Total No. of Questions—**12**] [Total No. of Printed Pages—**8**

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S.E. (Comp./IT/Electrical/Instru.)

(Second Sem.) EXAMINATION, 2011

ENGINEERING MATHEMATICS—III

(2008 PATTERN)

Time : Three Hours **Maximum Marks : 100**

***N.B.* :—** (*i*) In Section I, attempt Q. No. **1** *or* Q. No. **2**, Q. No. **3** *or* Q. No. **4**, Q. No. **5** *or* Q. No. **6**.

(*ii*) In Section II, attempt Q. No. **7** *or* Q. No. **8**, Q. No. **9** *or* Q. No. **10**, Q. No. **11** *or* Q. No. **12**.

(*iii*) Answers to the two Sections should be written in separate answer-books.

(*iv*) Figures to the right indicate full marks.

(*v*) Use of electronic pocket calculator is allowed (Non-Programmable).

(*vi*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 + 3D + 2)y = e^{e^x} + \cos e^x$

(*ii*) $(D^2 - 4D + 3)y = x^3 e^{2x}$

P.T.O.

(*iii*) $(D^2 + 4)y = \tan 2x$ (by using method of variation of parameters)

(*iv*) $(x+1)^2 \dfrac{d^2y}{dx^2} + (x+1)\dfrac{dy}{dx} - y = 2\log(x+1) + x - 1.$

(*b*) Solve :

$$\frac{dx}{dt} + 5x - 2y = t, \quad \frac{dy}{dt} + 2x + y = 0.$$ [5]

Or

2. (*a*) Solve any *three* of the following : [12]

(*i*) $(D^2 - D - 2)y = 2\log x + \dfrac{1}{x} + \dfrac{1}{x^2}$

(*ii*) $(D^2 - 2D)y = e^x \sin x$

(by using method of variation of parameters)

(*iii*) $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x$

(*iv*) $\dfrac{dx}{y} = \dfrac{dy}{-x} = \dfrac{dz}{2x - 3y}.$

(*b*) An electric circuit consists of a capacitor of 10^{-3}F is in series with an emf of 20 V and inductor of 0.4 H. If charge Q and current I are zero at time $t = 0$, find charge Q at time t. [5]

3. (*a*) If $v = 3x^2y - y^3$, find the analytic function $f(z) = u + iv$ in terms of z. [5]

(*b*) Find bilinear transformation which maps the points $1, \ i, -1$ of z-plane into the points i, 0, $-i$ of w-plane respectively. [5]

(*c*) Evaluate :

$$I = \oint_C \frac{4z^2 + z}{(z-1)^2} dz$$

where 'C' is contour $|z - 1| = 2$. [6]

Or

4. (*a*) Show that analytic function with constant modulus is constant. [6]

(*b*) Show that the map $w = \dfrac{2z+3}{z-4}$ transforms the circle $x^2 + y^2 - 4x = 0$ into the straight line $4u + 3 = 0$. [5]

(*c*) Evaluate :

$$\int_0^{2\pi} \frac{d\theta}{5 + 3\cos\theta},$$

using Cauchy's theorem. [5]

5. (*a*) Find Fourier transform of

$$f(x) = \begin{cases} 1 & -2 \le x < 0 \\ -1 & 0 < x \le 2 \end{cases}$$

Hence show that :

$$\int_0^{\infty} \frac{(\cos 2x - 1)\sin 2x}{x} dx = \frac{-\pi}{2}.$$ [6]

(*b*) Find Fourier cosine transform of the function

$$f(x) = 2e^{-5x} + 5e^{-2x}.$$ [5]

(*c*) Find z-transform of (any *two*) : [6]

(*i*) $f(k) = \sin(3k + 4) \ldots\ldots k \geq 0$

(*ii*)

(*iii*) $f(k) = (k + 2)2^k \ldots\ldots\ldots k \geq 0.$

Or

6. (*a*) Find inverse z-transform of (any *two*) : [8]

(*i*) $F(z) = \dfrac{1}{(z-2)(z-3)}$ (by Inversion integral method)

(*ii*) $F(z) = \dfrac{z(z+1)}{z^2 - 2z + 1}$ if $|z| > 1$

(*iii*) $F(z) = \dfrac{z^2}{\left(z - \frac{1}{4}\right)\left(z - \frac{1}{5}\right)}$ if $|z| < \dfrac{1}{5}$

(*b*) Solve the difference equation : [4]

$$f(k+1) - f(k) = 1, \; f(0) = 0.$$

(*c*) Solve the integral equation : [5]

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda} \quad \lambda > 0$$

SECTION II

7. (*a*) Find the first four moments about mean for the following distribution. Also find β_1 and β_2 : [8]

Marks	No. of Students
0–10	1
10–20	6
20–30	10
30–40	15
40–50	11
50–60	7

(*b*) Two lines of regression are given by $5y - 8x + 17 = 0$ and $2y - 5x + 14 = 0$. If $\sigma_y^2 = 16$, find :

(*i*) the mean value of x and y

(*ii*) σ_x^2

(*iii*) the coefficient of correlation between x and y. [9]

Or

8. (*a*) A husband and wife appear in an interview for two vacancies in the same post. The probability of husband selection is $\frac{1}{7}$ and that of wife's selection is $\frac{1}{5}$. What is the probability (*i*) both of them will be selected, (*ii*) only one of them will be selected and (*iii*) none of them will be selected ? [6]

(*b*) One percent of articles from a certain machine are defective. What is the probability of (*i*) no defective, (*ii*) one defective and (*iii*) two or more defective in a sample of 100. [6]

(*c*) Mean and variance of binomial distribution are 6 and 2 respectively. Find $P(r > 1)$. [5]

9. (*a*) Find the directional derivative of $\phi(x, y, z) = x^2yz + 4xz^2$ at $(1, -2, 1)$ in the direction of $2i - j - 2k$. Find the greatest rate of increase of ϕ. [5]

(*b*) A fluid motion is given by $\bar{v} = (y \sin z - \sin x) + (x \sin z + 2yz)\hat{j} + (xy \cos z + y^2)\hat{k}$. Is the motion irrotational. If so, find the velocity potential. [6]

(*c*) Find curl curl $\bar{F}$ at the point $(0, 1, 2)$, where

$$\bar{F} = x^2y\, i + xyz\, j + z^2y\, k.$$ [5]

Or

10. (*a*) Attempt any *two* : [6]

(*i*) Prove that :

$$\nabla \times \left(\frac{\bar{a} \times \bar{r}}{r^n} \right) = \frac{(2 - n)\bar{a}}{r^n} + \frac{n(\bar{a} \cdot \bar{r})}{r^{n+2}} \bar{r}.$$

(*ii*) Prove that :

$$\bar{b} \times \nabla (\bar{a} \cdot \nabla \log r) = \frac{\bar{b} \times \bar{a}}{r^2} - \frac{2(\bar{a} \cdot \bar{r})(\bar{b} \times \bar{r})}{r^4}.$$

(*iii*) Find the value of $\nabla^2 (r^n \log r)$.

(*b*) If

$$\bar{r} \cdot \frac{d\bar{r}}{dt} = 0,$$

then show that $\bar{r}$ has constant magnitude. [5]

$$\bar{F} = (2x + y^2)i + (3y - 4x)j$$

(*c*) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2). [5]

11. (*a*) If

then evaluate $\int_C \bar{F} \cdot d\bar{r}$ around the parabolic arc $y^2 = x$ joining (0, 0) and (1, 1). [5]

(*b*) Evaluate :

$$\iint_S (x^3 i + y^3 j + z^3 k) d\bar{S}$$

where S is the surface of the sphere $x^2 + y^2 + z^2 = 16$. [6]

(*c*) Evaluate

$$\oint_C \overline{F} \cdot d\overline{r}$$

by Stokes' theorem, where $\overline{F} = y^2\hat{i} + x^2\hat{j} - (x + z)\hat{k}$ and C is the boundary of the triangle with vertices (0, 0, 0), (1, 0, 0) and (1, 1, 0). [6]

Or

12. (*a*) Verify Green's Lemma in the plane for

$$\oint (3x^2 - 8y^2)dy + (4y - 6xy)dy$$

where C is the boundary defined by $x = 0$, $y = 0$ $x + y = 1$. [5]

(*b*) A vector field is given by $\overline{F} = (\cos y)i + x(1 - \sin y)\, j$. Evaluate the line integral over the circular path given by $x^2 + y^2 = a^2$, $z = 0$. [6]

(*c*) Evaluate :

$$\iint_S \nabla \times \overline{P} \cdot d\overline{S}$$

for $\overline{F} = y\hat{i} + z\hat{j} + x\hat{k}$, where S is the surface of the paraboloid $z = 1 - x^2 - y^2$, $z > 0$. [6]