Total No. of Questions—12] [Total No. of Printed Pages—8

[3762]-141

S.E. (E&TC/Elex) (I Sem.) EXAMINATION, 2010

SIGNALS AND SYSTEMS

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Your answers will be valued as a whole.

(*vi*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*vii*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Sketch and label the even and odd components of the signals shown in Fig. 1. [8]

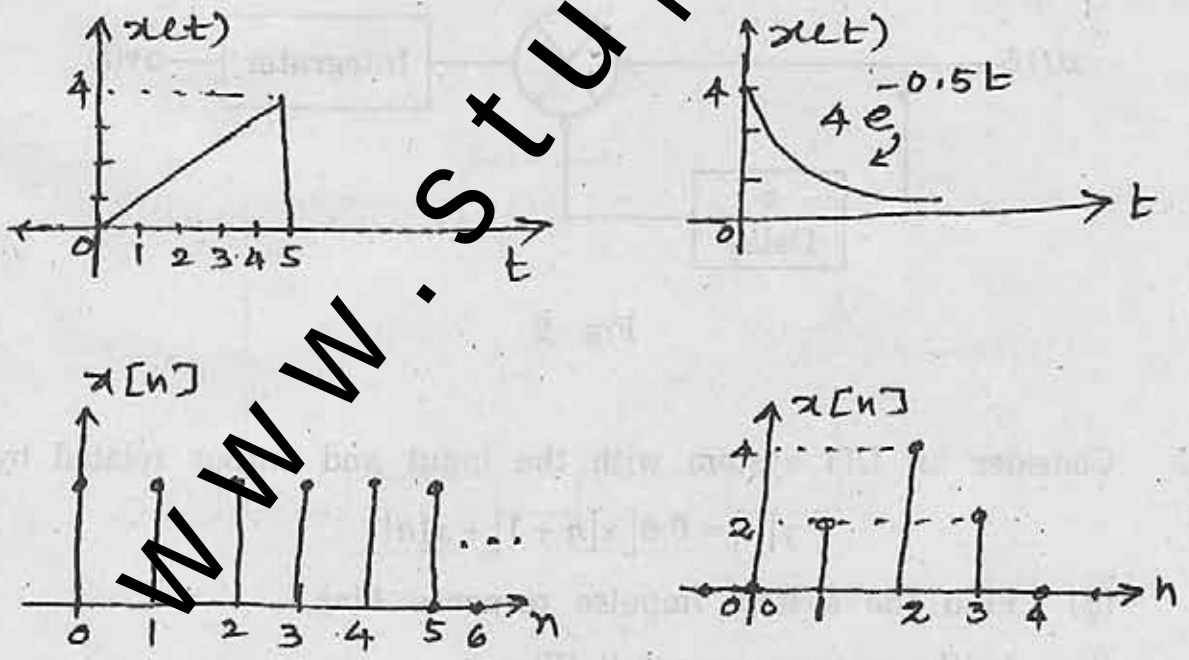


Fig. 1

P.T.O.

(b) Determine whether the following signals are energy, power signals or neither and find the corresponding value : [10]

(i) $x(t) = e^{-at}\ u(t)\ a > 0$

(ii) $x(t) = A\ \cos(\omega_0 t + \theta)$

(iii) $x(t) = t\ u(t)$

(iv) $x[n] = (-0.5)^n\ u[n]$

(v) $x[n] = 2e^{j3n}$.

Or

2. (a) Determine whether the system described by :

$$y(t) = e^{tx(t)}$$

is memoryless, invertible, causal, stable, time invariant, linear with justification. [12]

(b) Find the input-output relation of the system shown in Fig. 2. [6]

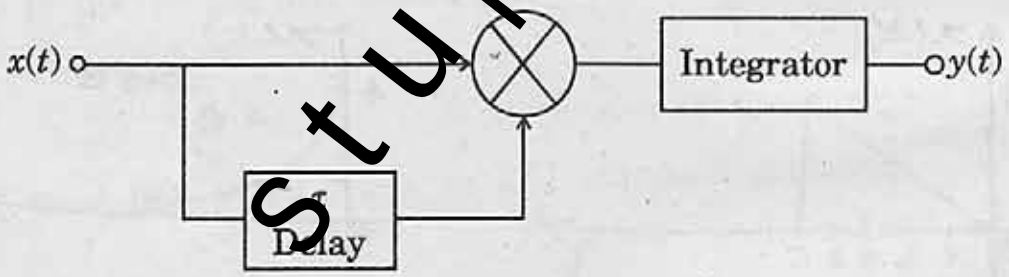


Fig. 2

3. Consider an LTI system with the input and output related by :

$$y[n] = 0.8\big[x[n+1] + x[n]\big]$$

(a) Find the system impulse response $h[n]$.

(b) Is the system causal ? Why ?

(c) Determine the system response $y[n]$ for the input shown in Fig. 3(a).

(d) Consider the interconnection of the LTI system given in Fig. 3(b) where $h[n]$ is the function found in Q. 3(a). Find the impulse response of the total system.

(e) Solve for the response of the system of Part Q. 3(d) for the input $x[n]$ in Q. 3(a). [16]

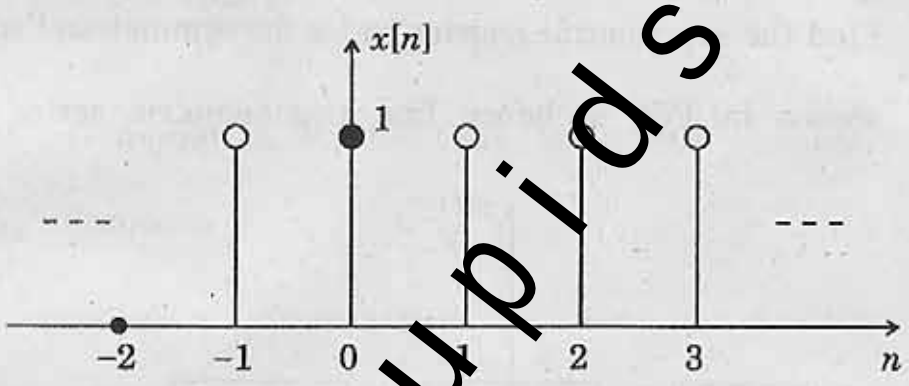


Fig. 3(a)

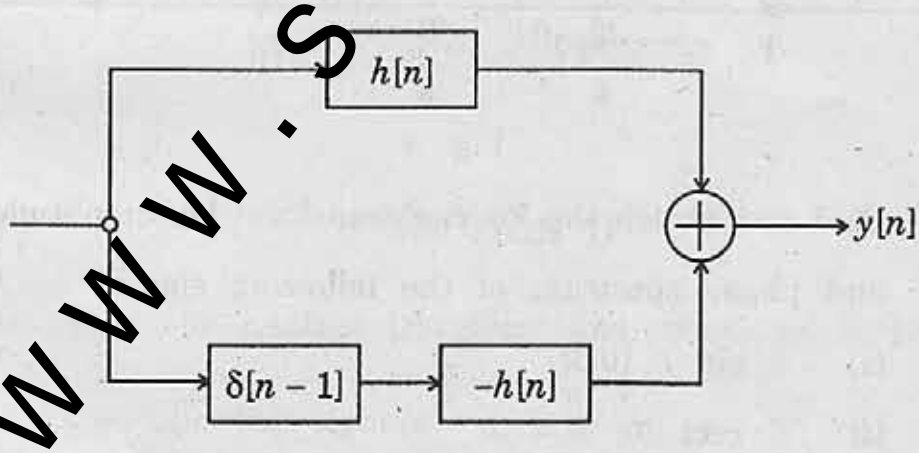


Fig. 3(b)

Or

4. (*a*) An LTI system has the impulse response :

$$h(t) = u(t + 1) - u(t - 3)$$

(*i*) Determine whether the system is causal and stable.

(*ii*) Find and sketch the system response to the input :

$$x(t) = e^{-at}\ u(t - 1).$$ [10]

(*b*) State and discuss the properties of convolution. [6]

5. (*a*) Find the exponential Fourier series for symmetrical square wave shown in Fig. 4, hence find trigonometric series also. [8]

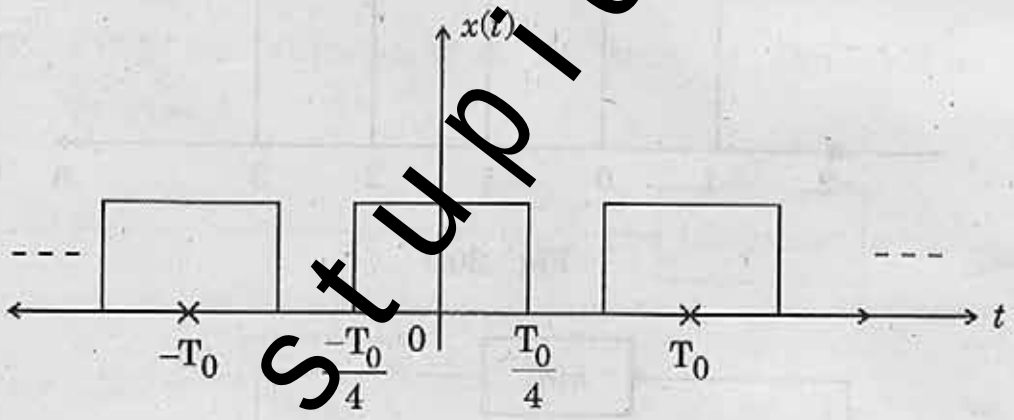


Fig. 4

(*b*) Find and sketch the Fourier transform i.e. amplitude spectrum and phase spectrum of the following signals :

(*i*) 6 sin *c* (0.5*t*)

(*ii*) 6 rect [(*t* − 4)/3]

(*iii*) 10 sin (100π*t*). [8]

Or

6. (*a*) Given :

$$e^{-|t|} \xleftrightarrow{F} \frac{2}{4\pi f^2 + 1}$$

Find the Fourier transforms of the following :

(*i*) $\frac{d}{dt} e^{-|t|}$

(*ii*) $\frac{1}{2\pi\left(t^2 + 1\right)}$

(*iii*) $\frac{4\cos(2t)}{t^2 + 1}$. [8]

(*b*) Consider a linear time invariant system with impulse response :

$$h(t) = \frac{0.5\sin(2t)}{t}.$$

Find the system response $y(t)$ if the input is $x(t) = \cos t + \sin 3t$ using Fourier transforms. [8]

SECTION II

7. (*a*) Find the Laplace transform and associated ROC for each of the following signals :

(*i*) $\delta(t - t_0)$

(ii) $u(t - t_0)$

(iii) $e^{-2t}\left[u(t) - u(t-5)\right]$

(iv) $\sum_{K=0}^{\infty} \delta(t - KT)$

(v) $\delta(at + b)$, a, b real constants. [10]

(b) State and prove the integration property of Laplace transform. [6]

Or

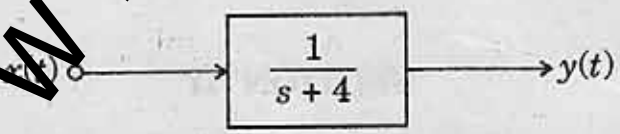
8. (a) Find inverse Laplace transform of the following given X(s) :

(i) $\frac{s^2 + 6s + 7}{s^2 + 3s + 2}\, Re(s) > -1$

(ii) $\frac{s^3 + 2s^2 + 6}{s^2 + 3s}\, Re(s) > 0$

(iii) $\frac{2 + 2se^{-2s} + 4e^{-4s}}{s^2 + 4s + 3}\, Re(s) > -1$. [10]

(b) In the system shown below $x(t) = \text{rect}(t/\tau)$. Find the response $y(t)$ using Laplace transform where $\tau = 2$: [6]



9. (a) Find the autocorrelation of the signal and power spectral density :

$$x(t) = 40 \cos (100\ \pi t + \phi).$$ [8]

(b) State and discuss the properties of autocorrelation and its applications. [8]

Or

10. (a) Find the cross-correlation between the signals :

$$x(t) = 4e^{-5t}\ u(t) \quad y(t) = 10e^{-10t}$$

Sketch the cross-correlation. [8]

(b) The signal $x(t) = 10 \sin c\ (10t)$ is applied to a system whose transfer function is :

$$H(f) = 3 \text{ rect } (f/4)\ e^{-j4\pi f}$$

Find output energy spectral density and energy. [8]

11. (a) In the experiment of rolling six face dice find the probability of occurrence of 4 if it is known that even face has appeared. [6]

(b) In a digital communication channel the probability of sending '0' or '1' is 0.5. If the probability of error due to noise in channel is 0.05, find the probability of sending '0' when the received bit is '1'. [6]

(c) Explain "the random variable is neither random nor variable" then what is Random variable. Give an example for discrete and continuous random variable. [6]

Or

12. (*a*) A random variable X is defined by the CDF :

$$F_X(x) = \begin{cases} 0 & x < 0 \\ \frac{1}{2}x & 1 \le x < 1 \\ K & x \ge 1 \end{cases}$$

(*i*) Find the value of K

(*ii*) Is this *k*V is discrete or continuous or mixed

(*iii*) What is the probability $P\left(\frac{1}{2} < X \le 1\right)$

(*iv*) $P\left(\frac{1}{2} < X < 1\right)$

(*v*) $P(X > 2)$

(*vi*) Find its PDF. [10]

(*b*) If the probability density function of a random variable is given by :

$$f_X(x) = (1 - x)^2 \text{ for } 0 \le x \le 1,$$

then find mean, variance and standard deviation. [8]