S.E. (COMP/IT/Electrical/Instru.) (II Sem.) EXAMINATION, 201

ENGINEERING MATHEMATICS-III

(2008 PATTERN)

Time: Three Hours

Maximum Marks: 100

- N.B. :- (i) In Section I attempt Q. No. 1 or Q. No. 2, Q. No. 3 or Q. No. 4, Q. No. 5 or Q. No. 6
 - In Section II attempt Q. No. 7 or Q. No. 8, Q. No. 9 (ii)or Q. No. 10, Q. No. 11 or Q. No. 12.
 - Answers to the two Sections anuld be written in separate (iii) answer-books.
 - Neat diagrams must be drawn wherever necessary. (iv)
 - (v) Figures to the right indicate full marks.
 - Use of electronic pocket calculator is allowed.
 - Assume suitable data, if necessary.

SECTION I

1. (a) Solve any three of the following:

[12]

(i)
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} - 3y = 3e^{-3x}\sin(e^{-3x}) + \cos(e^{-3x})$$

(ii) $(x+2)^2\frac{d^2y}{dx^2} + 3(x+2)\frac{dy}{dx} + y = \cos\log(x+2)$

(ii)
$$(x+2)^2 \frac{d^2y}{dx^2} + 3(x+2)\frac{dy}{dx} + y = \cos\log(x+2)$$

$$(iii) \frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = e^x \sin^2 x$$

(iv)
$$\frac{d^4y}{dx^4} - 2\frac{d^2y}{dx^2} - 8y = \cosh 2x + 2^{-x}$$
.

An uncharged condenser of capacity C is charged by applying (b) an e.m.f. E sin $\left(\frac{t}{\sqrt{LC}}\right)$, through leads of self-inductance L and negligible resistance. Prove that at any time t, the charge on one of the plates is

$$\frac{\mathrm{EC}}{2} \left\{ \sin \left(\frac{t}{\sqrt{\mathrm{LC}}} \right) - \frac{t}{\sqrt{\mathrm{LC}}} \mathsf{Cs} \left(\frac{t}{\sqrt{\mathrm{LC}}} \right) \right\}. \tag{4}$$

(a) Solve any three of the following: 2.

[12]

(i)
$$x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 4y = x \log x$$

(ii)
$$\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^{2x} \cdot \sec^2 x$$
 (by variation of parameters)

(iii)
$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 2y = x^2 + e^{-x} + 1$$

(iv) $\frac{xdx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}$.

$$(iv) \ \frac{xdx}{y^2z} = \frac{dy}{xz} = \frac{dz}{y^2}.$$

$$\frac{dx}{dt} + 5x - 2y = t$$

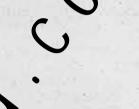
$$\frac{dy}{dt} + 2x + y = 0.$$



(a) Evaluate: 3.

$$\int_{\mathcal{C}} \frac{2z^3 + z + 5}{(z - 3)^3} dz,$$

where C is
$$\frac{x^2}{16} + \frac{y^2}{4} = 1$$
.



(b) If
$$f(z) = u(x, y) + iv(x, y)$$
 is analytic, find $f(z)$ if $u - v = x^3 + 3x^2y - 3xy^2 - y$. [6]

Find the map of the straight line 2y = x under the transformation

$$w=\frac{2z-1}{2z+1}.$$

[5]

[5]

$$f(z) = \frac{z}{(z-1)^2(z-2) \ (z-3)}$$
 at its poles and hence evaluate :
$$\int_C f(z) dz,$$

$$\int_{\mathcal{C}} f(z)dz,$$

where C is the circle
$$|z| = 4$$
.

[5]

- (b) Show that $u(x, y) = y + e^x$. cos y is harmonic function. Also find its harmonic conjugate. [6]
- (c) Find the bilinear transformation which maps the points 1 i, 2i of z-plane onto points -2i, 0, 1 of w-plane. [5]
- 5. (a) Using Fourier integral representation, show that [7]

$$e^{-5x} \cosh 3x = \frac{10}{\pi} \int_{0}^{\infty} \frac{\lambda^2 + 16}{(\lambda^2 + 4)(\lambda^2 + 64)} \cosh \lambda x \, d\lambda$$

(b) Solve the integral equation:

$$\int_{0}^{\infty} f(x) \sin \lambda x \, dx \in \begin{cases} \lambda + \lambda & 0 \le \lambda \le 1 \\ 2\lambda, & 1 < \lambda \le 2 \\ \lambda - 1, & 2 < \lambda \le 4 \\ 0, & \lambda > 4 \end{cases}$$

[5]

(c) Find the z-transform of the following (any two): [6]

(i)
$$f(k) = \left(\frac{1}{4}\right)^{|k|} \forall k$$
(ii)
$$f(k) = a^k \sin(bk + c), \quad k \ge 0$$
(iii)
$$f(k) = k \cdot 3^k \cdot 2^k \cdot 5^k, \quad k \ge 0$$

Find the inverse z-transform (any two): 6. (a)

[8]

(i)
$$F(z) = \frac{z^2}{z^2 - 7z + 12}$$
, $|z| > 4$

- (ii) $F(z) = \left(\frac{z}{z-2}\right)^2$ (by integral inversion method)
- (iii) $F(z) = \frac{z+2}{z^2-2z+1}, |z| > 1.$
- Solve the following by z-transform (b)

$$f(k + 2) + f(k + 1) + f(k) = 0, f(0) = f(1) = 1, k \ge 0.$$
 [5]

Find the Fourier transform of

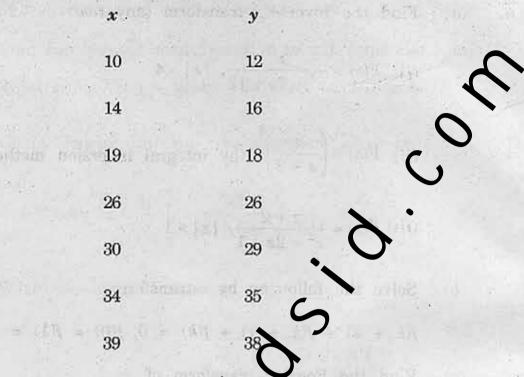
$$f(x) = \begin{cases} \cos x & 0 < x < \pi \\ 0 & \text{otherwise} \end{cases}$$

Also write Fourier integral representation of f(x).

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7. (a) The first four moments about the working mean 30.2 of a distribution are 0.255, 6.222, 30.211 and 400.25. Calculate the moments about the mean. Also calculate coefficients of skewness and kurtosis. [8]

(b) Find the lines of regression for the following data:



and estimate y for x = 145 and x for y = 29.5. [9]

- 8. (a) If 3 of 20 tubes are delective and 4 of them are randomly chosen for inspection, then what is the probability that only one of the delective tubes will be included? [5]
 - (b) A manufacturer of cotter pins knows that 2% of his product is defective. If he sells cotter pins in boxes of 100 pins and guarantees that not more than 5 pins will be defective in box, find the approximate probability that a box will fail to meet the guaranteed quality. Use Poisson distribution. [6]

- (c) In a certain examination test, 2000 students appeared in a subject of Mathematics. Average marks obtained were 50% with standard deviation 5%. How many students are expected to obtain more than 60% of marks, supposing that mark are distributed normally ? [z = 2, A = 0.4772].
- 9. (a) Find the directional derivative of $\phi = e^{2x} \cos(yz)$ at (0, 0, 0) in the direction of tangent to the curve $x = a \sin t$, $y = a \cos t$, z = at at $t = \frac{\pi}{4}$.
 - (b) For $\overline{u} = u_1 \overline{i} + u_2 \overline{j} + u_3 \overline{k}$ and ϕ any scalar function show that : $(i) \quad \nabla \cdot (\phi \overline{u}) = \phi(\nabla \cdot \overline{u}) + \nabla \phi \cdot \overline{u} \bullet$

(ii)
$$\nabla \times (\phi \overline{u}) = \phi (\nabla \times \overline{u}) + \nabla \phi \times \overline{u}$$
. [6]

(c) If a particle P moves such that at any point the position vector of P is perpendicular to the velocity vector, show that path of a particle is a circle. [4]

Or

10. (a) Show that

$$\overline{F} = (6xy + z^3)\overline{i} + (3x^2 - z)\overline{j} + (3xz^2 - y)\overline{k}$$

irrotational. Find scalar ϕ such that $\overline{F} = \nabla \phi$.

[5]

(b) If the sea white place cape and agree as

$$\nabla \phi = (y^2 + 2y + z)\overline{i} + (2xy + 2x)\overline{j} + x\overline{k},$$

find ϕ if $\phi(1, 1, 0) = 5$.

[5]

(c) Prove that:

$$(i) \quad \nabla \left(\overline{r} \, . \, \overline{u} \right) = \overline{r} \times (\nabla \times \, \overline{u}) + (\overline{r} \, . \, \nabla) \overline{u} \, + \overline{u}$$

$$(ii) \quad \nabla \times (\overline{r} \times \overline{u}) = \overline{r} \ (\nabla \cdot \overline{u}) - (\overline{r} \cdot \nabla) \overline{u} - 2\overline{u} \qquad \bullet \qquad [6]$$

11. (a) Verify Green's theorem for the field

$$\overline{F} = x^2 \overline{i} + xy \overline{j}$$

over the region R enclosed by $y = x^2$ and line y = x. [5]

(b) Evaluate:

$$\iint\limits_{S} 2x^2ydydx - y^2dzdx + 4xz^2dxdy$$

where S is the surface enclosing a region bounded by hemisphere $x^2 + y^2 + z^2 = 9$ above the xoy plane. [6]

(c) Verify Mokes' theorem for

$$\overline{\mathbf{F}} = xy^2 \, \overline{i} + y \, \overline{j} + z^2 x \, \overline{k}$$

for the surface of rectangular lamina bounded by x = 0,

$$y = 0, x = 1, y = 2, z = 0.$$

12. (a) Evaluate:

$$\iint\limits_{S} (2xy\overline{i} + yz^2\overline{j} + xz\overline{k}) \cdot d\overline{S}$$

over the surface of the region bounded by x = 0, z = 0 and x + y + z = 1. [6]

(b) If

$$\overline{E} = \nabla \varphi \quad \text{and} \quad \nabla^2 \varphi = - \, 4 \pi \rho \, . \label{eq:energy_energy}$$

prove that:

$$\iint_{S} \overline{E} \cdot d\overline{S} = -4\pi \iint_{V} V .$$
 [5]

(c) Evaluate:

$$\iint\limits_{\mathbf{S}} \operatorname{cod} \bar{\mathbf{F}} \cdot \hat{n} \, d\mathbf{S}$$

for the surface of the paraboloid

$$z = 9 - (x^2 + y^2)$$

where

$$\overline{F} = (x^2 + y - 4)\overline{i} + 3xy\overline{j} + (2xz + z^2)\overline{k}$$
. [6]