Total No. of Questions—**12**] [Total No. of Printed Pages—**8+1**

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S.E. (COMP/IT/Electrical/Instru.) (II Sem.) EXAMINATION, 2010

ENGINEERING MATHEMATICS-III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) In Section I attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

(*ii*) In Section II attempt Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*iii*) Answers to the two Sections should be written in separate answer-books.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Figures to the right indicate full marks.

(*vi*) Use of electronic pocket calculator is allowed.

(*vii*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* of the following : [12]

(*i*) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 3e^{-3x}\sin(e^{-3x}) + \cos(e^{-3x})$

(*ii*) $(x+2)^2\frac{d^2y}{dx^2} + 3(x+2)\frac{dy}{dx} + y = \cos\log(x+2)$

P.T.O.

(iii) $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} - 2y = e^x \sin^2 x$

(iv) $\dfrac{d^4y}{dx^4} - 2\dfrac{d^2y}{dx^2} - 8y = \cosh 2x + 2^{-x}$.

(b) An uncharged condenser of capacity C is charged by applying an e.m.f. $E \sin\left(\dfrac{t}{\sqrt{LC}}\right)$, through leads of self-inductance L and negligible resistance. Prove that at any time t, the charge on one of the plates is

$$\frac{EC}{2}\left\{\sin\left(\frac{t}{\sqrt{LC}}\right) - \frac{t}{\sqrt{LC}}\cos\left(\frac{t}{\sqrt{LC}}\right)\right\}.$$ [4]

Or

2. (a) Solve any *three* of the following : [12]

(i) $x^2\dfrac{d^2y}{dx^2} + 5x\dfrac{dy}{dx} + 4y = x \log x$

(ii) $\dfrac{d^2y}{dx^2} - 4\dfrac{dy}{dx} + 4y = e^{2x}.\sec^2 x$ (by variation of parameters)

(iii) $\dfrac{d^2y}{dx^2} - 2\dfrac{dy}{dx} + 2y = x^2 + e^{-x} + 1$

(iv) $\dfrac{xdx}{y^2z} = \dfrac{dy}{xz} = \dfrac{dz}{y^2}$.

(*b*) Solve the simultaneous equations : [4]

$$\frac{dx}{dt} + 5x - 2y = t$$

$$\frac{dy}{dt} + 2x + y = 0.$$

3. (*a*) Evaluate :

$$\int_C \frac{2z^3 + z + 5}{(z-3)^3} dz,$$

where C is $\frac{x^2}{16} + \frac{y^2}{4} = 1$. [5]

(*b*) If $f(z) = u(x, y) + iv(x, y)$ is analytic, find $f(z)$ if $u - v = x^3 + 3x^2y - 3xy^2 - y^3$. [6]

(*c*) Find the map of the straight line $2y = x$ under the transformation $w = \frac{2z-1}{2z+1}$. [5]

Or

4. (*a*) Find the residue of

$$f(z) = \frac{z}{(z-1)^2(z-2)(z-3)}$$

at its poles and hence evaluate :

$$\int_C f(z) dz,$$

where C is the circle $|z| = 4$. [5]

(b) Show that $u(x, y) = y + e^x \cdot \cos y$ is harmonic function. Also find its harmonic conjugate. [6]

(c) Find the bilinear transformation which maps the points 1, i, $2i$ of z-plane onto points $-2i$, 0, 1 of w-plane. [5]

5\. (a) Using Fourier integral representation, show that : [7]

$$e^{-5x} \cosh 3x = \frac{10}{\pi} \int_0^{\infty} \frac{\lambda^2 + 16}{(\lambda^2 + 4)(\lambda^2 + 64)} \cos \lambda x \, d\lambda.$$

(b) Solve the integral equation : [5]

$$\int_0^{\infty} f(x) \sin \lambda x \, dx = \begin{cases} \lambda + 1, & 0 \le \lambda \le 1 \\ 2, & 1 < \lambda \le 2 \\ \lambda - 1, & 2 < \lambda \le 4 \\ 0, & \lambda > 4 \end{cases}$$

(c) Find the z-transform of the following (any *two*) : [6]

(i) $f(k) = \left(\frac{1}{4}\right)^{|k|} \; \forall \; k$

(ii) $f(k) = a^k \sin (bk + c), \;\; k \ge 0$

(iii) $f(k) = k.3^k.2^k.5^k, \;\; k \ge 0$

Or

6. (*a*) Find the inverse z-transform (any *two*) : [8]

(*i*) $F(z) = \dfrac{z^2}{z^2 - 7z + 12}, \; |z| > 4$

(*ii*) $F(z) = \left(\dfrac{z}{z-2}\right)^2$ (by integral inversion method)

(*iii*) $F(z) = \dfrac{z+2}{z^2 - 2z + 1}, \; |z| > 1.$

(*b*) Solve the following by z-transform

$f(k + 2) + f(k + 1) + f(k) = 0, \; f(0) = f(1) = 1, \; k \geq 0.$ [5]

(*c*) Find the Fourier transform of

$$f(x) = \begin{cases} \cos x, & 0 < x < \pi \\ 0, & \text{otherwise} \end{cases}$$

Also write Fourier integral representation of $f(x)$. [5]

SECTION II

7. (*a*) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the moments about the mean. Also calculate coefficients of skewness and kurtosis. [8]

(b) Find the lines of regression for the following data :

x	y
10	12
14	16
19	18
26	26
30	29
34	35
39	38

and estimate y for $x = 14.5$ and x for $y = 29.5$. [9]

Or

8. (a) If 3 of 20 tubes are defective and 4 of them are randomly chosen for inspection, then what is the probability that only one of the defective tubes will be included ? [5]

(b) A manufacturer of cotter pins knows that 2% of his product is defective. If he sells cotter pins in boxes of 100 pins and guarantees that not more than 5 pins will be defective in a box, find the approximate probability that a box will fail to meet the guaranteed quality. Use Poisson distribution. [6]

(c) In a certain examination test, 2000 students appeared in a subject of Mathematics. Average marks obtained were 50% with standard deviation 5%. How many students are expected to obtain more than 60% of marks, supposing that marks are distributed normally ? [$z = 2$, $A = 0.4772$]. [6]

9. (a) Find the directional derivative of $\phi = e^{2x} \cos(yz)$ at (0, 0, 0) in the direction of tangent to the curve $x = a \sin t$, $y = a \cos t$, $z = at$ at $t = \frac{\pi}{4}$. [6]

(b) For $\overline{u} = u_1\overline{i} + u_2\overline{j} + u_3\overline{k}$ and ϕ any scalar function show that :

(i) $\nabla . (\phi\overline{u}) = \phi(\nabla . \overline{u}) + \nabla\phi . \overline{u}$.

(ii) $\nabla \times (\phi\overline{u}) = \phi(\nabla \times \overline{u}) + \nabla\phi \times \overline{u}$. [6]

(c) If a particle P moves such that at any point the position vector of P is perpendicular to the velocity vector, show that path of a particle is a circle. [4]

Or

10. (a) Show that :

$$\overline{F} = (6xy + z^3)\overline{i} + (3x^2 - z)\overline{j} + (3xz^2 - y)\overline{k}$$

is irrotational. Find scalar ϕ such that $\overline{F} = \nabla\phi$. [5]

(b) If

$$\nabla\phi = (y^2 + 2y + z)\bar{i} + (2xy + 2x)\bar{j} + x\bar{k},$$

find ϕ if $\phi(1, 1, 0) = 5$. [5]

(c) Prove that :

(i) $\nabla(\bar{r}.\bar{u}) = \bar{r} \times (\nabla \times \bar{u}) + (\bar{r}.\nabla)\bar{u} + \bar{u}$

(ii) $\nabla \times (\bar{r} \times \bar{u}) = \bar{r}(\nabla . \bar{u}) - (\bar{r} . \nabla)\bar{u} - 2\bar{u}$ [6]

11. (a) Verify Green's theorem for the field

$$\bar{F} = x^2\bar{i} + xy\bar{j}$$

over the region R enclosed by $y = x^2$ and line $y = x$. [5]

(b) Evaluate :

$$\iint_S 2x^2y\,dy\,dz - y^2\,dz\,dx + 4xz^2\,dx\,dy$$

where S is the surface enclosing a region bounded by hemisphere $x^2 + y^2 + z^2 = 9$ above the xoy plane. [6]

(c) Verify Stokes' theorem for

$$\bar{F} = xy^2\bar{i} + y\bar{j} + z^2x\bar{k}$$

for the surface of rectangular lamina bounded by $x = 0$, $y = 0$, $x = 1$, $y = 2$, $z = 0$. [6]

Or

12. (*a*) Evaluate :

$$\iint_S (2xy\bar{i} + yz^2\bar{j} + xz\bar{k}) \, . \, d\bar{S}$$

over the surface of the region bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. [6]

(*b*) If

$$\bar{E} = \nabla\phi \quad \text{and} \quad \nabla^2\phi = -4\pi\rho,$$

prove that :

$$\iint_S \bar{E} \, . \, d\bar{S} = -4\pi \iiint_V \rho \, dV.$$ [5]

(*c*) Evaluate :

$$\iint_S \text{curl}\, \bar{F} \, . \, \hat{n} \, dS$$

for the surface of the paraboloid

$$z = 9 - (x^2 + y^2)$$

where

$$\bar{F} = (x^2 + y - 4)\bar{i} + 3xy\bar{j} + (2xz + z^2)\bar{k}.$$ [6]