Total No. of Questions—12] [Total No. of Printed Pages—4+1

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S.E. (E&TC) (I Sem.) EXAMINATION, 2010

SIGNALS AND SYSTEMS

(2003 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answer *three* questions from Section I and *three* questions from Section II.

(*ii*) Answers to the two Sections should be written in separate answer-books.

(*iii*) Neat diagrams must be drawn wherever necessary.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of logarithmic tables, slide rule, Mollier charts, electronic pocket calculator and steam tables is allowed.

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Check whether the following systems are :

(*i*) memoryless

(*ii*) stable

(*iii*) causal

(*iv*) linear

(1) $y(t) = x(t/2)$

(2) $y(t) = x[2 - t]$

(3) $y(t) = x(t) \cos 200 \pi t$

(4) $y[n] = x[n] - x[n - 1]$. [8]

P.T.O.

(*b*) Sketch the following signals to the scale :

(*i*) $x[n] = \sum_{k=-6}^{+6} \delta[k-n]$, where $\delta[n]$ is an impulse signal.

(*ii*) $x[n] = \sin c\left[\frac{n}{4}\right]$ for $-4 \leq n \leq +4$. [8]

Or

2. (*a*) Find even and odd components of the following signals :

(1) $x(t) = \cos(t) + \sin(t) + \sin t \cdot \cos t$

(2) $x(t) = 2t^2 - 3t + 6$

(3) $\sin c(t) = x(t)$

(4) $x(t) = \sin\left(40\pi t + \frac{\pi}{6}\right)$. [8]

(*b*) Find whether the following signals are energy or power signals :

(1) $x[n] = a^n u[n]$ where $|a| > 1$

(2) $x[n] = 1 + \cos\left[\frac{\pi n}{2}\right]$

(3) $x(t) = e^{-at} u(t)$. [8]

3. (*a*) Determine whether the following systems described by impulse response :

(*i*) $h(t) = e^{-t} u(t + 1)$

(*ii*) $h(t) = e^{t} u(-t - 1)$

are stable and causal. [8]

(*b*) What is LTI system and its impulse response ? Explain how the impulse response is useful in finding out the LTI system output due to any arbitrary input signal. [8]

Or

4. (*a*) Find the convolution of two continuous functions :

$x(t) = e^{-|t|}$ for all t

$h(t) = Ae^{-2t}\ u(t)$. [8]

(*b*) State and prove any *four* properties of convolution of continuous time signal. [8]

5. (*a*) State and prove time-shifting and convolution properties of Fourier transform. [10]

(*b*) Find exponential Fourier series of direct delta [comb] function with period 'T_0' and draw its spectrum. [8]

Or

6. (*a*) Find continuous time Fourier transform of the following signals :

(1) $x(t) = e^{-at}\ u(t)$

(2) $x(t) = \sin\ \omega_c t\ u(t)$ [12]

(*b*) Impulse response of LTI system is given by $n(t) = k\delta(t)$ and input $x(t) = u(t)$. Find output $y(t)$ and $y(f)$. Also draw the input and output magnitude spectrum. [6]

SECTION II

7. (*a*) Find the Laplace transform of the following signal with ROC :

(1) $f(t) = e^{3t}\ u(t) + e^{-t}\ u(t)$

(2) $f(t) = e^{-at}\ \sin(bt)$. [8]

(*b*) Find Inverse Laplace transform of the following signals using partial function expansion :

$$X(s) = \frac{s+3}{s(s+1)(s+2)}.$$ [8]

Or

8. (*a*) Determine bilateral Laplace transform and ROC for the following signal :

(*i*) $x(t) = u(t - 2)$

(*ii*) $x(t) = \delta(t - t_0)$. [8]

(*b*) State and prove any *two* properties of Laplace transform. [8]

9. (*a*) Find autocorrelation and PSD of the following signal :

$$x(t) = 5 + 4 \cdot \sin(10\pi t + 30°).$$ [12]

(*b*) Determine energy spectral density and prove relation between autocorrelation and energy spectral density. [6]

Or

10. (*a*) State and prove Parseval's theorem. [6]

(*b*) Find cross-correlation function of :

$x(t) = Ae^{-at}$

$y(t) = Be^{-bt}$. [12]

11. (*a*) Write the expression of CDF and PDF of a uniform random variable and sketch the same and also illustrate an example of uniform random variable. [8]

(*b*) A biased coin is loaded such that :

$$P(H) = \frac{1+\varepsilon}{2} \text{ with } 0 < |\varepsilon| < 1$$

Show that probability of a match in two independent tosses will be greater than $\frac{1}{2}$. [8]

Or

12. (*a*) State and prove sampling theorem in frequency domain. [8]

(*b*) An experiment consisting of observing the sum of the numbers showing up when two dice are thrown. Events A = (sum = 7), B = {8 < sum ≤ 11}, C = {10 < sum}. Draw Venn diagram and find :

(*i*) P(A)

(*ii*) P(B)

(*iii*) P(C)

(*iv*) $P(A \cap B)$

(*v*) $P(A \cup B)$. [8]