) with x and multiply (2) with y

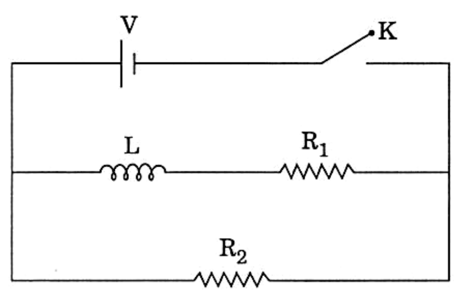
$x\frac{dx}{dt} = Kxy$ ……(3)

$y\frac{dy}{dt} = Kxy$ ……(4)

$x\frac{dx}{dt} = y\frac{dy}{dt} \Rightarrow x\,dx = y\,dy$

$\frac{x^2}{2} = \frac{y^2}{2} + C \Rightarrow x^2 = y^2 + C$ or $y^2 = x^2 + C$

60. In the circuit shown below, the key K is closed at t = 0. The current through the battery is



(1) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at $t = \infty$

(2) $\frac{V(R_1 + R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at $t = \infty$

(3) $\frac{VR_1R_2}{\sqrt{R_1^2 + R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at $t = \infty$

(4) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1 + R_2)}{R_1R_2}$ at $t = \infty$

Solution : (4)

At t = 0, current through inductor is zero

$\therefore$ Current through battery = $\frac{V}{R_2}$

At $t = \infty$, current through inductor is steady, hence the potential difference across the inductor is zero. Thus R_1 & R_2 are in parallel.

$\therefore$ Current through battery = $\frac{V}{\frac{R_1R_2}{R_1 + R_2}} = \frac{V(R_1 + R_2)}{R_1R_2}$

PART C : CHEMISTRY

61. A solution containing 2.675 g of $CoCl_3 .6\, NH_3$ (molar mass = 267.5 g mol^{-1}) is passed through a cation exchanger. The chloride ions obtained in solution were treated with excess of $AgNO_3$ to give 4.78 g of AgCl (molar mass = 143.5 g mol^{-1}). The formula of the complex is

(1) $[CoCl_3(NH_3)_3]$ (2) $[CoCl(NH_3)_5]Cl_2$ (3) $[Co(NH_3)_6]Cl_3$ (4) $[CoCl_2(NH_3)_4]Cl$

Solution :(3)

No. of moles of $CoCl_3 . 6\, NH_3$ $= \dfrac{2.675}{267.5} = 0.01$ mol.

No. of AgCl formed $= \dfrac{4.78}{143.5} = 0.033$

No. of moles of Cl outside the co−ordination sphere $= \dfrac{0.033}{0.01} = 3$

Hence, molecular formula $= [Co(NH_3)_6]Cl_3$

62. The standard enthalpy of formation of NH_3 is −46.0 kJ mol^{-1}. If the enthalpy of formation of H_2 from its atoms is −436 kJ mol^{-1} and that of N_2 is −712 kJ mol^{-1}, the average bond enthalpy of N−H bond in NH_3 is

(1) +1056 kJ mol^{-1} (2) −1102 kJ mol^{-1} (3) −964 kJ mol^{-1} (4) + 352 kJ mol^{-1}

Solution : (4)

$$N_2 + 3H_2 \rightarrow 2NH_3$$

$$\Delta H = 2\Delta H_{NH_3} - [\Delta H_{N_2} + 3\Delta H_{H_2}]$$

$$= 2x - [-712 - 3 \times (-436)] = 2 \times (-46)$$

$$= 2x + 2020 = -92$$

$$\Rightarrow 2x = 2112$$

$$\Rightarrow x = 1056$$

$$\Delta E_{N-H} = \frac{1056}{3} = 352 \text{ kJ/mol}$$

63. The time for half life period of a certain reaction A → Products is 1 hour. When the initial concentration of the reactant 'A', is 2.0 mol L^{-1}, how much time does it take for its concentration to come from 0.50 to 0.25 mol L^{-1} if it is a zero order reaction ?

(1) 0.25 h (2) 1 h (3) 4 h (4) 0.5 h

Solution : (1)

For zeroth order Rx^n,

$$x = kt$$

$$1 = k\,(1hr) \Rightarrow k = 1$$

$$(0.5 - 0.25) = 0.25 = kt = 1\,t$$

$$t = 0.25 \text{ hr}$$

64. If sodium sulphate is considered to be completely dissociated into cations and anions in aqueous solution, the change in freezing point of water (ΔT_f), when 0.01 mol of sodium sulphate is dissolved in 1 kg of water, is (K_f = 1.86 K kg mol^{-1})

(1) 0.0744 K (2) 0.0186 K (3) 0.0372 K (4) 0.0558 K

Solution : (4)

$$\Delta T_f = i\,k_f m$$

$$= 3 \times 1.86 \times 0.01 \quad [Na_2SO_4 \rightarrow 2Na^+ + SO_4^{2-}]$$

$$= 0.0558 \text{ K} \quad (i = 3)$$

65. From amongst the following alcohols the one that would react fastest with conc. HCl and anhydrous $ZnCl_2$, is

(1) 2−Methylpropanol (2) 1−Butanol (3) 2−Butanol (4) 2−Methylpropan−2−ol

Solution : (4)

2–methyl propanol $CH_3-\underset{\displaystyle CH_3}{\underset{|}{CH}}-CH_2-OH$ (1° alcohol)

1–butanol $CH_3CH_2CH_2CH_2-OH$ (1°alcohol)

2–butanol $CH_3-\underset{\displaystyle OH}{\underset{|}{CH}}-CH_2-CH_3$ (2° alcohol)

2 methyl propanol $CH_3-\overset{\displaystyle CH_3}{\overset{|}{\underset{\displaystyle CH_3}{\underset{|}{C}}}}-OH$ (3° alcohol)

Reactivity order of alcohols : 3° alcohol > 2° alcohol > 1° alcohol.

66. If $10^{-4} dm^3$ of water is introduced into a 1.0 dm^3 flask at 300 K, how many moles of water are in the vapour phase when equilibrium is established ?
(Given : Vapour pressure of H_2O at 300 K is 3170 Pa; R = 8.314 JK^{-1} mol^{-1})
(1) 4.46×10^{-2} mol (2) 1.27×10^{-3} mol (3) 5.56×10^{-3} mol (4) 1.53×10^{-2} mol

Solution : (2)

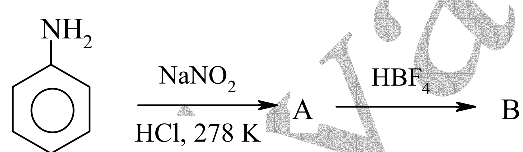
$P_{H_2O}V = n_{H_2O}RT$ | $V = 1 dm^3 = 1$ litre

$P_{H_2O} = 3170 \text{ Pa} = \frac{3170}{101325} = 0.031$ atm

$$\frac{3170}{101325} \times 1 = n_{H_2O}(0.0821)(300)$$

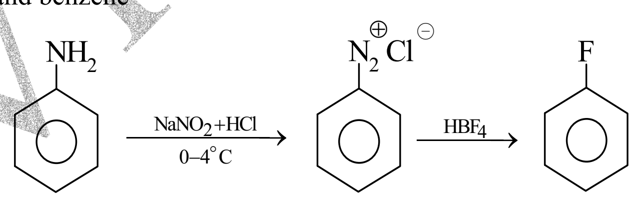
$\Rightarrow n_{H_2O} = 1.27 \times 10^{-3}$ mol

67. In the chemical reactions,



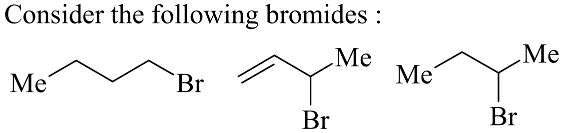
The compounds 'A' and 'B' respectively are,
(1) benzene diazonium chloride and fluorobenzene
(2) nitrobenzene and chlorobenzene
(3) nitrobenzene and fluorobenzene
(4) phenol and benzene

Solution : (1)



(diazotization Reaction) Benzene diazonium chloride Fluorobenzene

68. Consider the following bromides :



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The correct order of S_N1 reactivity is
(1) C > B > A (2) A > B > C (3) B > C > A (4) B > A > C

Solution : (3)

S_N1 Reactivity $\propto$ Stability of carbocation.

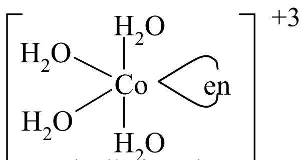
: B > C > A

69. Which one of the following has an optical isomer ?

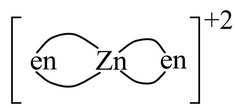
(1) $[Co(H_2O)_4(en)]^{3+}$ (2) $[Zn(en)_2]^{2+}$ (3) $[Zn(en)(NH_3)_2]^{2+}$ (4) $[Co(en)_3]^{3+}$

(en = ethylenediamine)

Solution : (4)



Optically inactive



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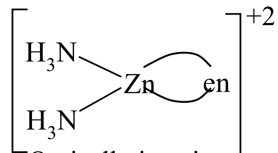

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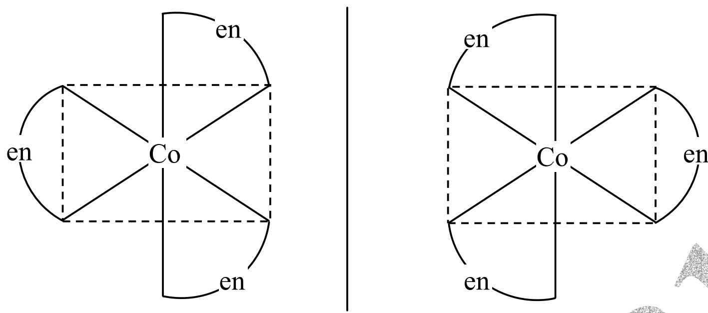


mirror

Optically Active

70. The Gibbs energy for the decomposition of Al_2O_3 at 500°C is as follows :

$$\frac{2}{3}Al_2O_3 \rightarrow \frac{4}{3}Al + O_2,\ \Delta_rG = +966 \text{ kJ mol}^{-1}.$$

The potential difference needed for electrolytic reduction of Al_2O_3 at 500°C is at least

(1) 2.5 V (2) 5.0 V (3) 4.5 V (4) 3.0 V

Solution : (1)

No. of e^- involved per mole of Al_2O_3 = 6

No. of e^- involved for $\frac{2}{3}$ mole of Al_2O_3 = $\frac{2}{3} \times 6$

= 4 mol

$$\Delta G = -nFE$$

$$E = \left|\frac{966 \times 1000}{4 \times 96500}\right| = 2.5 \text{ V}$$

71. On mixing, heptane and octane form an ideal solution. At 373 K, the vapour pressures of the two liquid components (heptane and octane) are 105 kPa and 45 kPa respectively. Vapour pressure of the solution obtained by mixing 25.0 g of heptane and 35 g of octane will be (molar mass of heptane = 100 g mol^{-1} and of octane = 114 g mol^{-1})

(1) 96.2 kPa (2) 144.5 kPa (3) 72.0 kPa (4) 36.1 kPa

Solution : (3)

$$n_A = \frac{25}{100} = 0.25$$

$$n_B = \frac{35}{114} = 0.307$$

$$X_A = \frac{0.25}{0.55} = 0.45$$

$$X_B = 0.55$$

$$
\begin{aligned}
P &= P_A^\circ x_a + P_B^\circ x_b \\
&= (105)(0.45) + (45)(0.55) \\
&= 47.72 + 24.75 \\
P &= 72.47 \text{ kPa}
\end{aligned}
$$

72. The edge length of a face centered cubic cell of an ionic substance is 508 pm. If the radius of the cation is 110 pm, the radius of the anion is

(1) 618 pm (2) 144 pm (3) 288 pm (4) 398 pm

Solution : (2)

For FCC, $r_+ + r_- = a/2$

$r_- = 254 - 110 = 144$ pm

73. The correct order of increasing basicity of the given conjugate bases ($R = CH_3$) is

(1) $RCO\bar{O} < \bar{N}H_2 < HC \equiv \bar{C} < \bar{R}$ (2) $RCO\bar{O} < HC \equiv \bar{C} < \bar{N}H_2 < \bar{R}$

(3) $RCO\bar{O} < HC \equiv \bar{C} < \bar{R} < \bar{N}H_2$ (4) $\bar{R} < HC \equiv \bar{C} < RCO\bar{O} < \bar{N}H_2$

Solution : (2)

Acidic Nature :

$RCOOH > H - C \equiv CH > NH_3 > R\text{–}H$

Basic Nature : $RCOO^- < HC \equiv C^- < NH_2^- < R^-$

74. For a particular reversible reaction at temperature T, ΔH and ΔS were found to be both +ve. If T_e is the temperature at equilibrium, the reaction would be spontaneous when

(1) T_e is 5 times T (2) $T = T_e$ (3) $T_e > T$ (4) $T > T_e$

Solution : (4)

$$T_e = \frac{\Delta H}{\Delta S} \quad \text{(At equilibrium)}$$

$$
\begin{aligned}
\Delta G &= \Delta H - T\Delta S \\
&= (+) - T(+ve)
\end{aligned}
$$

$$T > \frac{\Delta H}{\Delta S} = T_e \quad \text{(Spontaneous Reaction)}$$

75. Consider the reaction :

$Cl_2\,(aq) + H_2S\,(aq) \rightarrow S\,(s) + 2H^+\,(aq) + 2\,Cl^-\,(aq)$

The rate equation for this reaction is, rate = k $[Cl_2]\,[H_2S]$

Which of these mechanisms is/are consistent with this rate equation ?

A. $Cl_2 + H_2S \rightarrow H^+ + Cl^- + Cl^+ + HS^-$ (slow)

$Cl^+ + HS^- \rightarrow H^+ + Cl^- + S$ (fast)

B. $H_2S \Leftrightarrow H^+ + HS^-$ (fast equilibrium)

$Cl_2 + HS^- \rightarrow 2\,Cl^- + H^+ + S$ (slow)

(1) Neither A nor B (2) A only (3) B only (4) Both A and B

Solution : (2)

For mechanism A : Rate of a reaction = k $[Cl_2]\,[H_2S]$

Mechanism A is consistent with given reaction.

For mechanism B : Rate of a reaction $= k[Cl_2][HS^-]$

$$= k\,K_{eq}\frac{[Cl_2][H_2S]}{[H^+]} \quad \left(\because K_{eq} = \frac{[H^+][HS^-]}{[H_2S]}\right)$$

Hence, mechanism B is not consistent with the rate equation.

76. Percentages of free space in cubic close packed structure and in body centered packed structure are respectively.

(1) 32% and 48% (2) 48% and 26% (3) 30% and 26% (4) 26% and 32%

Solution : (4)

For BCC,	Packing fraction	=	68%,	Percentage of free space	=	32%
For CCP,	Packing fraction	=	74%,	Percentage of free space	=	26%

77. Out of the following, the alkene that exhibits optical isomerism is
(1) 3-methyl-1-pentene (2) 2-methyl-2-pentene
(3) 3-methyl-2-pentene (4) 4-methyl-1-pentene

Solution : (1)

$CH_2{=}CH{-}\underset{\displaystyle CH_3}{\underset{|}{CH}}{-}CH_2{-}CH_3$ Optically

$CH_3{-}\underset{\displaystyle CH_3}{\underset{|}{C}}{=}CH{-}CH_2{-}CH_3$ Optically Inactive

$CH_3{-}CH{=}\underset{\displaystyle CH_3}{\underset{|}{C}}{-}CH_2{-}CH_3$ Optically

$CH_2{=}CH{-}CH_2{-}\underset{\displaystyle CH_3}{\underset{|}{CH}}{-}CH_3$ Optically

78. One mole of a symmetrical alkene on ozonolysis gives two moles of an aldehyde having a molecular mass of 44 u. The alkene is
(1) 2-butene (2) ethane (3) propene (4) 1-butene

Solution : (1)

$$R-CH=CH-R \xrightarrow[Zn/H_2O]{O_3} 2RCHO$$

Molecular weight of RCHO = R + 12 + 1 + 16
= R + 29 = 44
$\Rightarrow$ R = 15
i.e., CH_3

Hence, symmetrical alkene is $CH_3-CH=CH-CH_3$ (2-butene).

79. 29.5 mg of an organic compound containing nitrogen was digested according to Kjeldahl's method and the evolved ammonia was absorbed in 20 mL of 0.1 M HCl solution. The excess of the acid required 15 mL of 0.1 M NaOH solution for complete neutralization. The percentage of nitrogen in the compound is
(1) 23.7 (2) 29.5 (3) 59.0 (4) 47.4

Solution : (1)

% of Nitrogen in the organic compound

$$= \frac{1.4\times[N_AV_A - N_BV_B]}{\omega_{O.C.}} = \frac{1.4\times[0.1\times20-0.1\times15]}{29.5\times10^{-3}}$$

$$= \frac{1.4\times[0.5]}{29.5\times10^{-3}} = 23.7\%$$

80. Ionisation energy of He^+ is 19.6×10^{-18} J atom^{-1}. The energy of the first stationary state (n = 1) of Li^{2+} is
(1) -2.2×10^{-15} J atom^{-1} (2) 8.82×10^{-17} J atom^{-1}
(3) 4.41×10^{-16} J atom^{-1} (4) -4.41×10^{-17} J atom^{-1}

Solution : (4)

$$I.E._{He^+} = \frac{E_H z^2}{n^2}$$

$$19.6\times10^{-18}\text{ J atm}^{-1} = \frac{E_H(2)^2}{1}$$

$$\frac{19.6\times10^{-18}}{4} = E_H$$

$$I.E._{Li^{+2}} = E_H\frac{(3)^2}{1}$$
$$= \left(\frac{19.6\times10^{-18}}{4}\right)(9)$$
$$= 44.1 \times 10^{-18} \text{ J/atom}$$
$$= 4.41 \times 10^{-17} \text{ J/atom}$$

81. The energy required to break one mole of Cl−Cl bonds in Cl_2 is 242 kJ mol^{-1}. The longest wavelength of light capable of breaking a single Cl−Cl bond is ($c = 3 \times 10^8$ ms^{-1} and $N_A = 6.02 \times 10^{23}$ mol^{-1})
(1) 700 nm (2) 494 nm (3) 594 nm (4) 640 nm

Solution : (2)

$$\Delta E = \frac{hc}{\lambda}$$
$$\frac{242\times10^3}{6.02\times10^{23}} = \frac{6.6\times10^{-34}\times3\times10^8}{\lambda}$$
$$\lambda = \frac{6.6\times10^{-34}\times3\times10^8\times6.02\times10^{23}}{242\times10^3}$$
$$= \left(\frac{6.6\times3\times6.02}{242}\right)\times10^{-6}$$
$$= 494 \text{ nm}$$

82. Solubility product of silver bromide is 5.0×10^{-12}. The quantity of potassium bromide (molar mass taken as 120 g mol^{-1}) to be added to 1 litre of 0.05 M solution of silver nitrate to start the precipitation of AgBr is
(1) 6.2×10^{-5} g (2) 5.0×10^{-8} g (3) 1.2×10^{-10} g (4) 1.2×10^{-9} g

Solution : (4)

$KBr + AgNO_3 \rightarrow AgBr + KNO_3$

Let 'a' mole of KBr be added into $AgNO_3$ to bring in precipitation

Thus, $[KBr] = \frac{a}{1} = a$ M, $[AgNO_3] = 0.05$ M

$[Br^-] = a$, $[Ag^+] = 0.05$

$\Rightarrow$ $[Br^-] = \frac{K_{sp}}{[Ag^+]} = \frac{5\times10^{-13}}{0.05} = 10^{-11}$

Weight of KBr $= 10^{-11} \times 120 = 1.2 \times 10^{-9}$ gm

83. The correct sequence which shows decreasing order of the ionic radii of the elements is
(1) $Na^+ > F^- > Mg^{2+} > O^{2-} > Al^{3+}$ (2) $O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$
(3) $Al^{3+} > Mg^{2+} > Na^+ > F^- > O^{2-}$ (4) $Na^+ > Mg^{2+} > Al^{3+} > O^{2-} > F^-$

Solution : (2)

Ionic Radius : Anion > Neutral > Cation

$O^{2-} > F^- > Na^+ > Mg^{2+} > Al^{3+}$

84. In aqueous solution the ionization constants for carbonic acid are
$K_1 = 4.2 \times 10^{-7}$ and $K^2 = 4.8 \times 10^{-11}$.
Select the correct statement for a saturated 0.034 M solution of the carbonic acid.
(1) The concentrations of H^+ and HCO_3^- are approximately equal.
(2) The concentration of H^+ is double that of CO_3^{2-}.
(3) The concentration of CO_3^{2-} is 0.034 M.
(4) The concentration of CO_3^{2-} is greater than that of HCO_3^-

Solution : (1)

$$[H^+] = \sqrt{K_1C}$$
$$= \sqrt{4.2\times10^{-7}\times0.034}$$
$$= \sqrt{0.1428\times10^{-7}} = \sqrt{1.428\times10^{-8}}$$
$$= 1.19 \times 10^{-4}\ M$$
$$\approx [HCO_3^-]$$
$$[CO_3^{2-}] = K_2$$
$$= 4.8 \times 10^{-11}$$

Statement (2), (3) and (4) are incorrect.

85. At 25 °C, the solubility product of $Mg(OH)_2$ is 1.0×10^{-11}. At which pH, will Mg^{2+} ions start precipitating in the form of $Mg(OH)_2$ from a solution of 0.001 M Mg^{2+} ions ?

(1) 11 (2) 8 (3) 9 (4) 10

Solution : (4)

$$K_{sp} = [Mg^{+2}]\,[OH^-]^2$$
$$[OH^-] = \sqrt{\frac{K_{sp}}{[Mg^{+2}]}}$$
$$= \sqrt{\frac{10^{-11}}{10^{-3}}} = \sqrt{10^{-8}}$$
$$= 10^{-4}\ M$$

i.e., pOH = 4

$\Rightarrow$ pH = 10

86. The polymer containing strong intermolecular forces e.g. hydrogen bonding, is

(1) polystyrene (2) natural rubber (3) Teflon (4) nylong 6, 6

Solution : (4)

Nylon 6, 6 = Fibre

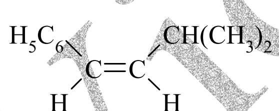
Natural Rubber = Elastomers

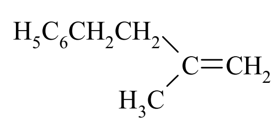
Teflon = Thermoplastics

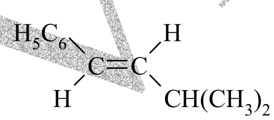
Polystyrene = Thermoplastics

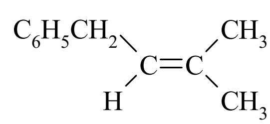
Strength : Fibre > Thermoplastics > Elastomers

87. The main product of the following reaction is : $C_6H_5CH_2CH(OH)CH(CH_3)_2 \xrightarrow{\text{conc.}H_2SO_4} ?$

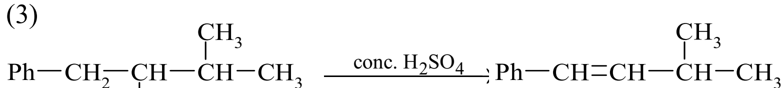
(1) 

(2) 

(3) 

(4) 

Solution : (3)



(trans – major)

88. Biuret test is ***not*** given by

(1) urea (2) proteins (3) carbohydrates (4) polypeptides

Solution : (3)

Biuret test is a chemical test for detecting the presence of peptide bond. In the presence of peptides, a copper (II) ion forms a violet – coloured complex in an alkaline solution.

89. The correct order of $E^{o}_{M^{2+}/M}$ values with negative sign for the four successive elements Cr, Mn, Fe and Co is

(1) Fe > Mn > Cr > Co
(2) Cr > Mn > Fe > Co
(3) Mn > Cr > Fe > Co
(4) Cr > Fe > Mn > Co

Solution : (3)

$E^{o}_{M^{+2}/M}$ order : Mn > Cr > Fe > Co

$E^{o}_{Mn^{+2}/Mn} = -1.18$ V

$E^{o}_{Fe^{+2}/Fe} = -0.44$ V

$E^{o}_{Co^{+2}/Co} = -0.27$ V

90. Three reactions involving $H_2PO_4^-$ are given below :

(i) $H_3PO_4 + H_2O \rightarrow H_3O^+ + H_2PO_4^-$

(ii) $H_2PO_4^- + H_2O \rightarrow HPO_4^{2-} + H_3O^+$

(iii) $H_2PO_4^- + OH^- \rightarrow H_2PO_4 + O^{2-}$

In which of the above does $H_2PO_4^-$ act as an acid ?

(1) (iii) only
(2) (i) only
(3) (ii) only
(4) (i) and (ii)

Solution : (3)

$H_2PO_4^-$ acts as base in reaction (i)

$H_2PO_4^-$ acts as acid in reaction (ii)

$H_2PO_4^-$ acts as base in reaction (iii)

❑ ❑ ❑ ❑ ❑ ❑