AIEEE 2011

Answers by

Aakash IIT-JEE

(Division of Aakash Educational Services Ltd.)

Q.N.	P	Q	R	S
01	2	1	4	1,3
02	3	1	4	1
03	1	2	2	3
04	4	3	4	1
05	3	3	1	3
06	2	1	1	3
07	1	3	3	3
08	3	4	3	1
09	4	3	3	2
10	1	1	4	1
11	1	1	1	3
12	3	1	2	2
13	4	4	2	1
14	1	3	2	2
15	3	3	2	3
16	4	3	1	3
17	3	1	4	3
18	3	1	4	1
19	3	1	2	2
20	1	2	4	4
21	4	4	3	2
22	2	1	3	1
23	3	2	3	4
24	4	1	3	4
25	4	1	3	1
26	1	3	3	1
27	2	4	3	2
28	4	1	4	2
29	1	2	2	2
30	1,3	3	1	3

Q.N.	P	Q	R	S
31	2	2	1	3
32	3	4	1	1
33	4	2	4	2
34	3	4	4	4
35	2	1	4	2
36	3	3	2	4
37	1	4	3	4
38	1	2	2	1
39	2	4	1	3
40	2	3	1	2
41	2	2	3	2
42	2	3	2	1
43	2	2	4	4
44	1	4	4	4
45	4	4	2	3
46	2	4	2,4	3
47	4	1	2	1
48	4	3	4	1
49	2	1	1	3
50	2	1	4	4
51	3	1	2	2
52	4	1	1	1
53	2	1	1	4
54	4	2	2	1
55	2	1	4	4
56	3	1	2	1
57	2	1	4	4
58	4	2	3	3
59	2	2	2	4
60	4	3	1	3

Q.N.	P	Q	R	S
61	4	2	2	4
62	1	1	3	1
63	4	4	3	2
64	2	3	1	4
65	3	2	1	3
66	4	2	2	4
67	3	4	3	2
68	3	4	4	4
69	3	4	4	3
70	1	4	1	4
71	3	2	1	1
72	2	4	4	2
73	4	4	2	2
74	2	2	3	4
75	2	2	4	1
76	1	1	1	4
77	1	3	3	4
78	1	2	3	2
79	3	4	1	4
80	1	3	1	1
81	2	3	1	3
82	3	3	3	4
83	3	1	3	2
84	2	2	3	4
85	2	2,4	1	4
86	2	3	3	2
87	4	3	4	3
88	2	3	3	2
89	1	4	3	2
90	2	2	3	4

Though every care has been taken to provide the answers correctly but the Institute shall not be responsible for typographical error, if any.

DATE : 01/05/2011

Code - R

Aakash IIT-JEE

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Time : 3 hrs. **Max. Marks: 360**

Solutions

for

AIEEE 2011

(Mathematics, Chemistry & Physics)

Important Instructions :

1. Immediately fill in the particulars on this page of the Test Booklet with Blue/Black Ball Point Pen. *Use of pencil is strictly prohibited*.

2. The Answer Sheet is kept inside this Test Booklet. When you are directed to open the Test Booklet, take out the Answer Sheet and fill in the particulars carefully.

3. The test is of **3 hours** duration.

4. The Test Booklet consists of **90** questions**.** The maximum marks are **360.**

5. There are ***three*** parts in the question paper A, B, C consisting of **Mathematics**, **Chemistry** and **Physics** having 30 questions in each part of equal weightage. Each question is allotted 4 **(four)** marks for each correct response..

6. *Candidates will be awarded marks as stated above in Instructions No. 5 for correct response of each question. ¼ (one-fourth) marks will be deducted for indicating incorrect response of each question.* ***No deduction*** *from the total score will be made* ***if no response*** *is indicated for an item in the answer sheet.*

7. There is only one correct response for each question. Filling up more than one response in each question will be treated as wrong response and marks for wrong response will be deducted accordingly as per instruction 6 above.

8. No candidate is allowed to carry any textual material, printed or written, bits of papers, pager, mobile phone, any electronic device, etc. except the Admit Card inside the examination hall/room.

9. On completion of the test, the candidate must hand over the Answer Sheet to the Invigilator on duty in the Room/Hall. However the candidates are allowed to take away this Test Booklet with them.

10. The **CODE** for this Booklet is **R.** Make sure that the **CODE** printed on Side-2 of the Answer Sheet is the same as that on this booklet. In case of discrepancy, the candidate should immediately report the matter to the Invigilator for replacement of both the Test Booklet and the Answer Sheet

11. Do not fold or make any stray marks on the Answer Sheet.

PART–A : MATHEMATICS

1. Consider 5 independent Bernoulli's trials each with probability of success p. If the probability of at least one failure is greater than or equal to $\frac{31}{32}$, then p lies in the interval

(1) $\left(\frac{11}{12}, 1\right]$ (2) $\left(\frac{1}{2}, \frac{3}{4}\right]$

(3) $\left(\frac{3}{4}, \frac{11}{12}\right]$ (4) $\left[0, \frac{1}{2}\right]$

Ans. (4)

Sol. Probability of atleast one failure

$$=1-no\ \ failure \geq \frac{31}{32}$$

$$\Rightarrow 1-p^5 \geq \frac{31}{32}$$

$$\Rightarrow p^5 \leq \frac{1}{32}$$

$$\Rightarrow p \leq \frac{1}{2}$$

Also $p \geq 0$

Hence $p \in \left[0, \frac{1}{2}\right]$

2. The coefficient of x^7 in the expansion of $(1 - x - x^2 + x^3)^6$ is

(1) 132 (2) 144

(3) –132 (4) –144

Ans. (4)

Sol. We have $(1-x-x^2+x^3)^6 = (1-x)^6(1-x^2)^6$

coefficient of x^7 in

$$\left(1-x-x^2+x^3\right)^6 = {}^6C_1.{}^6C_3 - {}^6C_3.{}^6C_2 + {}^6C_5.{}^6C_1$$

$$= 6 \times 20 - 20 \times 15 + 6 \times 6$$

$$= -144$$

3. $\lim\limits_{x \to 2}\left(\dfrac{\sqrt{1-\cos\{2(x-2)\}}}{x-2}\right)$

(1) Equals $\frac{1}{\sqrt{2}}$ (2) Does not exist

(3) Equals $\sqrt{2}$ (4) Equals $-\sqrt{2}$

Ans. (2)

Sol. $\lim\limits_{x \to 2}\left(\dfrac{\sqrt{1-\cos 2(x-2)}}{x-2}\right)$

$$= \lim_{x \to 2} \frac{\sqrt{2}\,|\sin(x-2)|}{x-2}$$

which doesn't exist as L.H.L. $= -\sqrt{2}$ whereas

R.H.L. $= \sqrt{2}$

4. Let R be the set of real numbers.

Statement-1 :

$A = \{(x, y) \in R \times R : y - x$ is an integer$\}$ is an equivalence relation on R.

Statement-2 :

$B = \{(x, y) \in R \times R : x = \alpha y$ for some rational number $\alpha\}$ is an equivalence relation on R.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is false

Ans. (4)

Sol. Statement-1 is true

We observe that

Reflexivity

xRx as $x - x = 0$ is an integer, $\forall x \in A$

Symmetric

Let $(x, y) \in A$

$\Rightarrow y - x$ is an integer

$\Rightarrow x - y$ is also an integer

Transitivity

Let $(x, y) \in A$ and $(y, z) \in A$

$\Rightarrow y - x$ is an integer and $z - y$ is an integer

$\Rightarrow y - x + z - y$ is also an integer

$\Rightarrow z - x$ is an integer

$\Rightarrow (x,z) \in A$

Because of the above properties A is an equivalence relation over R

Statement 2 is false as 'B' is not symmetric on $\mathbb{R}$

We observe that

$0Bx$ as $0 = 0.x \forall x \in \mathbb{R}$ but $(x,0) \notin B$

5. Let α, β be real and z be a complex number. If $z^2 + \alpha z + \beta = 0$ has two distinct roots on the line Re z = 1, then it is necessary that

(1) $\beta \in (1, \infty)$ (2) $\beta \in (0, 1)$

(3) $\beta \in (-1, 0)$ (4) $|\beta| = 1$

Ans. (1)

Sol. Let the roots of the given equation be $1 + ip$ and $1 - ip$, where $p \in \mathbb{R}$

$\Rightarrow \beta = $ product of roots

$= (1+ip)(1-ip) = 1 + p^2 > 1, \forall p \in \mathbb{R}$

$\Rightarrow \beta \in (1, \infty)$

6. $\dfrac{d^2x}{dy^2}$ equals

(1) $-\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-3}$ (2) $\left(\dfrac{d^2y}{dx^2}\right)^{-1}$

(3) $-\left(\dfrac{d^2y}{dx^2}\right)^{-1}\left(\dfrac{dy}{dx}\right)^{-3}$ (4) $\left(\dfrac{d^2y}{dx^2}\right)\left(\dfrac{dy}{dx}\right)^{-2}$

Ans. (1)

Sol. We have

$$\frac{d^2x}{dy^2} = \frac{d}{dy}\left(\frac{dx}{dy}\right) = \frac{d}{dy}\left(\frac{1}{\frac{dy}{dx}}\right)$$

$$= \frac{d}{dx}\left(\frac{1}{\frac{dy}{dx}}\right).\frac{dx}{dy} = -\frac{1}{\left(\frac{dy}{dx}\right)^2}.\frac{d^2y}{dx^2}.\frac{1}{\left(\frac{dy}{dx}\right)}$$

$$= -\frac{1}{\left(\frac{dy}{dx}\right)^3}.\frac{d^2y}{dx^2} = -\left(\frac{dy}{dx}\right)^{-3}\left(\frac{d^2y}{dx^2}\right)$$

7. The number of values of k for which the linear equations

$4x + ky + 2z = 0$

$kx + 4y + z = 0$

$2x + 2y + z = 0$

possess a non-zero solution is

(1) Zero (2) 3

(3) 2 (4) 1

Ans. (3)

Sol. For non-trivial solution of given system of linear equations

$$\begin{vmatrix} 4 & k & 2 \\ k & 4 & 1 \\ 2 & 2 & 1 \end{vmatrix} = 0$$

$\Rightarrow 8 + k(2-k) + 2(2k-8) = 0$

$\Rightarrow -k^2 + 6k - 8 = 0$

$\Rightarrow k^2 - 6k + 8 = 0$

$\Rightarrow k = 2, 4$

Clearly there exists two values of k.

8. **Statement-1 :**

The point $A(1, 0, 7)$ is the mirror image of the point $B(1, 6, 3)$ in the line : $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$.

Statement-2 :

The line : $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$ bisects the line segment joining $A(1, 0, 7)$ and $B(1, 6, 3)$.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is false

Ans. (3)

Sol. The mid point of $A(1, 0, 7)$ and $B(1,6,3)$ is $(1,3,5)$

which lies on the line $\dfrac{x}{1} = \dfrac{y-1}{2} = \dfrac{z-2}{3}$

Also the line passing through the points A and B is perpendicular to the given line, hence B is the mirror image of A, consequently the statement-1 is true.

Statement-2 is also true but it is not a correct explanation of statement-1 as there are infinitely many lines passing through the midpoint of the line segment and one of the lines is perpendicular bisector.

9. Consider the following statements

P : Suman is brilliant

Q : Suman is rich

R : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as

(1) $\sim (P \wedge \sim R) \leftrightarrow Q$ (2) $\sim P \wedge (Q \leftrightarrow \sim R)$

(3) $\sim (Q \leftrightarrow (P \wedge \sim R))$ (4) $\sim Q \leftrightarrow \sim P \wedge R$

Ans. (3)

Sol. Suman is brilliant and dishonest is $P \wedge \sim R$

Suman is brilliant and dishonest iff suman is rich is $Q \leftrightarrow (P \wedge \sim R)$

Negative of statement is expressed as

$\sim (Q \leftrightarrow (P \wedge \sim R))$

10. The lines $L_1 : y - x = 0$ and $L_2 : 2x + y = 0$ intersect the line $L_3 : y + 2 = 0$ at P and Q respectively. The bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statements 1 : The ratio $PR : RQ$ equals $2\sqrt{2} : \sqrt{5}$.

Statement 2 : In any traingle, bisector of an angle divides the triangle into two similar triangles.

(1) Statement-1 is true, Statement-2 is false

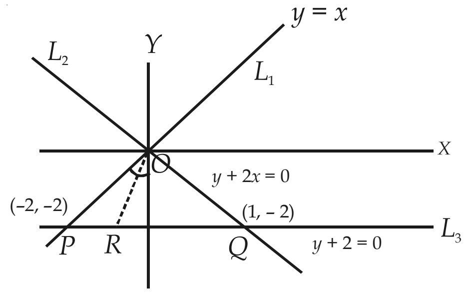
(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation of Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is the *not* the correct explanation of Statement-1

(4) Statement-1 is false, Statement-2 is true

Ans. (4)

Sol. The figure is self explanatory



$$\frac{PR}{RQ} = \frac{OP}{OQ} = \frac{2\sqrt{2}}{\sqrt{5}}$$

Statement-2 is false

11. A man saves Rs. 200 in each of the first three months of his service. In each of the subsequent months his saving increases by Rs. 40 more than the saving of immediately previous month. His total saving from the start of service will be Rs. 11040 after

(1) 21 months (2) 18 months

(3) 19 months (4) 20 months

Ans. (1)

Sol. Total savings = 200 + 200 + 200 + 240 + 280 + ... to n months = 11040

$$\Rightarrow 400 + \frac{n-2}{2}\left(400 + (n-3).40\right) = 11040$$

$\Rightarrow (n - 2)(140 + 20n) = 10640$

$\Rightarrow 20n^2 + 100n - 280 = 10640$

$\Rightarrow n^2 + 5n - 546 = 0$

$\Rightarrow (n - 21)(n + 26) = 0$

$\Rightarrow n = 21$ as $n \neq -26$

12. Equation of the ellipse whose axes are the axes of coordinates and which passes through the point (– 3, 1) and has eccentricity $\sqrt{\frac{2}{5}}$ is

(1) $5x^2 + 3y^2 - 32 = 0$

(2) $3x^2 + 5y^2 - 32 = 0$

(3) $5x^2 + 3y^2 - 48 = 0$

(4) $3x^2 + 5y^2 - 15 = 0$

Ans. (2)

Sol. Let the equation of the ellipse be $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$

Which will pass through (– 3, 1) if $\frac{9}{a^2} + \frac{1}{b^2} = 1$

and eccentricity = $e = \sqrt{1 - \frac{b^2}{a^2}} = \sqrt{\frac{2}{5}}$

$$\Rightarrow \frac{2}{5} = 1 - \frac{b^2}{a^2}$$

$$\Rightarrow \frac{b^2}{a^2} = \frac{3}{5}$$

$$\Rightarrow b^2 = \frac{3}{5}a^2$$

Thus $\frac{9}{a^2} + \frac{1}{b^2} = 1$

$\frac{9}{a^2} + \frac{5}{3a^2} = 1$

$\Rightarrow$ $27 + 5 = 3a^2 = 32$

$\Rightarrow$ $a^2 = \frac{32}{3},\ b^2 = \frac{3}{5} \times \frac{32}{3} = \frac{32}{5}$

Required equation of the ellipse is $3x^2 + 5y^2 = 32$

13. If $A = \sin^2 x + \cos^4 x$, then for all real x

(1) $\frac{3}{4} \le A \le \frac{13}{16}$ (2) $\frac{3}{4} \le A \le 1$

(3) $\frac{13}{16} \le A \le 1$ (4) $1 \le A \le 2$

Ans. (2)

Sol. We have,

$A = \sin^2 x + \cos^4 x = 1 - \cos^2 x + \cos^4 x$

$= 1 + \left(\cos^2 x - \frac{1}{2}\right)^2 - \frac{1}{4}$

$= \frac{3}{4} + \left(\cos^2 x - \frac{1}{2}\right)^2 \ge \frac{3}{4}$

Clearly $\frac{3}{4} \le A \le 1$

14. The value of $\int_0^1 \frac{8\log(1+x)}{1+x^2}dx$ is

(1) log 2 (2) π log2

(3) $\frac{\pi}{8}\log 2$ (4) $\frac{\pi}{2}\log 2$

Ans. (2)

Sol. We have,

$I = \int_0^1 \frac{8\log(1+x)}{1+x^2}dx$

Put $x = \tan\theta$

$\Rightarrow$ $I = \int_0^{\frac{\pi}{4}} 8.\frac{\log(1+\tan\theta)}{\sec^2\theta}\sec^2\theta\, d\theta$

$= 8\int_0^{\pi/4} \log(1+\tan\theta)d\theta$

$= 8\int_0^{\pi/4} \log\left(1+\frac{1-\tan\theta}{1+\tan\theta}\right)d\theta$

$= 8\int_0^{\pi/4} \log\left(\frac{2}{1+\tan\theta}\right)d\theta$

$= 8(\log 2).\frac{\pi}{4} - I$

$\Rightarrow$ $2I = 2\pi\log 2$

$\Rightarrow$ $I = \pi\log 2$

15. If the angle between the line $x = \frac{y-1}{2} = \frac{z-3}{\lambda}$ and the plane $x + 2y + 3z = 4$ is $\cos^{-1}\left(\sqrt{\frac{5}{14}}\right)$, then λ equals

(1) $\frac{5}{3}$ (2) $\frac{2}{3}$

(3) $\frac{3}{2}$ (4) $\frac{2}{5}$

Ans. (2)

Sol. The direction ratios of the given line

$\frac{x}{1} = \frac{y-1}{2} = \frac{z-3}{\lambda}$ are

1, 2, λ

Hence direction cosines of the line are

$\frac{1}{\sqrt{5+\lambda^2}}, \frac{2}{\sqrt{5+\lambda^2}}, \frac{\lambda}{\sqrt{5+\lambda^2}}$

Also the direction cosines of the normal to the plane are $\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}}$.

Angle between the line and the plane is θ,

then $\cos(90° - \theta) = \frac{1+4+3\lambda}{\sqrt{(5+\lambda^2)}\sqrt{14}} = \sin\theta$

$\Rightarrow$ $\frac{3}{\sqrt{14}} = \frac{5+3\lambda}{\sqrt{\lambda^2+5}\sqrt{14}}$

$\Rightarrow$ $9\lambda^2 + 45 = 9\lambda^2 + 30\lambda + 25$

$\Rightarrow$ $30\lambda = 20$

$\lambda = \frac{2}{3}$

16. For $x \in \left(0, \frac{5\pi}{2}\right)$, define

$$f(x) = \int_0^x \sqrt{t}\ \sin t\, dt$$

Then f has

(1) Local maximum at π and local 2π

(2) Local maximum at π and 2π

(3) Local minimum at π and 2π

(4) Local minimum at π and local maximum at 2π

Ans. (1)

Sol. We have,

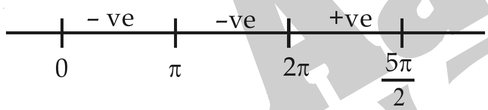
$$f(x) = \int_0^x \sqrt{t}\ \sin t\ dt$$

$\Rightarrow$ $f'(x) = \sqrt{x}\sin x$

For maximum or minimum value of $f(x)$, $f'(x) = 0$

$\Rightarrow$ $x = n\pi$, $n \in Z$

We observe that



$f'(x)$ changes its sign from +ve to – ve in the neighbourhood of π and – ve to +ve in the neighbourhood of 2π.

Hence $f(x)$ has local maximum at $x = \pi$ and local minima at $x = 2\pi$

17. The domain of the function $f(x) = \frac{1}{\sqrt{|x| - x}}$ is

(1) $(-\infty, \infty) - \{0\}$ (2) $(-\infty, \infty)$

(3) $(0, \infty)$ (4) $(-\infty, 0)$

Ans. (4)

Sol. The given function f is well defined only

when $|x| - x > 0$

$\Rightarrow$ $x < 0$

Required domain is $(-\infty, 0)$

18. If the mean deviation about the median of the numbers $a, 2a, \ldots\ldots, 50a$ is 50, then $|a|$ equals

(1) 5 (2) 2

(3) 3 (4) 4

Ans. (4)

Sol. From the given data, median $= \frac{25 + 26}{2}a = 25.5a$

Required mean deviation about median

$$= \frac{2\,|0.5 + 1.5 + 2.5 + \ldots + 24.5|}{50}|a| = 50$$

$\Rightarrow$ $|a| = 4$

19. If $\vec{a} = \frac{1}{\sqrt{10}}(3\hat{i} + \hat{k})$ and $\vec{b} = \frac{1}{7}(2\hat{i} + 3\hat{j} - 6\hat{k})$, then the value of $(2\vec{a} - \vec{b})\cdot[(\vec{a} \times \vec{b}) \times (\vec{a} + 2\vec{b})]$ is

(1) 3 (2) –5

(3) –3 (4) 5

Ans. (2)

Sol. We have

$(2\vec{a} - \vec{b}).\left[\left(\vec{a} \times \vec{b}\right) \times \left(\vec{a} + 2\vec{b}\right)\right]$

$= \left(2\vec{a} - \vec{b}\right).\left(\vec{b} - 2\vec{a}\right)$

$= -\left(2\vec{a} - \vec{b}\right)^2$

$= -\left[4|\vec{a}|^2 + |\vec{b}|^2 - 4\vec{a}.\vec{b}\right]$

$= -[4 + 1] = -5$

20. The values of p and q for which the function

$$f(x) = \begin{cases} \dfrac{\sin(p+1)x + \sin x}{x}, & x < 0 \\ q, & x = 0 \\ \dfrac{\sqrt{x + x^2} - \sqrt{x}}{x^{3/2}}, & x > 0 \end{cases}$$

is continuous for all x in R, are

(1) $p = \frac{1}{2}, q = \frac{3}{2}$ (2) $p = \frac{1}{2}, q = -\frac{3}{2}$

(3) $p = \frac{5}{2}, q = \frac{1}{2}$ (4) $p = -\frac{3}{2}, q = \frac{1}{2}$

Ans. (4)

Sol. The given function f is continuous at $x = 0$ if

$\lim_{h \to 0} f(0 - h) = f(0) = \lim_{h \to 0} f(0 + h)$

$\Rightarrow$ $p + 2 = q = \frac{1}{2}$

$\Rightarrow$ $p = -\frac{3}{2}, q = \frac{1}{2}$

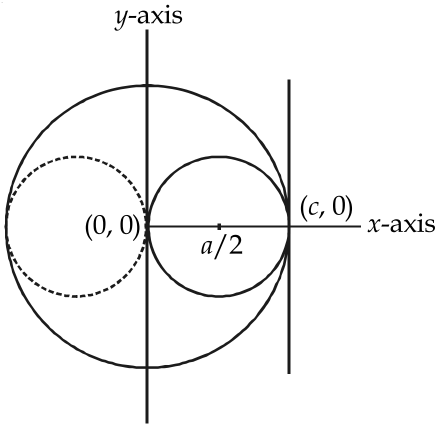
21. The two circles $x^2 + y^2 = ax$ and $x^2 + y^2 = c^2$ $(c > 0)$ touch each other, if

(1) $|a| = 2c$ (2) $2|a| = c$

(3) $|a| = c$ (4) $a = 2c$

Ans. (3)

Sol.



The figure is self explanatory

Clearly $c = |a|$

22. Let I be the purchase value of an equipment and $V(t)$ be the value after it has been used for t years. The value $V(t)$ depreciates at a rate given by differential equation $\frac{dV(t)}{dt} = -k(T-t)$, where $k > 0$ is a constant and T is the total life in years of the equipment. Then the scrap value $V(T)$ of the equipment is

(1) e^{-kT} (2) $T^2 - \frac{I}{k}$

(3) $I - \frac{kT^2}{2}$ (4) $I - \frac{k(T-t)^2}{2}$

Ans. (3)

Sol. $\int_{I}^{V(T)} dV(t) = \int_{t=0}^{T} -k(T-t)dt$

$\Rightarrow V(T) - I = k\left[\frac{(T-t)^2}{2}\right]_0^T$

$\Rightarrow V(T) - I = -k\left[\frac{T^2}{2}\right]$

$\Rightarrow V(T) = I - \frac{kT^2}{2}$

23. If C and D are two events such that $C \subset D$ and $P(D) \neq 0$, then the correct statement among the following is

(1) $P(C|D) = \frac{P(D)}{P(C)}$ (2) $P(C|D) = P(C)$

(3) $P(C|D) \geq P(C)$ (4) $P(C|D) < P(C)$

Ans. (3)

Sol. We have

$C \subset D$

$\Rightarrow C \cap D = C$

$\Rightarrow P\left(\frac{C}{D}\right) = \frac{P(C \cap D)}{P(D)} = \frac{P(C)}{P(D)} \geq P(C)$

as $0 < P(D) \leq 1$

24. Let A and B be two symmetric matrices of order 3.

Statement-1 : $A(BA)$ and $(AB)A$ are symmetric matrices.

Statement-2 : AB is symmetric matrix if matrix multiplication of A with B is commutative.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is false

Ans. (3)

Sol. Clearly both statements are true but statement-2 is not a correct explanation of statement-1.

25. If $\omega(\neq 1)$ is a cube root of unity, and $(1 + \omega)^7 = A + B\omega$. Then (A, B) equals

(1) (–1, 1) (2) (0, 1)

(3) (1, 1) (4) (1, 0)

Ans. (3)

Sol. $(1 + \omega)^7 = A + B\omega$

$\Rightarrow (-\omega^2)^7 = A + B\omega$

$\Rightarrow -\omega^2 = A + B\omega$

$\Rightarrow 1 + \omega = A + B\omega$

$\Rightarrow A = 1, B = 1$

$\Rightarrow (A, B) = (1, 1)$

26. **Statement-1 :** The number of ways of distributing 10 identical balls in 4 distinct boxes such that no box is empty is ${}^9C_3$.

Statement-2 : The number of ways of choosing any 3 places from 9 different places is ${}^9C_3$.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is true; Statement-2 is the correct explanation for Statement-1

(3) Statement-1 is true, Statement-2 is true; Statement-2 is ***not*** a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is false

Ans. (3)

Sol. Number of ways to distribute 10 identical balls in four distinct boxes such that no box remains empty $= {}^{10-1}C_{4-1} = {}^9C_3$

Number of ways to select 3 different places from 9 places $= {}^9C_3$

Clearly statement-2 is not a correct explanation of statement-1

27. The shortest distance between line $y - x = 1$ and curve $x = y^2$ is

(1) $\frac{4}{\sqrt{3}}$ (2) $\frac{\sqrt{3}}{4}$

(3) $\frac{3\sqrt{2}}{8}$ (4) $\frac{8}{3\sqrt{2}}$

Ans. (3)

Sol. The equation of the tangent to $x = y^2$ having slope 1 is $y = x + \frac{1}{4}$

Hence shortest distance $= \left|\frac{1-\frac{1}{4}}{\sqrt{2}}\right|$

$= \frac{3}{4\sqrt{2}} = \frac{3\sqrt{2}}{8}$ units

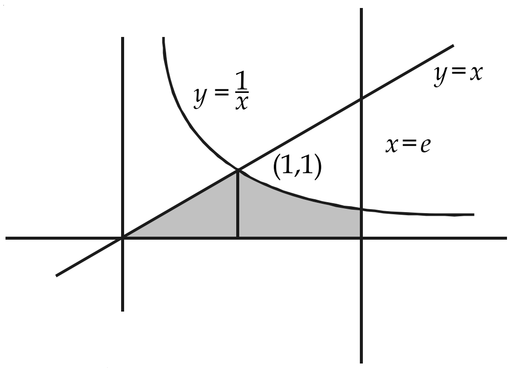
28. The area of the region enclosed by the curves $y = x$, $x = e$, $y = \frac{1}{x}$ and the positive x-axis is

(1) $\frac{5}{2}$ square units (2) $\frac{1}{2}$ square units

(3) 1 square units (4) $\frac{3}{2}$ square units

Ans. (4)

Sol.



Required area

$= \frac{1}{2}\times1\times1+\int_1^e \frac{1}{x}dx$

$= \frac{1}{2}+[\ln x]_1^e$

$= \frac{1}{2}+1-0=\frac{3}{2}$ sq. units

29. If $\frac{dy}{dx} = y+3>0$ and $y(0) = 2$, then $y(\ln 2)$ is equal to

(1) –2 (2) 7

(3) 5 (4) 13

Ans. (2)

Sol. We have

$\frac{dy}{dx} = y+3$

$\Rightarrow \frac{1}{y+3}dy = dx$

$\Rightarrow$ ln $|(y + 3)| = x + k$, where k is a constant of integration

$\Rightarrow$ $(y + 3) = c\, e^x$

Initially when $x = 0$, $y = 2$

$\Rightarrow$ $c = 5$

Finally the required solution is $y + 3 = 5e^x$

$\Rightarrow$ y (ln2) $= 5e^{\ln 2} - 3 = 10 - 3 = 7$

30. The vectors $\vec{a}$ and $\vec{b}$ are not perpendicular and $\vec{c}$ and $\vec{d}$ are two vectors satisfying $\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$ and $\vec{a}\cdot\vec{d} = 0$. Then the vector $\vec{d}$ is equal to

(1) $\vec{c}-\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ (2) $\vec{b}-\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

(3) $\vec{c}+\left(\frac{\vec{a}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{b}$ (4) $\vec{b}+\left(\frac{\vec{b}\cdot\vec{c}}{\vec{a}\cdot\vec{b}}\right)\vec{c}$

Ans. (1)

Sol. We have

$\vec{b}\times\vec{c} = \vec{b}\times\vec{d}$

$\Rightarrow \vec{a}\times(\vec{b}\times\vec{c}) = \vec{a}\times(\vec{b}\times\vec{d})$

$\Rightarrow (\vec{a}\cdot\vec{c})\vec{b}-(\vec{a}\cdot\vec{b})\vec{c} = (\vec{a}\cdot\vec{d})\vec{b}-(\vec{a}\cdot\vec{b})\vec{d}$

$\Rightarrow (\vec{a}\cdot\vec{b})\vec{d} = -(\vec{a}\cdot\vec{c})\vec{b}+(\vec{a}\cdot\vec{b})\vec{c}$

$\Rightarrow \vec{d} = -\frac{(\vec{a}\cdot\vec{c})}{(\vec{a}\cdot\vec{b})}\vec{b}+\vec{c}$

PART–B : CHEMISTRY

31. In context of the lanthanoids, which of the following statements is **not** correct?

(1) Availability of 4f electrons results in the formation of compounds in +4 state for all the members of the series

(2) There is a gradual decrease in the radii of the members with increasing atomic number in the series

(3) All the members exhibit +3 oxidation state

(4) Because of similar properties the separation of lanthanoids is not easy

Ans. (1)

Sol. Lanthanoid generally show the oxidation state of +3.

32. In a face centred cubic lattice, atom A occupies the corner positions and atom B occupies the face centre positions. If one atom of B is missing from one of the face centred points, the formula of the compound is

(1) A_2B_5 (2) A_2B

(3) AB_2 (4) A_2B_3

Ans. (1)

Sol. $Z_A = \frac{8}{8}$

$Z_B = \frac{5}{2}$

So formula of compound is $AB_{5/2}$

i.e., A_2B_5

33. The magnetic moment (spin only) of $[NiCl_4]^{2-}$ is

(1) 1.41 BM (2) 1.82 BM

(3) 5.46 BM (4) 2.82 BM

Ans. (4)

Sol. Hybridisation of Ni is sp^3

Unpaired e^- in $3d^8$ is 2

So $\mu = \sqrt{n(n+2)}$ BM

$= \sqrt{2\times 4} = \sqrt{8} = 2.82$ BM

34. Which of the following facts about the complex $[Cr(NH_3)_6]Cl_3$ is **wrong**?

(1) The complex gives white precipitate with silver nitrate solution

(2) The complex involves d^2sp^3 hybridisation and is octahedral in shape

(3) The complex is paramagnetic

(4) The complex is an outer orbital complex

Ans. (4)

Sol. Cr^{+3} in octahedral geometry always form inner orbital complex.

35. The rate of a chemical reaction doubles for every 10°C rise of temperature. If the temperature is raised by 50°C, the rate of the reaction increases by about

(1) 64 times (2) 10 times

(3) 24 times (4) 32 times

Ans. (4)

Sol. Temperature coefficient $(\theta) = \frac{\text{rate at } (t+10)°C}{\text{rate at } t°C}$

$\theta = 2$ here, so increase in rate of reaction $= (\theta)^n$

When n is number of times by which temperature is raised by 10°C.

36. 'a' and 'b' are van der Waals' constants for gases. Chlorine is more easily liquefied than ethane because

(1) a for Cl_2 > a for C_2H_6 but b for Cl_2 < b for C_2H_6

(2) a and b for Cl_2 > a and b for C_2H_6

(3) a and b for Cl_2 < a and b for C_2H_6

(4) a for Cl_2 < a for C_2H_6 but b for Cl_2 > b for C_2H_6

Ans. (2)

Sol. Both a and b for Cl is more than C_2H_6.

37. The hybridisation of orbitals of N atom in NO_3^-, NO_2^+ and NH_4^+ are respectively

(1) sp^2, sp^3, sp (2) sp, sp^2, sp^3

(3) sp^2, sp, sp^3 (4) sp, sp^3, sp^2

Ans. (3)

Sol. $NO_3^- - sp^2$; $NO_2^+ - sp$; $NH_4^+ - sp^3$

38. Ethylene glycol is used as an antifreeze in a cold climate. Mass of ethylene glycol which should be added to 4 kg of water to prevent it from freezing at –6°C will be: (K_f for water = 1.86 K kgmol^{-1} and molar mass of ethylene glycol = 62gmol^{-1})

(1) 304.60 g (2) 804.32 g

(3) 204.30 g (4) 400.00 g

Ans. (2)

Sol. $\Delta T_f = K_f.m$ for ethylenglycol in aq. solution

$$\Delta T_f = K_f \frac{w}{\text{mol. wt.}} \times \frac{1000}{\text{wt. of solvent}}$$

$$6 = \frac{1.86 \times w \times 1000}{62 \times 4000}$$

$\Rightarrow$ w = 800 g

So, weight of solute should be more than 800 g.

39. The outer electron configuration of Gd (Atomic No.: 64) is

(1) $4f^7\,5d^1\,6s^2$ (2) $4f^3\,5d^5\,6s^2$

(3) $4f^8\,5d^0\,6s^2$ (4) $4f^4\,5d^4\,6s^2$

Ans. (1)

Sol. Gd = $4f^{7}5d^{1}6s^{2}$

40. The structure of IF_7 is

(1) Pentagonal bipyramid

(2) Square pyramid

(3) Trigonal bipyramid

(4) Octahedral

Ans. (1)

Sol. Hybridisation of iodine is sp^3d^3

So, structure is pentagonal bipyramid.

41. Ozonolysis of an organic compound gives formaldehyde as one of the products. This confirms the presence of :

(1) An acetylenic triple bond

(2) Two ethylenic double bonds

(3) A vinyl group

(4) An isopropyl group

Ans. (3)

Sol. $>C=CH_2 \xrightarrow{\text{Ozonolysis}} HCHO$

Vinylic group

42. The degree of dissociation (α) of a weak electrolyte, A_xB_y is related to van't Hoff factor (*i*) by the expression:

(1) $\alpha = \dfrac{x+y+1}{i-1}$ (2) $\alpha = \dfrac{i-1}{(x+y-1)}$

(3) $\alpha = \dfrac{i-1}{x+y+1}$ (4) $\alpha = \dfrac{x+y-1}{i-1}$

Ans. (2)

Sol. Van't Hoff factor (i)

$$= \frac{\text{Observed colligative property}}{\text{Normal colligative property}}$$

$$\underset{1-\alpha}{A_xB_y} \longrightarrow \underset{x\alpha}{xA^{+y}} + \underset{y\alpha}{yB^{-x}}$$

Total moles = 1 – α + xα + yα

= 1 + α(x + y – 1)

$$i = \frac{1+\alpha(x+y-1)}{1}$$

$$\Rightarrow \quad \alpha = \frac{i-1}{x+y-1}$$

43. A gas absorbs a photon of 355 nm and emits at two wavelengths. If one of the emissions is at 680 nm, the other is at :

(1) 518 nm (2) 1035 nm

(3) 325 nm (4) 743 nm

Ans. (4)

Sol. $\dfrac{1}{\lambda} = \dfrac{1}{\lambda_1} + \dfrac{1}{\lambda_2}$

$$\frac{1}{355} = \frac{1}{680} + \frac{1}{\lambda_2}$$

$$\frac{1}{\lambda_2} = \frac{680-355}{355\times 680}$$

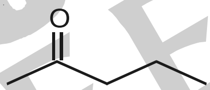
$\Rightarrow \lambda_2 = 743$ nm

44. Identify the compound that exhibits tautomerism.

(1) Phenol (2) 2–Butene

(3) Lactic acid (4) 2–Pentanone

Ans. (4)

Sol. 

2-Pentanone

has α-hydrogen & hence it will exhibit tautomerism.

45. The entropy change involved in the isothermal reversible expansion of 2 moles of an ideal gas from a volume of 10 dm^3 at 27°C is to a volume of 100 dm^3

(1) 42.3 J mol^{-1} K^{-1} (2) 38.3 J mol^{-1} K^{-1}

(3) 35.8 J mol^{-1} K^{-1} (4) 32.3 J mol^{-1} K^{-1}

Ans. (2)

Sol. $\Delta S = nR\ln\dfrac{V_2}{V_1}$

$$\Delta S = 2.303\times 2\times 8.314\,\log\frac{100}{10}$$

ΔS = 38.3 J/mole/K

46. Silver Mirror test is given by which one of the following compounds?

(1) Benzophenone (2) Acetaldehyde

(3) Acetone (4) Formaldehyde

Ans. (2, 4)

Sol. Both formaldehyde and acetaldehyde will give this test

$$HCHO \xrightarrow{[Ag(NH_3)_2^+]} \underset{\text{Silver mirror}}{Ag\downarrow} + \text{Organic compound}$$

$$CH_3-CHO \xrightarrow{[Ag(NH_3)_2]^+} \underset{\text{Silver mirror}}{Ag\downarrow} + \text{Organic Compound}$$

47. Trichloroacetaldehyde was subject to Cannizzaro's reaction by using NaOH. The mixture of the products contains sodium trichloroacetate and another compound. The other compound is :

(1) Chloroform

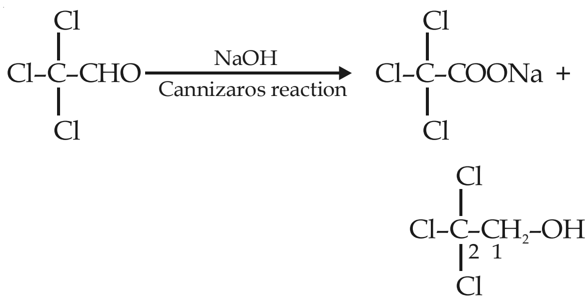
(2) 2, 2, 2-Trichloroethanol

(3) Trichloromethanol

(4) 2, 2, 2-Trichloropropanol

Ans. (2)

Sol.



2, 2, 2-trichloroethanol

48. The reduction potential of hydrogen half-cell will be negative if :

(1) $p(H_2)$ = 2 atm and $[H^+]$ = 2.0 M

(2) $p(H_2)$ = 1 atm and $[H^+]$ = 2.0 M

(3) $p(H_2)$ = 1 atm and $[H^+]$ = 1.0 M

(4) $p(H_2)$ = 2 atm and $[H^+]$ = 1.0 M

Ans. (4)

Sol. $H^+ + e^- \longrightarrow \frac{1}{2}H_2$

Apply Nernst equation

$$E = 0 - \frac{0.059}{1}\log\frac{P_{H_2}^{\frac{1}{2}}}{[H^+]}$$

$$E = -\frac{0.059}{1}\log\frac{2^{1/2}}{1}$$

Therefore E is negative.

49. Phenol is heated with a solution of mixture of KBr and $KBrO_3$. The major product obtained in the above reaction is :

(1) 2, 4, 6-Tribromophenol

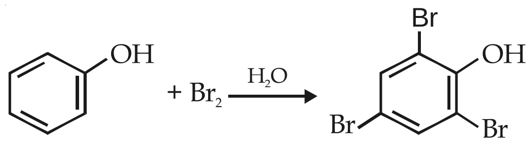
(2) 2-Bromophenol

(3) 3-Bromophenol

(4) 4-Bromophenol

Ans. (1)

Sol. $KBr + KBrO_3 \longrightarrow Br_2 + ...$



2, 4, 6-Tribomophenol

50. Among the following the maximum covalent character is shown by the compound

(1) $MgCl_2$ (2) $FeCl_2$

(3) $SnCl_2$ (4) $AlCl_3$

Ans. (4)

Sol. According to Fajans rule, cation with greater charge and smaller size favours covalency.

51. Boron cannot form which one of the following anions?

(1) BO_2^- (2) BF_6^{3-}

(3) BH_4^- (4) $B(OH)_4^-$

Ans. (2)

Sol. Due to absence of low lying vacant d orbital in B. sp^3d^2 hybridization is not possible hence BF_6^{3-} will not formed.

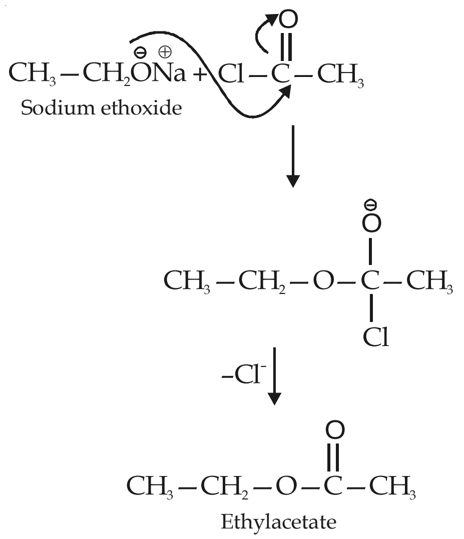
52. Sodium ethoxide has reacted with ethanoyl chloride. The compound that is produced in the above reaction is

(1) Ethyl ethanoate (2) Diethyl ether

(3) 2–Butanone (4) Ethyl chloride

Ans. (1)

Sol.



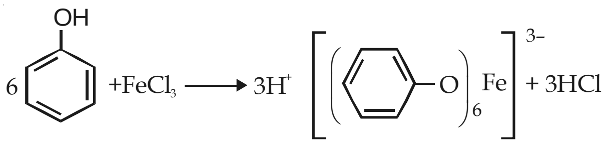
53. Which of the following reagents may be used to distinguish between phenol and benzoic acid?

(1) Neutral $FeCl_3$ (2) Aqueous NaOH

(3) Tollen's reagent (4) Molisch reagent

Ans. (1)

Sol. Neural $FeCl_3$ gives violet colored complex



Violet coloured complex

54. A vessel at 1000 K contains CO_2 with a pressure of 0.5 atm. Some of the CO_2 is converted into CO on the addition of graphite. If the total pressure at equilibrium is 0.8 atm, the value of K is

(1) 0.18 atm (2) 1.8 atm

(3) 3 atm (4) 0.3 atm

Ans. (2)

Sol. $CO_2(g) + C(s) \rightleftharpoons 2CO$

at 1000 K P = 0.5 atm 0

0.5 – p 2p

According to given condition 0.5 + p = 0.8

p = 0.3

$$\text{So, } K_P = \frac{(0.6)^2}{(0.2)} = \frac{0.36}{0.2} = 1.8 \text{ atm}$$

55. The strongest acid amongst the following compounds is :

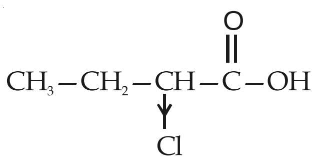
(1) $ClCH_2CH_2CH_2COOH$

(2) CH_3COOH

(3) HCOOH

(4) $CH_3CH_2CH(Cl)CO_2H$

Ans. (4)

Sol. 

Presence of electron withdrawing group nearest to the carboxylic group increase the acidic strength to maximum extent

56. Which one of the following orders presents the correct sequence of the increasing basic nature of the given oxides?

(1) $K_2O < Na_2O < Al_2O_3 < MgO$

(2) $Al_2O_3 < MgO < Na_2O < K_2O$

(3) $MgO < K_2O < Al_2O_3 < Na_2O$

(4) $Na_2O < K_2O < MgO < Al_2O_3$

Ans. (2)

Sol. While moving from left to right in periodic table basic character of oxide of elements will decrease.

$$\therefore \xrightarrow[\text{Increasing basic strength}]{Al_2O_3 < MgO < Na_2O}$$

and while descending in the group basic character of corresponding oxides increases.

$$\therefore \xrightarrow[\text{Increasing basic strength}]{Na_2O < K_2O}$$

$\therefore$ Correct order is

$Al_2O_3 < MgO < Na_2O < K_2O$

57. A 5.2 molal aqueous solution of methyl alcohol, CH_3OH is supplied. What is the mole fraction of methyl alcohol in the solution?

(1) 0.050 (2) 0.100

(3) 0.190 (4) 0.086

Ans. (4)

Sol. $x = \dfrac{\text{No. of moles of compound}}{\text{Total no. of mole of solution}}$

For 1 kg solvent

$$x = \frac{5.2}{55.5 + 5.2} = \frac{5.2}{60.7} = 0.086$$

58. The presence or absence of hydroxy group on which carbon atom of sugar differentiates RNA and DNA?

(1) 4^{th} (2) 1^{st}

(3) 2^{nd} (4) 3^{rd}

Ans. (3)

Sol. Fact.

59. Which of the following statement is **wrong**?

(1) N_2O_4 has two resonance structures

(2) The stability of hydrides increases from NH_3 to BiH_3 in group 15 of the periodic table

(3) Nitrogen cannot form $d\pi$-$p\pi$ bond

(4) Single N - N bond is weaker than the single P - P bond

Ans. (2)

Sol. Stability of hydrides from NH_3 to BiH_3 decreases due to decreasing bond strength.

60. Which of the following statements regarding sulphur is incorrect?

(1) The oxidation state of sulphur is never less than +4 in its compounds

(2) S_2 molecule is paramagnetic

(3) The vapour at 200°C consists mostly of S_8 rings

(4) At 600°C the gas mainly consists of S_2 molecules

Ans. (1)

Sol. The oxidation state of S may be less than +4.

i.e., in H_2S it is –2.

PART–C : PHYSICS

61. A carnot engine operation between temperatrues T_1 and T_2 has efficiency $\frac{1}{6}$. When T_2 is lowered by 62 *K*, its efficiency increases to $\frac{1}{3}$. Then T_1 and T_2 are respectively

(1) 310 K and 248 K (2) 372 K and 310 K

(3) 372 K and 330 K (4) 330 K and 268 K

Ans. (2)

Sol. $\frac{1}{6} = 1 - \frac{T_2}{T_1} \Rightarrow \frac{T_2}{T_1} = \frac{5}{6}$

Also, $\frac{T_2 - 62}{T_1} = \frac{2}{3}$

$\frac{T_2}{T_1} = \frac{5}{6}$

$\Rightarrow \frac{T_2 - 62}{T_2} = \frac{4}{5}$

or $T_2 = 62 \times 5$

or $T_2 = 310$ K

$\Rightarrow T_1 = 372$ K

62. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion if reversed is

(1) More than 9

(2) Less than 3

(3) More than 3 but less than 6

(4) More than 6 but less than 9

Ans. (3)

Sol. $T = F \times r = 40t - 10t^2$

$\alpha = 4t - t^2$

$2t^2 - \frac{t^3}{3} = 0$

$\Rightarrow t = 6$ s

Also, $\theta = \frac{2t^3}{3} - \frac{t^4}{12}$

$= \frac{2\times6\times6\times6}{3} - 108$

$n = \frac{\theta}{2\pi} = 5.73$

63. Three perfect gases at absolute temperatures T_1, T_2 and T_3 are mixed. The masses of molecules are m_1, m_2 and m_3 and the number of molecules are n_1, n_2 and n_3 respectively. Assuming no loss of energy, the final temperature of the mixture is

(1) $\frac{n_1^2T_1^2 + n_2^2T_2^2 + n_3^2T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$ (2) $\frac{T_1 + T_2 + T_3}{3}$

(3) $\frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$ (4) $\frac{n_1T_1^2 + n_2T_2^2 + n_3T_3^2}{n_1T_1 + n_2T_2 + n_3T_3}$

Ans. (3)

Sol. $U = \frac{\frac{n_1}{N}T_1 + \frac{n_2}{N}T_2 + \frac{n_3}{N}T_3}{\frac{n_1}{N} + \frac{n_2}{N} + \frac{n_3}{N}}$

$= \frac{n_1T_1 + n_2T_2 + n_3T_3}{n_1 + n_2 + n_3}$

64. A boat is moving due east in a region where the earth's magnetic field is 5.0×10^{-5} NA^{-1} due north and horizontal. The boat carries a vertical aerial 2 m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the induced emf in the wire of aerial is

(1) 0.15 mV (2) 1 mV

(3) 0.75 mV (4) 0.50 mV

Ans. (1)

Sol. $\varepsilon = Bvl$

$= 5.0 \times 10^{-5} \times 2 \times 1.50$

$= 15 \times 10^{-5}$

$= 0.15$ mV

65. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect the angular speed of the disc

(1) First increase and then decrease

(2) Remains unchanged

(3) Continuously decreases

(4) Continuously increases

Ans. (1)

Sol. $L = I\omega$ = Constant

I first decreases, then increases.

$\Rightarrow$ ω first increases and then decreases

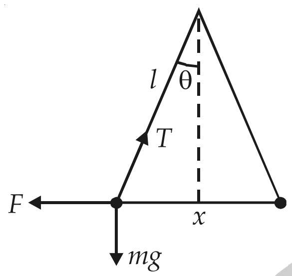
66. Two identical charged spheres suspended from a common point by two massless strings of length l are initially a distance $d(d << l)$ apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them

(1) $v \propto x$ (2) $v \propto x^{-\frac{1}{2}}$

(3) $v \propto x^{-1}$ (4) $v \propto x^{\frac{1}{2}}$

Ans. (2)

Sol. $\tan\theta = \dfrac{F}{mg}$



or $\dfrac{x}{2l} = \dfrac{kq^2}{mgx^2}$

$\dfrac{x^3}{2l} = \dfrac{kq^2}{mg}$

$\dfrac{3x^2 \dfrac{dx}{dt}}{2l} = \dfrac{2kq\dfrac{dq}{dt}}{mg}$

Also, $q \propto x^{3/2}$

$\Rightarrow \dfrac{dx}{dt} \propto \dfrac{x^{3/2}}{x^2}$, *i.e.*, $v \propto x^{-1/2}$

67. 100 g of water is heated from 30°C to 50°C. Ignoring the slight expansion of the water, the change in its internal enetgy is (specific heat of water is 4184 J/kg/K)

(1) 2.1 kJ (2) 4.2 kJ

(3) 8.4 kJ (4) 84 kJ

Ans. (3)

Sol. $Q = \Delta U + W$

As $W = 0 \Rightarrow \Delta U = Q = 4184 \times \dfrac{100}{1000} \times 20$

$= 8368$

$= 8.368$ kJ

≈ 8.4 kJ

68. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\dfrac{2}{3}$ of its has decayed and time t_1 when $\dfrac{1}{3}$ of its had decayed is

(1) 28 min (2) 7 min

(3) 14 min (4) 20 min

Ans. (4)

Sol. At $t = t_2,\ N_2 = \dfrac{N}{3}$ (Left)

$t = t_1,\ N_1 = \dfrac{2N}{3}$

$\Rightarrow t_2 - t_1$ = half life = 20 min

69. Energy required for the electron excitation in Li^{++} from the first to the third Bohr orbit is

(1) 122.4 eV (2) 12.1 eV

(3) 36.3 eV (4) 108.8 eV

Ans. (4)

Sol. $E = -13.6\,(3)^2\left[\dfrac{1}{3^2} - \dfrac{1}{1^2}\right]$

$= 13.6 \times 9 \times \dfrac{8}{9}$

$= 108.8$ eV

70. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre ; a, b are constants. Then the charge density inside the ball is

(1) $-6\,a\varepsilon_0$ (2) $-24\,\pi\,a\varepsilon_0 r$

(3) $-6\,a\varepsilon_0 r$ (4) $-24\,\pi\,a\varepsilon_0$

Ans. (1)

Sol. $\phi = ar^2 + b$

$\dfrac{d\phi}{dr} = -2ar \Rightarrow E = -2ar$

$E \times 4\pi r^2 = \dfrac{q}{\varepsilon_0}$

$\Rightarrow q = -8\pi\varepsilon_0 a r^3$

$\Rightarrow \rho = \dfrac{dq}{dV}$

$= \dfrac{-24\pi\varepsilon_0 r^2 dr}{4\pi r^2 dr}$

$= -6\varepsilon_0 a$

71. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5 cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1})

(1) 0.4 π m J (2) 4 π m J

(3) 0.2 π m J (4) 2 π m J

Ans. (1)

Sol. $W = \Delta SE$

$= 2 \times T \times 4\pi(5^2 - 3^2) \times 10^{-4}$

$= 2 \times 0.03 \times 4\pi(16) \times 10^{-4}$

$= 3.84\pi \times 10^{-4}$

$= 0.4\pi$ mJ

72. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5 s after the switch has been closed. ($\log_{10}2.5 = 0.4$)

(1) $3.3 \times 10^7\ \Omega$ (2) $1.3 \times 10^4\ \Omega$

(3) $1.7 \times 10^5\ \Omega$ (4) $2.7 \times 10^6\ \Omega$

Ans. (4)

Sol. Charge on capacitor $q = q_0(1 - e^{-t/RC})$

or $V = \dfrac{q_0}{C}(1 - e^{-t/RC})$

$V = 200(1 - e^{-t/RC})$

$120 = 200(1 - e^{-t/RC})$

$e^{-t/RC} = 0.4$

$$\frac{-t}{RC} = \ln(0.4)$$

$$\frac{t}{RC} = \ln\left(\frac{10}{4}\right) = 2.303 \times 0.4$$

$$R = \frac{5}{2 \times 10^{-6} \times 2.303 \times 0.4}$$

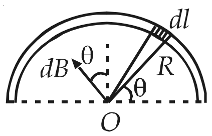
$= 2.7 \times 10^6\ \Omega$

73. A current I flows in an infinitely long wire with corss section in the form of a semi-circular ring of raidus R. The magnitude of the magnetic induction along its axis is

(1) $\dfrac{\mu_0 I}{4\pi R}$ (2) $\dfrac{\mu_0 I}{\pi^2 R}$

(3) $\dfrac{\mu_0 I}{2\pi^2 R}$ (4) $\dfrac{\mu_0 I}{2\pi R}$

Ans. (2)

Sol. 

$$B = \int dB \sin\theta$$

$$B = \int \frac{\mu_0 di}{2\pi R}\sin\theta$$

$$= \frac{\mu_0}{2\pi R}\left(\frac{i}{\pi R}\right) R \int_0^{\pi} \sin\theta\, d\theta$$

$$= \frac{\mu_0 i}{\pi^2 R}$$

74. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be

(1) 8 s (2) 1 s

(3) 2 s (4) 4 s

Ans. (3)

Sol. $\displaystyle\int_v^0 \frac{dv}{\sqrt{v}} = -\int_0^t 2.5\, dt$

$2\sqrt{v} = 2.5t$

$2\sqrt{6.25} = 2.5t$

$t = 2$ s

75. ***Direction :*** *The question has a paragraph followed by two statements,* ***Statement-1*** *and* ***Statement-2.*** *Of the given four alternatives after the statements, choose the one that describes the statements.*

A thin air film is formed by putting the covex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (coNvex) surface and the bottom (glass plate) surface of the film.

Statement-1 : When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2 : The centre of the interference pattern is dark.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1

(4) Statement-1 is true, Statement-2 is true and Statement-2 is ***not*** the correct explanation of Statement-1

Ans. (4)

Sol. The centre of interference pattern is dark, showing that the phase difference between two interfering waves is π. But this does not imply statement-1.

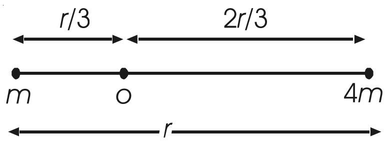
76. Two bodies of masses m and 4 m are placed at a distance r. The gravitational potential at a point on the line joining them where the gravitational field is zero is

(1) $-\frac{9Gm}{r}$ (2) Zero

(3) $-\frac{4Gm}{r}$ (4) $-\frac{6Gm}{r}$

Ans. (1)

Sol. Field is zero at O.



$$V = -\frac{GM}{r/3} - \frac{4GM}{2r/3}$$

$$= -\frac{9GM}{r}$$

77. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : Sky wave signals are used for long distance radio communication. These signals are in general, less stable than ground wave signals.

Statement-2 : The state of ionosphere varies from hour to hour, day to day and season to season.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1

(4) Statement-1 is true, Statement-2 is true and Statement-2 is ***not*** the correct explanation of Statement-1

Ans. (3)

Sol. Because of variation in composition of ionosphere, the signals are unstable.

78. A fully charged capacitor C with initial charge q_0 is connected to a coil of self inductance L at $t = 0$. The time at which the energy is stored equally between the electric and the magnetic fields is

(1) $\sqrt{LC}$ (2) $\pi\sqrt{LC}$

(3) $\frac{\pi}{4}\sqrt{LC}$ (4) $2\pi\sqrt{LC}$

Ans. (3)

Sol. $q = q_0 \cos \omega t$

energy is halved when $q = \frac{q_0}{\sqrt{2}}$

$$\Rightarrow \omega t = \frac{\pi}{4}$$

$$\Rightarrow \frac{1}{\sqrt{Lc}} t = \frac{\pi}{4}$$

$$\Rightarrow t = \frac{\pi}{4}\sqrt{Lc}$$

79. This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.

Statement-1 : A metallic surface is irradiated by a monochromatic light of frequency $v > v_0$ (the threshold frequency). The maximum kinetic energy and the stopping potential are K_{max} and V_0 respectively. If the frequency incident on the surface is doubled, both the K_{max} and V_0 are also doubled.

Statement-2 : The maximum kinetic energy and the stopping potential of photoelectrons emitted from a surface are linearly dependent on the frequency of incident light.

(1) Statement-1 is false, Statement-2 is true

(2) Statement-1 is true, Statement-2 is false

(3) Statement-1 is true, Statement-2 is true and Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true and Statement-2 is ***not*** the correct explanation of Statement-1

Ans. (1)

Sol. $k_{max} = hf - \phi$

$k'_{max} = 2hf - \phi$

$k'_{max} = 2k_{max} + \phi$

$k'_{max} > 2k_{max}$

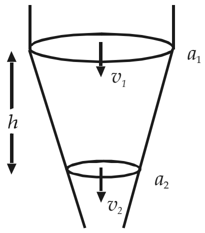
80. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to

(1) 3.6×10^{-3} m   (2) 5.0×10^{-3} m

(3) 7.5×10^{-3} m   (4) 9.6×10^{-3} m

Ans. **(1)**

Sol. $a_1v_1 = a_2v_2$



$v_2^2 = v_1^2 + 2gh$

$v_2^2 = (0.4)^2 + 2 \times 10 \times 0.2$

$= (0.4)^2 + 4$

$v_2^2 = 4.16$

$v_2 = \sqrt{4.16}$

$v_2 = 2.04$ m/s

Also, $a_1V_1 = a_2V_2$

$d_1^2V_1 = d_2^2V_2$

$$d_2 = d_1\sqrt{\frac{V_1}{V_2}}$$

$$= 8\times10^{-3}\sqrt{\frac{0.4}{2.04}}$$

$= 3.54 \times 10^{-3}$ m

81. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is

(1) $\left(\frac{M+m}{M}\right)^{1/2}$ (2) $\frac{M}{M+m}$

(3) $\frac{M+m}{M}$ (4) $\left(\frac{M}{M+m}\right)^{1/2}$

Ans. **(1)**

Sol. At mean position, initial velocity = $A_1\omega_1$

$$\text{New velocity} = \frac{MA_1\omega_1}{M+m}$$

$$\Rightarrow A_2\omega_2 = \frac{MA_1\omega_1}{M+m}$$

$$\frac{A_2}{A_1} = \left(\frac{M}{M+m}\right)\frac{\omega_1}{\omega_2}$$

$$\omega_1 = \sqrt{\frac{k}{M}}$$

$$\omega_2 = \sqrt{\frac{k}{M+m}}$$

$$\Rightarrow \frac{A_2}{A_1} = \sqrt{\frac{M}{m+M}}$$

$$\text{or } \frac{A_1}{A_2} = \sqrt{\frac{m+M}{M}}$$

82. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is

(1) $\frac{\pi}{6}$ (2) $\frac{\pi}{2}$

(3) $\frac{\pi}{3}$ (4) $\frac{\pi}{4}$

Ans. **(3)**

Sol. $x_1 = A \sin \omega t$

$x_2 = x_0 + A \sin(\omega t + \phi)$

Separation $= x_0 + A \sin(\omega t + \phi) - A \sin \omega t$

$$= x_0 + 2A\sin\left(\frac{\phi}{2}\right)\cos\omega t$$

$$\text{Maximum separation} = x_0 + 2A\sin\frac{\phi}{2} = x_0 + A$$

$$\Rightarrow \phi = \frac{\pi}{3}$$

83. If a wire is stretched to make it 0.1% longer, its resistance will

(1) Decrease by 0.05% (2) Increase by 0.05%

(3) Increase by 0.2% (4) Decrease by 0.2%

Ans. **(3)**

Sol. $R \propto l^2$

$$\frac{\Delta R}{R} = \frac{2\Delta l}{l}$$

84. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is

(1) $\pi\frac{v^2}{g^2}$ (2) $\pi\frac{v^2}{g}$

(3) $\pi\frac{v^4}{g^2}$ (4) $\frac{\pi}{2}\frac{v^4}{g^2}$

Ans. **(3)**

Sol. Area $= \pi R_{max}^2$

$$R_{max} = \frac{v^2\sin 90^\circ}{g} = \frac{v^2}{g}$$

$$\Rightarrow \text{Area} = \frac{\pi v^4}{g^2}$$

85. A thermally insulated vessel contains an ideal gas of molecular mass M and ratio of specific heats γ. It is moving with speed v and is suddenly brought to rest. Assuming no heat is lost to the surroundings, its temperature increases by

(1) $\frac{(\gamma-1)}{2R}Mv^2$ K (2) $\frac{(\gamma-1)}{2(\gamma+1)R}Mv^2$ K

(3) $\frac{(\gamma-1)}{2\gamma R}Mv^2$ K (4) $\frac{\gamma Mv^2}{2R}$ K

Ans. (1)

Sol. $\frac{1}{2}mv^2 = \frac{n\times R\Delta T}{(\gamma-1)}$

$\Rightarrow \Delta T = \frac{MV^2(\gamma-1)}{2R}$

86. A screw gauge gives the following reading when used to measure the diameter of a wire.

Main scale reading : 0 mm

Circular scale reading : 52 divisions

Given that 1 mm on main scale corresponds to 100 divisions of the circular scale.

The diameter of wire from the above data is

(1) 0.005 cm (2) 0.52 cm

(3) 0.052 cm (4) 0.026 cm

Ans. (3)

Sol. Diameter = main scale reading + l.c. × circular scale reading

$= 0 + 52 \times \frac{1}{100}$ mm

$= 0.52$ mm $= 0.052$ cm

87. A mass m hangs with the help of a string wrapped around a pulley on a frictionless bearing. The pulley has mass m and radius R. Assuming pulley to be a perfect uniform circular disc, the acceleration of the mass m, if the string does not slip on the pulley, is

(1) $\frac{g}{3}$ (2) $\frac{3}{2}g$

(3) g (4) $\frac{2}{3}g$

Ans. (4)

Sol. $a = \frac{mg}{m+\frac{I}{R^2}} = \frac{mg}{m+\frac{m}{2}} = \frac{2g}{3}$

88. The transverse displacement $y(x, t)$ of a wave on a string is given by

$y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}\,xt\right)}$

This represents a

(1) Standing wave of frequency $\frac{1}{\sqrt{b}}$

(2) Wave moving in +x direction with speed $\sqrt{\frac{a}{b}}$

(3) Wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$

(4) Standing wave of frequency $\sqrt{b}$

Ans. (3)

Sol. $y(x,t) = e^{-(\sqrt{a}\,x+\sqrt{b}\,t)^2}$

This is a wave travelling in –x direction with speed $\frac{\sqrt{b}}{\sqrt{a}}$

89. A car is fitted with a convex side-view mirror of focal length 20 cm. A second car 2.5 m behind the first car is overtaking the first car at a relative speed of 15 m/s. The speed of the image of the second car as seen in the mirror of the first one is

(1) 15 m/s (2) $\frac{1}{10}$ m/s

(3) $\frac{1}{15}$ m/s (4) 10 m/s

Ans. (3)

Sol. For mirror

$\frac{1}{v}+\frac{1}{u}=\frac{1}{f}$

$\frac{-1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0$

$\frac{dv}{dt}=\frac{-v^2}{u^2}\frac{du}{dt}=-\left(\frac{f}{f-u}\right)^2\frac{du}{dt}$

$=-\left(\frac{20}{20+280}\right)^2\frac{du}{dt}$

$=\left(\frac{1}{15}\right)^2\frac{du}{dt}=\frac{1}{15}$ m/s

90. Let the *x*-*z* plane be the boundary between two transparent media. Medium 1 in z ≥ 0 has a refractive index of $\sqrt{2}$ and medium 2 with z < 0 has a refractive index of $\sqrt{3}$. A ray of light in medium 1 given by the vector $\vec{A} = 6\sqrt{3}\hat{i} + 8\sqrt{3}\hat{j} - 10\hat{k}$ is incident on the plane of separation. The angle of refraction in medium 2 is

(1) 75° (2) 30°

(3) 45° (4) 60°

Ans. (3)

Sol. The data is inconsistant

Taking boundary as *x*-*y* plane instead of *x*-*z* plane, the angle of incidence is given as

$$\cos\theta = \frac{(-\hat{k}).(6\sqrt{3}\hat{i}+8\sqrt{3}\hat{j}-10\hat{k})}{\sqrt{108+192+100}}$$

$$= \frac{1}{2} \quad \Rightarrow \theta = 60°$$

Now, $\sqrt{2}\sin 60° = \sqrt{3}\sin r$

$$\Rightarrow \quad \sin r = \frac{1}{\sqrt{2}}$$

$$\Rightarrow \quad r = 45°$$

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