MODEL QUESTION PAPERS

For candidates admitted from 2007-2008 and onwards

M.Sc Degree Examination

First Semester

Time:3Hours **Paper I -ALGEBRA** **Max.Marks:75**

Answer All Questions

Section A –(10*1=10 marks)

Choose the correct answer:

1.The number of conjugate class in S5 is

a)5 b) 7 c)3 d)4

2.If O(G)=28 then G has a normal subgroup of order

a) 6 b) 8 c) 7 d) 9

3.An element a in a Euclidean ring R is a unit if

a) d(a)=1 b)d(a)=0 c) d(a)=d(1) d) d(a)=d(0)

4.The units in Z[i] are

a) ± 1b)$\pm$ i c) $\pm 1, \pm$ i d)1,i

5.If L,K ,F are the finite fields such that L$\subset$ K$\subset$F then [L:F] is

a)[L:K][K:F] b)[L:F][F:K] c)[K:L][L:F] d)[F:K][K:L]

6.If F is the of rational numbers and if f(x)= x^3-2 then [f(2):f] is

a)2 b) 3 c)1/3 d)1/2

7. If F is the field of real numbers and K is the field of complex numbers then O(G(K,F)) is

a)2 b) 3 c)1 d)0

8.If H is the subgroup of G(K,F) and K_H is the fixed field of H then [K:K_H] is

a)O(K)b)O(H)c)O(K/K_H) d)O(K_H)

9.If T$\in$ A(V)is such that (vT,v)=0$\forall$ v$\in$ V then T is

a) I b) o c)T-1 d) none of these

10. If A transformation T is normal if

a) TT* =TT* b) TT* =I c) TT* =0 d) T=T*

Section B (5$\times$5=25 marks)

11. a)Prove that the relation conjuncy is an equivalence relation

or

b) If O(G) =55 then Prove that its 11-sylow subgroup is normal

12. a)Prove that every Euclidean ring has a unit element

or

b) State and prove Euclid's lemma

13. a)Prove that the set of algebraic elements in K over F form a subfield of K

or

b)If f(x)=F[x] is of degree n$\geq$1 then Prove that there is an extension E of F of degree atmost $\lfloor$ n in which f(x) has n roots

14 a) If F is field of real numbers and K is the field of complex numbers then prove that K is an extension of F.

or

b)If F and K are the two finite fields $\ni$F$\subset$K and if O(F)=q then Prove that O(K)=q^n where n=[K:F]

15.a) If A,B $\in$ Fn then prove that tr(AB)=tr(BA)

or

. b) If T$\in$ A(V) then prove that T* is also in A(V)

Section C–(5× 8=40marks)

16.a) If G is the finite group then prove that O(C(a))=O(G)/O(N (a))

or

b) State and prove Sylow's second theorem

17 a) Let R be an Euclidean ring and A be an ideal of R .Then prove that A=(a_0) for some $a_o \in$ A

or

b) Prove that J[i] is an Euclidean ring.

18 a)If a∈ K is an algebraic over F of degree n then prove that [F(a):F]=n

or

b) Prove that a polynomial of n degree over then F can have atmost n roots in any extension field .

19 a) If K is a field and if $\sigma_1, \sigma_2, \ldots\ldots, \sigma_n$ are distinct automorphisms of K then prove that it is impossible to find elements a1,a2, ….,an not all of them 0 such that

$a_1\sigma_1(u)+ a_2\sigma_2(u)+\ldots\ldots+ a_n\sigma_n(u) =0 \ \forall u$

or

b)If K is a finite extension of field F then prove that O(G(K,F)) ≤[K:F]

20 a) If T∈ A(V) has all its characteristic roots in F then prove that there is a basis of V in which the matrix of T is regular

or

b) If F is a field of characteristic 0 and if T∈ A(V) if such that $trT^i=0 \forall i \geq 1$then prove that T

is nilpotent.

MODEL QUESTION PAPER

For candidates admitted from 2007-2008 and onwards

M.Sc Degree Examination

Third Semester

MATHEMATICS

Time:3Hours TOPOLOGY Max.Marks:75

Answer All Questions

Section A –(10*1=10 marks)

Choose the correct answer:

1. In a topology of a set X, both X and ϕ are

   a) only open b) only closed c) both open and closed d) neither open nor closed

2.Assume that Y is a subspace of X .If U is open in Y and Y is open in X then U

a) closed in X b) open in X c) Y is both open and closed in X d) none of these

3. Let C, D be a separation of X,Y be a connected subset of X then

a)Y lies in C b) Y lies in D c) Y lies in C or D d) none of these

4. If L is a linear continuum in the order of topology then L is

a)disconnected b)connected c) empty d)the whole space X

5. The set $A=\{x\times 1/x \,/ 0<x\leq 1\}$ in $R^2$is

a) compact b) closed and compact c) closed but not compact d) none of these

6. A space X is said to be separable if it has

a) a countable topology b) a countable basis c) a countable sub basis d) none of these

7. A product of normal spaces

a) is normal b) need not be normal c)not regular d) none of these

8.Two subsets A and B of a space X are said to be separated by a continuous function $f:X \to [0,1]$ such that

a)f(A)=f(B)=0 b) f(A)= 0 ,f(B)=1 c f(A)=f(B)=1 d) none of these$\times$

9. The space $S_{\bar{\Omega}} \times S_{\Omega}$ is

a) completely regular and normal b)normal c) not normal d) completely regular and not normal

10. The Stone –Check compactification $\beta(X)$ is

a)X b) unique c)not unique d) $X \subseteq \beta(X)$ and $\beta(X)$is unique

Section B ($5\times5=25$ marks)

11 a) Assume that β and β' are respectively bases for the topologies $\mathcal{J}$ and $\mathcal{J}'$. Show that ' is finer than

iff for each $x \in X$ and basis element $B \in \beta'$ containing x there is a basis element $B' \in \beta'$ such that

$x \in B' \in \beta'$.

or

b)Show that $\bar{A} = A \cup A'$

12. a) Show that the union of a collection of connected sets that a have a point in common is connected

or

b) State and prove Intermediate value theorem.

13 a) Show that every compact Hausdorff space is normal

or

b) Show that a subspace of a regular space is regular and product of regular spaces is regular

14 a)Let $A \subset X$ and $f:A \rightarrow Z$ be a continuous map of A into Hausdorff space Z .Show that there is atmost

one extension of f to a continuous function $g: \bar{\pi} \rightarrow Z$

or

b)If Y is complete under d ,Show that Y^J is complete in the union metric $\bar{\rho}$

15 a)If X is locally compact or if X satisfies the first axiom of countability , show that X is compactly

generated .

or

b)If $C_1 \supset C_2 \supset C_3 \supset$ ….. is a nested sequence of nonempty closed sets in a complete metric space X,

and diam $C_n \rightarrow 0$,show that $\cap C_n \neq \phi$.

Section C–(5*8=40marks)

16 a) Let $f:X \rightarrow Y$ be a map between two spaces X and Y .Show that the following statements are

equivalent

i) f is continuous

ii) $f(\bar{A}) \subset \overline{f(A)}$,for every subset A of X

iii) f-1(B) is closed in X , when ever B is closed in Y

or

b) Show that the topologies on R_n , induced by d and are the same as the product topology on R_n .

17 a) Show that Cartesian product of connected spaces is connected .

or

b) Show that the product of finitely many compact spaces is compact.

18 a) Show that every regular space with a countable basis is normal.

b) State and prove the Uryshon ‘s lemma

19 a)Show that there is an isomorphic imbedding of a metric space (X ,d) into a complete metric space .

or

b) State and prove the Ascoli’s theorem.

20 a) Let h : [0,1] $\rightarrow$ R be a continuous function .For any $\in$ >0,show that there is a function

g:[0,1] $\rightarrow$R with $|h(x)-g(x)| < \in$ for all $x \in X$,such that g is continuous and no where differentiable .

or

b) State and prove Tietz –extension theorem.

MODEL QUESTION PAPER

For candidates admitted from 2007-2008 and onwards

M.Sc Degree Examination

Second semester

MATHEMATICS

Time:3Hours **PARTIAL DIFFERENTIAL EQUATIONS** **Max.Marks:75**

Answer All Questions

Section A –(10*1=10 marks)

Choose the correct answer:

1. In u_{tt} = c2 u_{xx} which describes the vibration of a stretched string , is

   a) P/T b) T/P c)PT d) none of these

2. Along the curve ξ = constant it is true that dy/dx =

   a) $-\xi_x / \xi_y$ b) ξ_x / ξ_y c) ξ_y / ξ_x d) ξ_y / ξ_x

3. The two characteristics of u_{xx} _ u_{yy} =1 are

   a) straight lines b) parabolas c) ellipses d)rectangular hyberbolas

4. A sine wave traveling with speed C in the negative x-direction without changing its shape is
given by

   a)sin(x-ct) b)sin(x - (t/c)) c)sin(x + ct) d)sin(x + (t/c))

5. The inequality $\left| -a_n (n\pi/l)^2 K e^{-(n\pi/l)^2 kt} \sin(n\pi x/l) \right| \leq C(n\pi/l)^2 K e^{-(n\pi/l)^2 kt_0}$ holds if

   a) $t<t_0$ b) $tt_0 \leq 1$ c) $t \geq t_0$ d) $tt_0 \geq 1$

6. The solution of X ” + α^2 X =0 satisfying X’(0)=X’(a)=0 is

   a) A sin($n\pi x/a$) b) A cos($n\pi x/a$) c) A sin($nx/\pi a$) d) A cos($nx/\pi a$)

7. If u (x,y) is harmonic in a bounded domain D and continuous in D = D+ B , then u attains

its minimum on

a) D b) the boundary B of D c) any point in D d) none of these.

8. In the Neumann problem for a rectangular there is the compatibility condition to be satisfied

a) False b) Always True c) Occasionally true d) None of these

9. If $\delta(x-\xi)$ and $(y-\eta)$ are one dimensional delta functions,

then $\iint_R F(x,y)\,\delta(x-\xi)\,\delta(y-\eta)\,dxdy$ is

(a) F(x , y) b) F(x, η) c) F(ξ ,y) d) F(ξ ,η)

10 The equation $\nabla^2 u + k^2 u=0$

a) Laplace b) Poisson c) Helmholtz d) D' Alembert s

Section B (5×5=25 marks)

11 a) List any four assumptions made in the derivations of the equations of the vibrating membrane.

Or

b)Find the general solution of $x^2 u_{xx} + 2xy\, u_{xy} + y^2 u_{yy} = 0$

12 a) Define the Cauchy data for the equation $A u_{xx} + Bu_{xy} + Cu_{yy} = F(x, y, u, u_x, u_y)$

Or

b) Interprêt the D' Alembert s formula when g(x) = 0

13 a) Obtain u (x , t) = X(x)T(t) for $u_{tt} + a^2 u_{xxxx} = 0$

Or

b)Obtain u (x , t) = X(x)T(t) for $u_{tt} = k\,u_{xx}$, $0 < x < l$ satisfying

u (o ,t) = u (, t) , $t \geq 0$

14 a)Prove the solution of the Direchlet problem , if it exists , is unique .

Or

b) Explain the method of solution of the Direchlet problem involving the Poisson equation

15 a) state the three properties to be satisfied by the Green's function for the Dirchilet problem involving the Laplace operator

Or

b) Show that $\partial G/\partial n$ is discontinuous at (ξ,η) and and $\lim_{\in \to 0} \int_{C\in} \partial G/\partial n \, ds = 1$,

$C\in : (x-\xi)^2 + (y-\eta)^2 = \in^2$

SECTION – C (5×8 =40 marks)

16 a) Reduce $x^2 u_{xx} + 2xy\,u_{xy} + y^2 u_{yy} = 0$ to the canonical form

Or

b) Reduce $u_{xx} + x^2 u_{yy} = 0$ to the canonical form

17 a) Solve $u_{xx} - u_{yy} = 1$, $u(x, 0) = \sin x$, $u_y(x,0) = 0$

Or

b) Solve $u_{tt} = c^2 u_{xx}$, $0<x<l$,$t>0$ satisfying $u(x, 0) = \sin(\pi x/l)$, $u_t(x,0) = 0$,

$0 \leq x \leq l$ and $u(0,t) = u(l,t) = 0$, $t \geq 0$

18 a)Prove that there exist atmost one solution of the wave equation $u_{tt} = c^2 u_{xx}$

$0<x<l$, $t>0$ satisfying initial conditions $u(x, 0) = f(x)$, $u_t(x,0) = g(x)$, $0 \leq x \leq l$ and the boundary conditions $u(l, t) = 0$,$t \geq 0$ where u(x, t)is a twice continuously differentiable function with respect to both x and t

Or

b) solve $\Delta^2 u = 0$, $0<x<a$ $0<y<b$ given that $u(x, 0) = f(x)$, $0 \leq x \leq a$

$u(x, b) = u_x(0,y) = u_x(a,y) = 0$,

19 a) Find the solution of the Direchlet problem $\Delta^2 u = -2$, $r < a$, $0 < \theta < 2\pi$,

u(a ,0)=0

Or

b) prove that the solution of the Direchlet problem depends continuously on the boundary data

20 a) Apply the eigen function method to obtain Green's function of the Direchlet problem in the rectangular domain

Or

b) Solve the Direchlet problem in the rectangular domain

MODEL QUESTION PAPER

For candidates admitted from 2007-2008 and onwards

M.Sc Degree Examination

First semester

MATHEMATICS

Time:3Hours **NUMERICAL METHODS** **Max.Marks:75**

Answer All Questions

Section A –(10 $\times$1=10 marks)

Choose the correct answer:

1) Newton's method is also called____________

a)Newton's Raphson b) Picard's rule c) Euler d) none of these

2) Bairstows method is used to find ___________ of a polynomial

a) complex roots b) real roots c) repratred roots d) none of these

3) ____________ is a direct method to solve a system of equation

a) Fixed rule b) Runge kutta c) Gauss Elimination d) none of these

4) ____________ iteration method to solve a system equation converges faster

a) Gauss Jordan b) Gauss seidal c) Taylor's d) none of these

5) Value of y at specifed values of x can be found from ____________ methods coming under I Category

a) Adams Bashforth b) Euler c)Bairstow d) none of these

6) ____________ is a mulistep method

a) Milne b) Adam Moulton c) Euler d) none of these

7)1-D heat equation is an example of ____________ equation

a)parabolic b) Exponential c) Hyberbolic d) none of these

8) $u_{xx} + u_{yy} = f(x, y)$ is called ____________ equation

a)Parabola b)Eponential c)Hyberbolic d) none of these

9) ____________ method is used to find the largest eigen value

a) Power b) Relaxation c) Poission d) none of these

10) Pictorial operator $\Delta^2 u_{ij} =$ ____________

a) $\left\{\begin{matrix} & 1 & \\ 1 & -4 & 1 \\ & 1 & \end{matrix}\right\} u_{ij}$ b) $\left\{\begin{matrix} & 0 & \\ 0 & 1 & 0 \\ & 0 & \end{matrix}\right\} u_{ij}$ c) $\left\{\begin{matrix} & -1 & \\ -1 & -4 & -1 \\ & -1 & \end{matrix}\right\} u_{ij}$

d) None of these

SECTION B(5×5 =25)

11 a) Using Newton's method find the root between 0 &1 of x3=6x-4 correct to 3 decimal places

Or

b) Evaluate $\int_0^1 (1/1+x2)dx$ using Romberg method to find the approximate value of π

12 a) By Gauss Elimination method solve x+2y+z=3; 2x+3y+3z=10 ; 3x-y+2z =13

Or

b)Find the inverse of $A=\begin{pmatrix} 1 & -2 & 2 \\ 4 & 1 & 2 \\ 2 & 3 & -1 \end{pmatrix}$

13 a) Using Taylor's series method solve dy/dx = x+y given y(1)= 0 , find y(1.1)andy(1.2)

Or

b) Using Modified Euler method solve $dy/dx = x^2+y^2$ given y(0)= 1 , find y(0.1)andy(0.2)

14 a) Solve the boundary value problem $d^2x/dt^2 - (1-(t/5))x = t$ x(1)=2,x(3)=-1 using Shooting method assuming ,x'(1)=-1.5 and x'(2)= -3 *or*

b) find the largest eigen value and vector of $A=\begin{pmatrix} 2 & -1 & 0 \\ -1 & 2 & -1 \\ 0 & -1 & 2 \end{pmatrix}$

Or

15 a) Derive the Laplace equation to the pictorial form

Or

b) Solve the elliptic equation $u_{xx} + u_{yy} = 0$ for

1 2

	u1	u2
	u3	u4

Left side: 1, 2; right side: 4, 5

4 5

SECTION – C (5×8 =40 marks)

16 a) The population of a town is as follows .Estimate the population increase during 1946 to 1976

year	x	:	1941	1951	1961	1971	1981	1991
poulation lakhs	y	:	20	24	29	36	46	51

Or

b) Using Bairstows method obtain the quadratic factor of $x^4-5x^3+20x^2-40x+60 =$ (taking $p_0=-4$,$q_0=8$)

17 a) Using Gauss Sedial method solve x+6y+10z = -3; 4x-10y+3z=-3 ; 10x-5y-2z =3

Or

b)By Relation method solve 10 x-2y-2z=6; -x+10y-2z=7 ;-x-y+10z =8

18 a) Using Runge gutta method solve dy/dx = x+y given y(0)= 1 , find y(0.2)

Or

b) Using Milne's method find y(4.4) given $5xy'+y^2 -2 = 0$,y(4)=1, y(4.1)=1.0049 y(4.2)=1.0097, y(4.3)=1.0143

19 a) $d^2y/dx^2 =y$, y'(1)=1.1752, y'(3)=10.0179 convert this to a difference equation normalize to [0,1]with h =0.25

Or

b) Solve the Characteristic value problem $d^2y/dx^2 +k^2y=0$; y(0)=0; y(1)=0 . Convert this to a difference equation .What can you say about k ?

20 a) Solve the Laplace equation for the square region in the fig with boundary values (up to 3 iterations only) 11.1 17.0 19.2 18.6

u_{11}	u_{12}	u_{13}	
u_{21}	u_{22}	u_{23}	

0		u_{31}	u_{32}	u_{33}		21.0
0						21.0
0						17.0
0	9.0					
		8.7	12.1	12.8		

Or

b) Solve the heat flow problem

$u(x,0)=\sin \pi x/2$

$u(0, t)= 0$, $u(2 ,t)=0$

M.Sc. DEGREE MODEL QUESTION PAPER

(For the candidates admitted from 2007 onwards)

FIRST SEMESTER

Mathematics

Time : 3hours Ordinary Differential Equations Max. marks:75

SECTION A- (10 × 1=10 marks)

Choose the best option

1) The power series solution for $x'=\exp(-t^{-2}),x(0)=0$

a)exists b) does not exist c) exist and is not unique d) none of these

2) The Legendre polynomials form an orthogonal set of functions with weight function

a) unity on [0,1] b)unity on [0,1] c)t on [0,1] d) none of these

3) For a differentiable matrix X, dX-1/dt is

a) $-X^{-2}$ dX/dt b) –dX/dt X^{-2} c) –XdX/dt X^{-1} d) none of these

4) The fundamental system of solutions for the system $x^{-1} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$

a) $\begin{bmatrix} e^t \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ e^{2t} \end{bmatrix}$ b) $\begin{bmatrix} 0 \\ e^t \end{bmatrix}, \begin{bmatrix} e^{2t} \\ 0 \end{bmatrix}$ c) $\begin{bmatrix} e^t \\ e^{2t} \end{bmatrix}, \begin{bmatrix} 0 \\ e^t \end{bmatrix}$ d) none of these

5) The solution of matrix differential X'= - AX , X(0) = -E is

a) e^{-tA} b) $-e^{tA}$ c) $-e^{-tA}$ d) $-Ee^{tA}$

6) A linear system $x^{-1} = A x$ admits a non-zero periodic solution of period w iff $E-e^{Aw}$ is

a) singular b)non-singular c) both d) none o f these

7) If $A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$,then e^{At} is

a) $\begin{pmatrix} \cosh t & \sinh t \\ \sinh t & \cosh t \end{pmatrix}$ b) $\begin{pmatrix} \cos t & \sin t \\ \sin t & \cos t \end{pmatrix}$ c) $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ d) none o f these

8) The first approximation x1(t) of the IVP $x' = x/(1+x^2)$, x(0)=1 is

a) 1+t b) 1 c) $1/(1+t^2)$ d) t/2 +1

9) The equation x'' + x=0 ,t ≥ 0 is

a) oscillatory b) non-oscillatory c) none o f these

10) If x(t) is a solution of x''+a(t) x=0 ,t ≥ 0 where a(t)<0 is a continuous function for t ≥ 0 then x(t) has

a) atmost one zero b) no zero c)atleast one zero d) none of these

SECTION B- (5 × 5=25 MARKS)

11 a) Prove that $P_n(t)=1/2^n\, n!\, d^n/dt^n\,(t^2-1)n$

Or

b) Prove that $t^{1/2} J_{1/2}(t) = \sqrt{2}\ \sin t / \Gamma(1/2)$

12 a) Solve x’’-2x’ +x=0 , x(0)=0 , x’(0)=1

Or

b) Prove that a solution matrix ϕ of X’= A(t)X, $t\in I$ is a fundamental matrix of $x^{-1}=A(t)\bar{x}$ on I iff det $\phi(t)\neq 0$

13 a) Find the fundamental matrix of $x^{-1}=A\bar{x}$,where A= $\begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix}$ Or

b) State and prove Floquet theorem

14 a) Compute the first three successive approximations for the solution the following equation: $x' = x^2$, x(0)=1.

Or

b) Find the constants L, k, h for the IVP $x' = x^2 + \cos^2 t$, x(0 =1, R={(t, x): $0 \le t \le a$, $|x| \le b$, $a \ge ½$, b>0.

15 a) State and prove Strum’s separation theorem.

Or

b) If a(t) in $x'' + a(t)x = 0$ is continuous on $(0, \infty)$ and if $a^* = \lim_{t\to\infty} \sup t f(t) < ¼$ then prove that $x'' + a(t)x = 0$ is non oscillatory.

SECTION C- (5 x 8 = 40 MARKS)

16 a) Solve: $x''-2tx'+2x = 0$

Or

b) If $a_1, a_2, \ldots$, be the positive zeros of the Bessel function $J_p(t)$, then prove that

$$\int_0^1 t\, J_p(amt)\, J_p(ant)\, dt = \begin{cases} 0, & m \neq n \\ 1/2\, J^2_{p+1}(a_n), & m = n \end{cases}$$

16 a) State and prove existence and uniqueness theorem on IVP.

Or

b)Find the four approximations of a solution to

$x'' - 2x' + x = 0,\ x(0)=0,\ x'(0)=1.$

18 a) Find the general solution of $x^{-1} = \begin{pmatrix} 0 & 0 & 6 \\ 1 & 0 & -11 \\ 0 & 1 & 6 \end{pmatrix} \bar{x}$.

Or

b) Determine e^{tA} and a fundamental matrix for the system

$$x^{-1} = \begin{pmatrix} -1 & 2 & 3 \\ 0 & -2 & 1 \\ 0 & 3 & 0 \end{pmatrix} \bar{x}$$

19 a) State and prove Picard's theorem. (or)

b) State and prove the theorem on non-local existence of solution of IVP

$\bar{x}' = f(t, x),\ x(t_0) = x_o.$

20 a) Show that $x'' + a(t)x' + b(t)x = 0,\ t \geq 0$, where $a'(t)$ exists and is continuous for $t \geq 0$ is oscillatory iff $x'' + c(t)x = 0,\ c(t) = b(t) - 1/4\, a^2(t) - 1/2 a'(t)$ is oscillatory.

Or

b) State and prove Hille – Wintner comparison theorem.

M.Sc. DEGREE MODEL QUESTION PAPER

(For the candidates admitted from 2007 onwards)

THIRD SEMESTER

BRANCH I - Mathematics

Time : 3hours ELECTIVE I - NUMBER THEORY Max. marks:75

Answer All questions

Section A- (10 $\times$ 1=10 marks)

Choose the best option

1. The g.c.d of a and a+3 for any integer a is

   a) a b) a or 3 c) 1 or 3 d) 1or a

2. If (a, m) = g, ax $\not\equiv$ b (mod m) and g does not divide b then the number of solutions of this congruences is

   a) -1 b) 2 c)1 d)0

3. State Euclidean algorithm.

4. The number of solutions modulo 35

   15x ≡ 25 (mod 35) is ______________.

5. (963, 657) is

   a) 9 b) 27 c) 3 d) 12

6. The value of Φ(1896) is

   a) 246 b) 426 c) 624 d) 524.

7. Reduced residue system modulo 30 is

   a) 1 b)6 c)3 d)none of these

8. The degree of congruence 6x+7 ≡ 0 (mod 3) is 2

   a) 8 b) 2 c)1 d)0

9. The number of primitive roots of 29 is

   a) 28 b) 12 c)1 d)0

10. G = { 7, -2, 17, 30, 8, 3} is a group under addition modulo 6. The additive inverse of 8 in this group is

    a) 7 b)17 c) -2 d) 3

SECTION – B(5 X 5 = 25 marks)

11. a) If (a, m) = (b, m) = 1. Prove that (ab, m) = 1.

Or

b) State and prove Euclid's theorem.

12. a) Let p be a prime, show that $x^2 \equiv (-1)$ (mod p) has a solution iff p= 2 or p ≡ 1(mod 4).

Or

b) Solve the congruence $6x \equiv 3 \pmod 9$.

13. a) If a∈ exponent modulo h, prove that h | (j-k).

Or

b) Let m>1 be a positive integer. Prove that any reduced residue system modulo is a group under multiplication modulo m.

14. a) Evaluate $\left(\frac{-23}{83}\right)$

Or

b) If Q is add and $\phi > 0$, prove that

$$\left(\frac{-1}{Q}\right) = (-1)^{\frac{Q-1}{2}} \text{ and } \left(\frac{2}{Q}\right) = (-1)^{\frac{Q^2-1}{8}}$$

15. a) State and prove Moebius inversion formula.

Or

b) Show that an arithmetic function has a multiplicative inverse iff f(1)≠0. If the inverse exists, is it unique?

SECTION – C (5 X 8 = 40 marks)

16. a) If g is the g.c.d. of b and c, prove that there exists integers x_0, and y_0 such that $g = (b, c) = bx_0 + c\, y_0$.

Or

b) (i) If m>0, prove that [ma, mb] = m [a, b] and [a, b] = $|ab|$

(ii) Show that every integer n>1 can be expressed as product of primes.

17. a) State and prove Wilson's theorem.

Or

b) Solve $x^2 + x + 7 \equiv 0 \pmod{189}$

18. a) Prove that the set $z_m = \{0, 1, \ldots, m-1\}$ for m>1 is a ring with respect to

addition and multiplication defined modulo m. prove also that z_m is a field iff m is a prime.

Or

b) State and prove lemma of Gauss.

19. a) State and prove the Gaussian reciprocity law.

Or

b) Prove that $\frac{(ab)!}{a!(b!)^a}$ is an integer.

20. a) Let f(n) be a multiplicative function and let $F(n) = \sum_{d\backslash n} f(d)$. Prove that F(n) is mulplicative.

Or

b) If $F_0 = 1, F_1 = 1, F_{n+1} = F_n + F_{n-1,} n \geq 2$ find an expression for F_n.

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

FOURTH SEMESTER

MATHEMATICS

FUNCTIONAL ANALYSIS

TIME: 3HRS **MAXMARKS:75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. In a Banach space $x_n \rightarrow x$; $y_n \rightarrow y$ implies that $x_n + y_n \rightarrow$

   (a)x+y (b)x/y (c)x-y (d)xy

2. l_p^n is

   (a) linear space (b) Banach space (c) not Banach space

   (d) none of these

3. In a Hilbert space $|(x,y)|$

   (a) $\leq \|x\| \|y\|$ (b) $\leq \|x\|$ (c) $\leq \|y\|$ (d) $= \|x\| / \|y\|$

4.. A closed convex subset C of a Hilbert space H contains a unique vector

   (a)of smallest norm (b) which is negative (c) which is negative

   (d) none of these

5. An orthonormal set is a Hilbert space is

   (a) dependent (b) linearly independent (c) generates H

   (d) none of these

6. $\|TT^*\|$ is equal to

   (a) $\|T\|$ (b) $\|T^*\|$ (c) $\|T\|^2$ (d) $\|T\| / \|T^*\|$

7. If A is a positive operator then I + A is

   (a) singular (b) singular and onto (c) non singular (d) none of these

8. An operator U on H is unitary it

(a) UU* = U*U (b) UU* = U*U=I (c) UU* = U (d) UU* = U*

9. det ([δij]) =

(a) 0 (b) 1 (c) ½ (d) 2

10. For elements x and x_0 in G the value of $\| x_0^{-1}x - 1\|$ is

(a) < 0 (b) =0 (c) < ½ (d) < 1

SECTION-B (5X5=25 MARKS)

11. (a) Prove that addition and scalar multiplication are continuous in a Banach space

(or)

(b) Prove that the mapping $x \to F_x$ is an isometric isomorphism of N into N**

12. (a) State and prove the Schwartz inequality.

(or)

(b) If {ei}is an orthogonal sets in H , and if x in H, then prove that x-Σ (x, ei) ei ⊥ ej for j.

13. (a) Prove that ‖ T*T‖ = ‖ T‖2

(or)

(b) Prove that the self adjoint operators on H satisfy:

(i) $A_1 \leq A_2 \to A_1 + A \leq A_2 + A$ for every A:

(ii) $A_1 \leq A_2$ and $\alpha \geq 0 \Rightarrow \alpha A_1 \leq \alpha A_2$

14. (a) For a fixed real number θ , prove that the using two matrices are similar :

$$\begin{pmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{pmatrix} \text{ and } \begin{pmatrix} e^{i\theta} & 0 \\ 0 & e^{-i\theta} \end{pmatrix}$$ (or)

(b) For a self-adjoint operator A on H , prove that A = ∫ λ d Eλ .

15. (a) For a regular element x in a Banach algebra,prove that

$$x^{-1} = 1 + \sum_{n=1}^{\infty} (1-x)^n.$$

(or)

(b) Prove that σ(x) is nonempty.

SECTION-C- (5X8=40 MARKS)

16. (a) If M is a closed linear subspace of a Banach space N,prove that N/M is a Banach space.

(or)

(b) State and prove the Hahn-Banach theorem.

17. (a) Prove that the mapping T→T* is an isometric isomorphism of B(N) into B(N*).

(or)

(b) If M is a proper closed linear subspace of H, prove that there exists a non-zero Z_0 in H such that $Z_0 \perp M$.

18. (a) For an arbitrary functional f in H* , prove that there exists a unique vector y in H such that f (x) = (x, y) for every x in H.

(or)

(b) State and prove the conditions under which sum of projections is also a projection.

19. (a) Prove that two matrices in An are similar and only if they are the matrices of a single operator on H relative to different bases.

(or)

(b) For an arbitrary operator on H, prove that the eigen values of T constitute a non-empty finite subset of the complex plane.

20. (a) Prove that the boundary of S is a subset of Z .

(or)

(b) Prove that $r(x) = \lim \|x^n\|^{1/n}$.

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

THIRD SEMESTER

MATHEMATICS

FLUID DYNAMICS

TIME: 3HRS **MAXMARKS:75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. The condition for incompressible flow is

   (a) $\nabla \times \bar{q} = 0$ (b) $\nabla \cdot \bar{q} = 0$ (c) $\nabla \bar{q} = 0$.

2. The equation of motion of an inviscid fluid is

   (a) $\frac{d\bar{q}}{dt} = \bar{F} - \frac{1}{p}\nabla p$ (b) $\frac{d\bar{q}}{dt} = \frac{\nabla p}{p} - \bar{F}$ (c) $\frac{d\bar{q}}{dt} = -\bar{F} - \frac{1}{p}\nabla p$

3. For steady motion the Bernoulli's equation is

(a) $\int \frac{dp}{p} + \frac{q^2}{2} + \Omega = \text{constant}$ (b) $\int \frac{dp}{p} - \frac{q^2}{2} + \Omega = \text{constant}$

(c) $\int \frac{dp}{p} - \frac{q^2}{2} - \Omega = \text{constant}$

4. The vorticity vector is

(a) $\nabla \times \bar{a}$ (b) $\nabla \cdot \bar{a}$ (c) $\nabla \times \bar{a}$

5. The velocity potential of a source of strength m is

(a) mlnr (b) –mlnr (c) m^2lnr

6. The image of a sink is

(a) source (b) sink (c) doublet.

7. If the velocity vector $\bar{q}$ is $=x\,\bar{i} - y\,\bar{j}$, then the equation of stream line is

(a) xy = k (b) x log y = k (c) y log x = k

8. The velocity profile for a Poiseuille flow is

(a) circular (b) parabolic (c) cycloid.

9. The formula for boundary layer thickness is

(a) $\int_0^\infty \left(1 + \frac{u}{U_\infty}\right) dy$ (b) $\int_{-\infty}^\infty \left(1 - \frac{u}{U_\infty}\right) dy$ c) $\int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy$

10. Prandtl number is the ratio of

(a) kinematic viscocity to thermal diffusivity

(b) dynamic pressure to shearing stress (c) density to velocity.

SECTION-B (5X5=25 MARKS)

11. (a) Derive the equation of continuity.

(or)

(b) Derive Laplace equation for a liquid in irrotational motion.

12. (a) Obtain the expression for the equation of motion for conservative forces.

(or)

(b) Derive the energy equation when the forces are conservative.

13. (a) Show that both Φ and ψ satisfy Laplace equations.

(or)

(b) Define a doublet and obtain complex velocity potential for it.

14. (a) Define and explain vorticity and circulation in a viscuous flow.

(or)

(b) Explain the significance of Reynold's number.

15. (a) Explain displacement thickness.

(or)

(b) Explain momentum thickness.

SECTION-C- (5X8=40 MARKS)

16. (a) Derive the expression for the rate of change of linear momentum.

(or)

(b) Prove that the pressure at any point in an inviscid fluid is independent of direction.

17. (a) Derive the general form of Bernoulli's equation for a fluid in steady motion.

(or)

(b) State and prove Kelocin's theorem.

18. (a) State and prove Blasiu's theorem .

(or)

(b) Describe the flow of a uniform stream past a circular cylinder of radius having a circulation K per unit arc around it.

19. (a) Derive Navier-stoke's equation.

(or)

(b) Discuss the steady flow between parallel planes.

20. (a) Derive the integral equations of the boundary layer.

(or)

(b)Obtain the boundary layer equations for a two dimensional flow along a plane wall.

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

SECOND SEMESTER

MATHEMATICS

COMPLEX ANALYSIS

TIME: 3HRS **MAXMARKS:75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. The order of the rational function R(z) = P(z)/Q(z) with deg P = m and deg Q = n is
   (a) m + n (b) mn (c) max(m, n) (d) min(m, n)
2. The radius of convergence of a polynomial considered as a power series is
   (a) 0 (b) 1 (c) degree of the polynomial (d) ∞.
3. If γ(t) is a curve with parametric interval [a, b] then the parametric interval of -γ(t) is
   (a) [-b, -a] (b) [b, a] (c) [-a, -b] (d) [a, b]
4. The value of $\int_\gamma f\,\overline{dz}$ is
   (a) $\int_\gamma \overline{f}dz$ (b) $\overline{\int_\gamma \overline{f}dz}$ (c) $\int_\gamma fd\overline{z}$ (d) $\overline{\int_\gamma fdz}$
5. The residue of $1/z^3$ at z = 0 is
   (a) 1 (b) 0 (c) ∞ (d) not defined.
6. The function log | z | is harmonic in the region
   (a) **C** (b) **C** \ {0} (c) **C** \ {1, 2} (d) **C** \ {1}.
7. The value of $\lim_{x\to 0}\frac{\log(1+z)}{z}$ is
   (a) 1 (b) 0 (c) not defined. (d) ∞.
8. The function sin z is bounded in
   (a) **C** (b) outside a disc (c) inside a disc (d) **C**/ {0}.
9. An analytic branch of $\sqrt{z-a}$ can be defined in a region Ω if
   (a) Ω simply connected (b) Ω simply connected and a ∈ Ω
   (c) Ω is the whole plane (d) Ω is any region and a ∈ Ω.
10. The Riemann mapping theorem is not valid if the region Ω is
   (a) an open disc (b) an open rectangle (c) a half plane (d) the whole plane.

SECTION-B (5X5=25 MARKS)

11.(a) State and prove Luca's theorem .

(or)

(b) Show that every rational function has a partial fraction expansion.

12.(a)Define the index n(γ ,a) of a closed curve γ (a∉ γ) and prove that it is an integer.

(or)

(b) Using local correspondence theorem show that a non-constant analytic function region is an open map.

13.(a) State and prove Residue theorem.

(or)

(b) State and prove Poisson's formula for functions harmonic in a closed disc.

14.(a) Show that every series $\sum_{n=-\infty}^{\infty} A_n z^n$ represents an analytic function in an annular region $R_1<|z|<R_2$ under certain condition ,to be specified.

(or)

(b) Using Miffag-Leffler's theorem prove

$$\frac{\pi^2}{\sin^2 z}=\sum_{-\infty}^{\infty}\frac{1}{(z-n)^2}$$

15.(a) Define $\{z_n\}$ or z(t) tending to the boundary of Ω and prove that if f: $\Omega\to\Omega_1$ is topological then if $\{z_n\}$ or z(t) tends to $\partial\Omega$ then $\{f(z_n)\}$or f(z(t)) tends to $\partial\Omega_1$.

(or)

(b) Show that the Riemann mapping function of a simply connected region Ω can be extended to a one sided free boundary arc analytically

SECTION-C- (5X8=40 MARKS)

16.(a) Show that if f(z)= u(z) + iv $|z|$ is analytic in a region Ω (u ,v, Real and imaginary parts gf) then show that f is constant if either u or v or $u^2 + v^2$ or $u^2 - v^2$ or uv.

(or)

(b) Show that A $\Rightarrow$ B if,

(i) The image of the real axis under any linear fractional transformation is a circle or a straight line

(ii) The cross ratio of four points (a, b, c, d) is real iff the four points a, b, c, d lie on a circle or a straight line.

17.(a) Show that zeros of non-constant analytic functions are isolated and deduce that f(z) and g(z) are analytic in a Ω and if f(z) = g(z) over a set of points A $\subset$ Ω with a limit point in Ω then f(z) $\equiv$ g(z) for all z in Ω.

(or)

(b) Describe isolated singularities of f analytic in $0<|z-a|<\delta$ at z=a using algebraic order and state and prove Weierstrass theorem on isolated essential singularities.

18.(a) Compute $\int_0^\pi \log\sin\theta d\theta$ using residue calculus.

(or)

(b) Prove the following:

(i) If u_1 and u_2 are harmonic in a region Ω then $\int_\gamma u_1 * du_2 - u_2 * du_1 = 0$ where γ ~ 0 in Ω.

(ii) If u is a harmonic in an annulus $R_1 \subset |z| \subset R_z$ then $\frac{1}{2\pi}\int_{|z|=r} ud\theta = \alpha\log r + \beta$ for $R_1 \subset r \subset R_2$ and α= θ if u is harmonic in a disc.

19. (a) Show that every analytic function in a annulus $R_1 < |z-a| < R_z$ can be expanded in a Laurent series.

(or)

(b) State and prove Mittag Leffler's theorem.

20. (a) Construct a bijective bi-continuous map of $|z|$ <1 onto the whole w-plane. Can this be analytic justify.

(or)

(b) In the content of Riemann mapping theorem if Ω is symmetric with respect to real axis and if z is real then prove that Riemann mapping function f(z)

satisfies $f(z) = \overline{f(\bar{z})}$

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

THIRD SEMESTER

MATHEMATICS

MATHEMATICAL STATISTICS

TIME: 3HRS **MAXMARKS:75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. Two events A and B are mutually exclusive if

   (a)$A \cup B = \phi$ (b) $A \cap B = \phi$ (c) $A \cup B^c = \phi$ (d) $A^c \cap B = \phi$

2. If the distribution function of (X,Y) , X and Y are independent if

   (a) $F(x, y) = F_1(x)/F_2(y)$ (a) $F(x, y) = F_1(x) + F_2(y)$ (a) $F(x, y) = F_1(x)F_2(y)$

   (d) none of these.

3. If $\phi(t)$ is the characteristic function of the random variable X then $\phi(t)$ =

   (a) $E[e^{tx}]$ (b) $E[x^k]$ (a) $E[e^{itx}]$ (d) none.

4. The standard deviation of the Binomial distribution with parameters n and p is

   (a) np (b) npq (c) $\sqrt{np(1-p)}$ (d) None of these.

5. If X is a normal variate and if μ_{2k+1} is the $(2k + 1)^{th}$ central moment of X, then μ_{2k+1}

   (a) 1 (b) 0 (c) 2k+1 (d) $2k^2+1$.

6. If X is the random variable with p.d.f

$$f(x) = \begin{cases} 0 & \text{for } x \le 0 \\ \dfrac{x^{n-1}e^{-x}}{\Gamma n} & \text{for } x > 0 \end{cases}$$

   then the characteristic function of X is

   (a) $(1-t)^n$ (b) $(1-it)^n$ (c) $(1-t)^{-n}$ (d) (i-it)-n

7. The sum of squares of n independent standard normal variate is a ______ variate

   (a) t (b) x^2 (c) F (d) none of these.

8. For large value of degrees of freedom the t-distribution tends to a _____ distribution

   (a) normal (b) chi-square (c) F (d) none of these.

9. An estimator Un of the parameters Q is called consistent if ______ for every $\in >0$

   (a) $\lim_{n\to\infty} P(|U_n - Q| < \varepsilon) = 0$ (b) $\lim_{n\to\infty} P(|U_n - Q| > \varepsilon = 0$

   (c) $\lim_{n\to 0} P(|U_n - Q| < \varepsilon) = 0$ (d) None of these

10. If U_n is an estimator of Q and if $E[U_n] = Q$, then U_n is called_________

    (a) consistent (b) unbiased (c) efficient (d) none of these.

SECTION-B (5X5=25 MARKS)

11. (a) State and prove Baye's theorem on probability.

    (or)

    (b) Define distribution function throwing an unbiased die , find the distribution function.

12. (a) If the l^{th} moment of a random variable X exists then prove that it is given by the l^{th} derivative of the characteristic function $\phi(t)$ of X at t=0.

    (or)

(b) Find the mean and variance of Binomial distribution with parameters n and p.

13. (a) Define Gamma distribution and find its characteristic function.

(or)

(b) State and prove Bernoulli's law of large number.

14. (a) Define chi-square distribution. Find its characteristic function.

(or)

(b) Define Student's t- distribution. Find its mean and variance.

15. (a) Define a sufficient estimator .Give an example of sufficient estimator.

(or)

(b)Describe the method of maximum likelihood for construction of the estimator's.

SECTION-C- (5X8=40 MARKS)

16. (a) The content's of urns I , II , III are as follows:

1 white balls , 2 black balls and 3 red balls

2 white balls , 1 black balls and 1 red balls

4 white balls , 5 black balls and 3 red balls

(or)

(b) The joint distribution function of X and Y is given by,

$$f(x, y) = 4xye^{-(x^2+y^2)} \quad , \ x \geq 0 \ \ y \geq 0.$$

Test whether X and Y are independent.

17. (a) State and prove Levy's theorem on characteristic function.

(or)

(b) State and prove additive property of poisson variates.

18. (a) Find the characteristic function Cauchy distribution. State and prove addition theorem for Cauchy distribution.

(or)

(b) State and prove Lindeberg-Levy theorem.

19. (a) Derive the joint distribution of the statistic $(\overline{X}, S)$

(or)

(b) The heights of 6 randomly chosen sailors are in inches 63, 65, 68, 69, 71, 72. Those of 10 randomly chosen soldiers are 61, 62, 65, 66, 69, 69, 70, 71, 72,73 Discuss the height that these data throw on the suggestion that soldiers are on the average taller than soldiers.

20. (a) State and prove Rao- Cramers inequality.

(or)

(b) What is meant by confident interval? Find the 99% confidence interval for the unknown mean of a normal population when its S.D σ is unknown.

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

FIRST SEMESTER

MATHEMATICS

REAL ANALYSIS

TIME: 3HRS **MAXMARKS: 75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. If $\Delta f_i = f(x_i) + (x_{i-1})$ and $\Delta f_i > 0$, then f is

   (a) monotonically increasing function (b) monotonically decreasing function

   (c) strictly increasing function (d) strictly decreasing function.

2. If P is a partition of [a, b] and $c \in [a, b]$ then the partition of the interval [c, b] is

   (a) $P \cap [a, b]$ (b) $P \cap [a, c]$ (c) $P \cap [c, b]$ (d) $P \cup [c, b]$

3. If x is a real axis n = 0,1,2,….. then $\sum_{n=0}^{\infty} \frac{x^2}{(1+x^2)^n}$ is

   (a) $(1 + x^2)^{-1}$ (a) $(1 + x^2)$ (a) $(1 + 1/x^2)$ (d) $(1 - x^2)$

4. Under what conditions, the limit function f of a sequence of continuous functions $\{f_n\}$ is also continuous?

   (a) f_n ‘s are all uniformly continuous functions

   (b) f_n ‘s are all uniformly monotonically increasing functions

   (c) f_n converges to f monotonically (d) f_n converges to f uniformly

5. Let the vector space X is spanned by r vectors and dimX = n, then

   (a) $n \leq r$ (b) $n \geq r$ (c) n = r (d) n and r are not comparable.

6. Let $\phi: (X, d) \rightarrow (X, d)$ and c < 1. If $d(\phi(x), \phi(x))cd(x, y)$ then Q is known as

   (a) continuous function (b) contraction mapping (c) open mapping

   (d) closed mapping.

7. If A= {1, 2, 5, 7}, then m*A =_______

   (a) 6 (b) 4 (c) 0 (d) 3.5

8. Which one of the followings is not true

   (a) $m^*B \le m^*(A \cup B)$ (b) $m^*A \ge m^*(A \cap B)$ (c) $m^*B = m^*(A - B) + m^*(A \cap B)$

   (d) none of these.10

9. The Lebesgue interval of f over E is defined by the equation , $\int_E f(x)\,dx =$______

   (a) $\inf_{\psi \le f} \int \psi(x)dx$ (b) $\sup_{\psi \ge f} \int \psi(x)dx$ (c) $\inf_{\psi \ge f} \int \psi(x)dx$ (d) none of these

10. Fatou's Lemma is applied only for the sequence {} of

    (a) measurable functions (b) non-negative measurable functions

    (c)increasing non-negative measurable functions

    (d)non-negative integrable functions.

SECTION-B (5X5=25 MARKS)

11. (a) If f is monotonic on [a, b] and α is continuous on [a, b] , then prove that f $\in \mathbf{R}(\alpha)$ on [a, b].

    (or)

    (b) State and prove the fundamental theorem of calculus

12. (a) Limit of an integral need not be equal to integral of the limit .Give an example.

    (or)

    (b) If f $\in$ **B** then show that $|f| \in$ **B**.

13. (a) A linear operator A on a finite dimensional vector space X is one-to-one iff the range of A is all of X.

    (or)

    (b) Show that the determinant of the matrix of a linear operator doesnot depend on the basis.

14 (a) Let $\{E_i\}$be a sequence of measurable sets. Then prove that $m(\cup E_i) \le \sum mE_i$. Suppose E_n are pairwise disjoint. Then prove that $m(\cup E_i) = \sum mE_i$.

    (or)

    (b) Show that the cf and f + g are measurable, when f and g are measurable, C is a compact

15. (a) State and prove the relationship between Riemann integral and Lebesgue integral.

(or)

(b) State and prove Fatou's Lemma.

SECTION-C- (5X8=40 MARKS)

16 (a) Let α increasing on [a, b] and α′ ∈R on [a, b] . Let f be a bounded real function on [a, b] . Then f∈R(α) on [a, b] iff fα′ ∈R, further $\int_a^b f d\alpha = \int_a^b f(x)\, \alpha'(x) dx$

(or)

(b) Define the rectifiable curve. If $\gamma'(t)$ is continuous on [a, b] , then γ ′ is rectifiable and $\Lambda(\gamma) = \int_a^b |\gamma'(t)| dt$.

17 (a) State and prove the relationship between uniform convergence and differentiation.

(or)

(b) State and prove Weierstrass theorem

18.(a) State and prove implicit function.

(or)

(b) State and prove inverse function theorem.

19.(a) Prove that the outer measure of an interval is its length.

(or)

(b) (i) Show that (a, ∞) is measurable.

(ii) If f is measurable and f =g a.e , then prove that g is also measurable.

20 (a) Let f be bounded on a measurable set E with mE<∞.Then

$$\inf_{f \le \psi} \int \psi = \sup_{f \le \phi} \int \phi \text{ iff } f \text{ is measurable.}$$

(or)

(b) State and prove

(i) Monotone convergence theorem

(ii) Lebesgue convergence theorem.

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

SECOND SEMESTER

MATHEMATICS

MECHANICS

TIME: 3HRS **MAXMARKS: 75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. Constraint of the form f(t, r1, r2,….) = 0 is called __________
   (a) Holonomic (b) Scleronomic (c) Non Holonomic (d) None
2. Number of coordinates minus the number of independent equation of constraints equals ____________
   (a) units (b) Dimensions (c) degrees of freedom (d) none of these
3. Hamiltonian function is equal to total energy in
   (a) Conservative system (b) Scleronomic system (c) Scleronomic and natural systems (d) a holonomic conservative system.
4. State true (or) false
   For a non-conservative system the Lagranges equation remains the same.
5. The Hamiltonian equals total energy when

(a)Generalised Co-ordinates don't depend on time

(b)Forces are derivable from a conservative potential V.

(c)Both (a) and (b) (d) None

6. When the Lagrangian is not an explicit function of time in steady motion, the cyclic Co-ordinate are

   (a) Linear function of time (b) Non-linear function of time (c) Not an explicit function of time (d) None of these

7. State true (or) false

   If Q_i and P_i are to be canonical co-ordinates then modified Hamilton's principle is $\delta\int_{t_1}^{t_2}$ (P_i Q_i -K(Q, P, t)) dt = 0

8. The poisson bracket of u, v with respect to (q, p) is

   (a) $\frac{\partial u}{\partial q_i}\frac{\partial v}{\partial p_i}$ - $\frac{\partial u}{\partial p_i}\frac{\partial v}{\partial q_i}$ (b) $\frac{\partial u}{\partial q_i}\frac{\partial v}{\partial q_i}$ - $\frac{\partial u}{\partial p_i}\frac{\partial v}{\partial p_i}$ (c) $\frac{\partial u}{\partial p_i}\frac{\partial v}{\partial p_i}$ - $\frac{\partial u}{\partial q_i}\frac{\partial v}{\partial p_i}$ (d) None

9. The ________ function plays the role of the Hamiltonian in the new co-ordinate set (P, Q)

   (a) Routhian (b) Ignorable co-ordinates (c) Poisson bracket (d) None

10. Solution of Hamilton- Jacobi equation is called ___________

    (a) Gibb's function (b) Quadratic function (c)Routhian function (d)None

SECTION-B (5X5=25 MARKS)

11. (a) Write short notes on degrees of freedom holonomic and non-holonomic system

    (or)

    (b) Derive D-Alemberts Principle

12. (a) Find the shortest distance between two points in a plane .

    (or)

    (b) Write Short notes on cyclic. (or) ignorable co-o0rdinates , what can you say about corresponding generalized momentum .

13. (a) Prove that for a Conservative holonomic system the Hamilton H is a constant

    (or)

    (b) Discuss Routh's procedure and oscillations about steady motion

14. (a) Show that the transformation $P=\frac{1}{2}(p^2+q^{-2})$ $Q=\tan^{-1}\left(\frac{q}{p}\right)$ is canonical .

(or)

(b) Prove that the Lagrange brackets are invariant under contact transformation

15. (a) Write short notes on the physical significance of Hamilton Principal function

(or)

(b) Derive Hamilton- Jacobi equation in the form

$$\left(q_1, q_2, ..., q_n \frac{\partial F_2}{\partial q_1}, ..., \frac{\partial F_2}{\partial q_n} t\right) + \frac{\partial F_2}{\partial t} = 0 .$$

SECTION-C- (5X8=40 MARKS)

16. (a) Explain the motion of one particle using plane polar Co-ordinates .

(or)

(b) Show that the rate of dissipation of energy by frivtion is equal to twice the Rayleigh';s dissipation function

17. (a) Solve the Branchistochrone problem.

(or)

(b) Find the curve for which some given line integral has a stationary value

18. (a) Stater and prove the principle of least action

(or)

(b) Derive Hamilton's equation from a variational principle .

19.(a) Show that the integral $J=\iint_s \sum_i$ dqi dpi is invariant under canonical transformation

(or)

(b) Find the relation between Lagrange and poisson brackets.

20.(a) Discuss about the H-J equation for Hamilton's characteristic function

(or)

(b) By an example solve H-J equation by separation of variables.

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

FOURTH SEMESTER

MATHEMATICS

COMPUTER PROOGRAMMING -II

TIME: 3HRS **MAXMARKS: 75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. Which of the following are good reasons to use an object oriented languages
   (a) Define an own data type
   (b) Program statement are sympler than the procedural language
   (c) An OOP can be taught to connect its mown errors
   (d) its easier to computize an OOP
2. A normal C_{++} operator that acts in special ways on newly defined data types is set tobe
   (a) glorified (b) encapsulated (c) classified (d) overloaded
3. Operator overloading is
   (a) making C_{++} operators works with objects
   (b) Giving C_{++} operators more than they can handle
   (c) Giving new meanings to existing C_{++} operators
   (d) making new C_{++} operators
4. __________ is used to allocate memory in the constructors
   (a) new (b) set (c) assign (d) none
5. ___________ class contains basic facilities that are used by all input output classes
   (a) is (b) ios (c) os (d) iostream
6. ___________ is a sequence of bytes that serves are source or destination for an I/O data
   (a) statements (b) stream (c) Cin and Cout
7. ________ is a special member function whose task is to initialize the objects of its class
   (a) prototype (b) static (c) Abstract (d) constructors
8. The write( ) function handle the data is

(a) ascii form (b) ansi form (c) binary form

9. ___________ achieve the runtime polymorphism

(a) Virtus (b) Friend functions (c) Abs Functions (d) universal

10. In C_{++} the class variables are called as ________

(a) objects (b) Functions (c) prototype (d) static

SECTION-B (5X5=25 MARKS)

11. (a) What are the basic concepts of OOP?

(or)

(b) Explain software crisis

12. (a) Write note on "Operator Overloading"

(or)

(b) How do you declare variables in C_{++} with suitable examples?

13. (a) Discuss C_{++} stream classes

(or)

(b) What is the meaning of call by reference?

14. (a) Explain nesting of member functions

(or)

(b) Discuss multiple constructors in a class

15. (a) Write hierarchical inheritance in C_{++}

(or)

(b) Discuss the overloading unary and binary operators in C_{++}

SECTION-C- (5X8=40 MARKS)

16. (a) (i) What are the benefits of OOP?

(ii) Discuss OOP paradigm

(or)

(b) (i) Write the applications of OOP.

(ii) Discuss object oriented languages.

17. (a) How do you declare operators in C and C_{++} , with examples ,that are used for memory management ?

(or)

(b) Discuss in details identifiers and constants.

18. (a) (i) Write the term “Friend and Virtual Functions”

(ii) Explain the math library function

(or)

(b) Discuss the formatted I/O operations.

19. (a) Explain memory allocation for objects

(or)

(b) How do you declare private member functions and static member functions with examples?

20. (a) Explain data conversions with example which is basic to class type and class to basic type

(or)

(b) Write short notes on :

(i) Virtual base classes

(ii) Abstract classes.

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

FOURTH SEMESTER

MATHEMATICS

MATHEMATICAL METHODS

TIME: 3HRS **MAXMARKS: 75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. Fourier sine transform of f(t)sinwt is
   (a) $\frac{1}{2}[F_c(\xi+\omega)-F_s(\xi-\omega)]$ (b) $\frac{1}{2}[F_c(\xi-\omega)-F_c(\xi+\omega)]$
   (c) $\frac{1}{2}[F_c(\xi-\omega)+F_c(\xi+\omega)]$ (d) $\frac{1}{2}[F_s(\xi+\omega)-F_c(\xi-\omega)]$
2. $e^{-\frac{1}{2}t^2}$ is a self – reciprocal function under
   (a) Fourier sine transform (b) Fourier cosine transform
   (c) Fourier transform (d) both (b) and (c)
3. The Hankel inversion theorem is valid when
   (a) $\upsilon \geq -\frac{1}{2}$ (b) $\upsilon > -\frac{1}{2}$ (c) $\upsilon > 0$ (d) $\upsilon > -1$
4. The Hankel transform of order υ is equal to its inverse transform , when
   (a) $\upsilon = 0$ (b) $\upsilon = -1,0$ (c) $\upsilon = -1,1$ (d) $\upsilon = 0, -\frac{1}{2}$
5. The eigen value of $g(s)=\lambda\int_0^1 e^s s^t g(t)dt$ is __________
   (a) zero (b) non-zero (c) both (a) and (b) (d) None of above
6. The Newmann series for $g(s)=(1+s)+\int_0^s (s-t)g(t)dt$ is
   (a) e^{-s} (b) e^{s} (c) e^{st} (d) $e^{\frac{s}{t}}$
7. The boundary value problem y ''(t) + y '(t) + y(t) = f(t), y(0) = y(1) = 0 leads to Integral equation with
   (a) asymmetric Kernal (b) parametric Kernal (c) continuous Kernal
   (d) non-parametric Kernal
8. The integral equation $g(s)=f(s)+\lambda\int_0^\infty e^{-|s-t|}g(t)dt$ is of

(a) Fredholm type (b) Volterra type (c) singular type (d) None of the above

9. The variational problem $v[y(x)] = \int_a^b ydx + xdy$, $y(a) = y_0$, $y(b) = y_1$ has

(a) two solutions corresponding to (a, y_0) and (b, y_1) (b) unique solution

(c) no solution (d) none of the above.

10. The solution of minimum surface problem is

(a) Sphere (b) catenoid (c) ellipsoid (d) none of the above.

SECTION-B (5X5=25 MARKS)

11. (a) Find the Fourier transform of $e^{ia|t|}$, a > 0

(or)

(b) If a is a real find the Fourier transform F[f(at);ξ]

12. (a) Prove $H_0^{-1} = H_0$ -Hankel transform of order zero

(or)

(b) Obtain the parseval relation for Hankel transform

13. (a) Show that the equation

$\psi(s) = f(s) + \lambda \int k(t,s)\psi(t)dt$ has a unique

(or)

(b) Find the approximate solution of $g(s) = e^s - s - \int_0^1 S(e^{st} - 1)g(t)dt$

14. (a) Show that boundary value problems in ordinary differential equations lead to Fredholm-type integral equations

(or)

(b) Solve $S = \int_0^s \frac{g(t)dt}{(s-t)^{1/2}}$

15. (a) State and prove the fundamental lemma of the calculus of variations

(or)

(b) Give an example to show that there is no extremal that satisfies the boundary Conditions.

SECTION-C- (5X8=40 MARKS)

16. (a) Deduce the graphs of $\delta_n(x)$ and $\Delta_n(x)$ for various values of n

(or)

(b) Find the Lap lace's equations in a half-plane

17. (a) Find the relations between Fourier and Hankel transform

(or)

(b) Solve the axisymmetric Dirchilet problem for a thick plate

18. (a) Solve $g(s) = f(s) + \lambda \int_0^{2\pi} \cos(s+t) g(t) dt$ $g(s) = f(s) + \lambda \int_0^{2\pi} \cos(s+t) g(t) dt$

(Or)

(b) Solve $g(s) = 1 + \lambda \int_0^1 (s+t) g(t) dt$

19. (a) State and solve the transverse oscillations of a homogeneous elastic bar

(Or)

(b) Solve $f(s) = \int_a^s \frac{g(t)dt}{(s^2 - t^2)^\alpha}$, 0 < α<1 ; a < s < b

20. (a) Derive Euler's equation

(Or)

(b) Investigate the following functional for an extremum:

$V[z(x,y)] = \iint_D F = \left(x, y, z, \frac{\partial z}{\partial x}, \frac{\partial z}{\partial y} \right) dxdy$; the values of the function z(x, y) are

the given on the boundary C of domain D, a spatial path

MODEL QUESTION PAPER

FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS

M.SC., DEGREE EXAMINATIONS

SECOND S SEMESTER

MATHEMATICS

OPERATIONS RESEARCH

TIME: 3HRS **MAXMARKS: 75**

ANSWER ALL QUESTIONS

SECTION A-(10 X 1=10 MARKS)

CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. The behaviour of the optimum solution has been studied in

(a) Problem definition (b) Sensitivity analysis (c) implementation of the solution (d) Validation of the model

2. If the type of the objective function is maximization then the sign of coefficient of an artificial variable in the objective function is

(a) Negative (b) positive (c) M (d) zero

3. $(10)\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix}$ = ________ (a) $\begin{pmatrix} 10 & 20 & 30 \\ 4 & 5 & 6 \end{pmatrix}$ (b) $\begin{pmatrix} 10 & 2 & 3 \\ 40 & 5 & 6 \end{pmatrix}$

(c) Either (a) or (b) (d) $\begin{pmatrix} 10 & 20 & 30 \\ 40 & 50 & 60 \end{pmatrix}$

4. VAM is an improved version of

(a) North-west corner method (b) row-minima method (c) least cost method

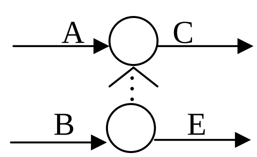
(d) Modi method

5. The algorithm used for the construction of paved roads that links several rural towns is

(a) Minimal spanning tree algorithm (b) shortest route method algorithm

(c) Critical path method algorithm (d) maximal flow algorithm

6. The predecessor(s) of the activity C in the following network is (are)



(a) A (b) A and B (c) A and D (d) A, B and D

7. If the line segment joining any two distinct points in the set, also falls in the set, then The set is known as a

   (a) Concave set (b) convex set (c) extreme point set (d) linear point set

8. The net evaluation is given by the equation $Z_j - C_j =$ __________

   (a) $P_j B^{-1} C_B - C_j$ (b) $C_B P_j B^{-1} - C_j$ (c) $C_B B^{-1} P_j - C_j$ (d) $B^{-1} P_j C_B - C_j$

9. Acceptance-rejection method is applied for generating successive __________

   (a) Probabilistic samples (b) Probabilistic tables

   (c) Random sample (d) convolution samples

10. If u0 =11, b = 9, c = 5 and m = 12, then by multiplicative congruence method the Value of u_1 is

    (a) 0.4167 (b) 0.1667 (c) 0.7776 (d) 0.6667

SECTION-B (5X5=25 MARKS)

11. (a) Solve graphically

    Maximize Z= $5x_1 + x_2$

    Subject constraints:

    $6x_1 + 4x_2 \le 24$; $x_2 \le 2$ and $x_1, x_2 \ge 0$

    (Or)

    (b) Write down the four steps to be adopted in solving a LPP.

12. (a) Explain how the dual problem is constructed from the primal.

    (Or)

    (b)Write down the mathematical formulation of the following transportation problem

	1	2	supply
A	80	215	1000
B	100	108	1500
C	102	68	1200
Demand	2300	1400	

(or)

13. (a) Write down Dijkstra's algorithm

    (or)

    (b) Calculate mean and variance of each of the following activities:

Activity:	A	B	C	D	E
Times :	(3, 5, 7)	(4, 6, 8)	(1, 3, 5)	(5, 8, 11)	(1, 2, 3)

14. (a) Classify all the basic solutions of the following systems of equations

$$\begin{pmatrix} 1 & 3 & -1 \\ 2 & -2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$$

(Or)

(b) Describe revised algorithm

15. (a) Write a short note on inverse method

(Or)

(b) Explain multiplication congruential method with an example.

SECTION-C- (5X8=40 MARKS)

16. (a) Ozark farm uses at least 800kg of special feed daily. It is a mixture of corn and Soyabean meal with the following compositions:

	Kg. per Kg. of feedstuff		
Feed stuff	Protein	Fiber	Cost (in Rs.PerKg)
Corn	0.09	0.02	30
Soyabean Meal	0.60	0.06	90

The dietary requirements of the special of the feed are at least 30% protein and at most 5% fiber . Determine the daily minimum-cost feed mix.

(Or)

(b) Solve

Minimize $Z= 4x_1 + x_2$

Subject to constraints:

$3x_1 + x_2 = 3$; $4x1 + 3x_2 \geq 6$; $x_1 + 2x_2 \leq 4$ and $x_1, x_2 \geq 0$

17. (a) Apply dual simplex method to Solve ,

Minimize Z= 3x1 + 2x2 + x3

Subject to constraints:

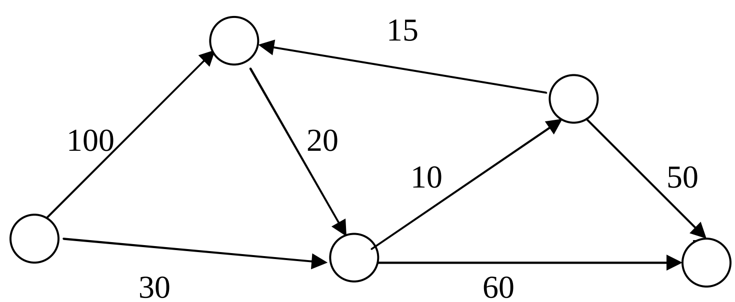
$3x_1 + x_2 + x_3 \geq 3$; $-3x_1 + 3x_2 + x_3 \geq 6$; $x_1 + x_2 + x_3 \leq 3$ and $x_1, x_2, x_3 \geq 0$

(Or)

(b) Solve the following transportation Problem:

	1	2	3	4	supply
1	10	2	20	11	15
2	12	7	9	20	25
3	4	14	16	18	10
Demand	5	15	15	15	

18. (a) The following network gives the permissible routes and their lengths in Km between city1 and four other cities. Determine the shortest routes between city1 and each of the remaining four cities.



(b) Determine the critical path for the following Project Network:

Activity:	(1, 2)	(1, 3)	(2, 3)	(2, 4)	(3, 5)	(3, 6)	(4, 6)	(5, 6)
Duration:	5	6	3	8	2	11	1	12

19. (a) Consider the following LP,

Maximize $Z = x_1 + 4x_2 + 7x_3 + 5x_4$

Subject to constraints:

$2_{x1} + x_2 + 2x_3 + 4x_4 = 10$, $3x_1 - x_2 - 2x_3 + 6x_4 = 5$ and $x_1, x_2, x_3, x_4, \geq 0$.

Generates the simplex tables associated with the bases $B = (P_1, P_2)$ and $B = (P_3, P_4)$

(Or)

(b) Solve the following LP by the revised simplex method:

Maximize $Z = 6x_1 - 2x_2 + 3x_3$

Subject to constraints:

$2x_1 - x_2 + 2x_3 \leq 2$, $x_1 + 4x_3 \leq 4$ and $x_1, x_2, x_3 \geq 0$

20. (a) Use Monte- Carlo Sampling to estimate the area of the circle

$$(x-1)^2 + (y-2)^2 = 25$$

(Or)

(b) Describe acceptance-rejection method .Illustrate it, wrong the beta distribution $f(x) = 6x\,(1- x)$ for $0 \le x \le 1$.

MODEL QUESTION PAPER
FOR CANDIDATES ADMITTED FROM 2007-2008 AND ONWARDS
M.SC., DEGREE EXAMINATIONS
FOURTH SEMESTER
MATHEMATICS
GRAPH THEORY

TIME: 3HRS **MAXMARKS: 75**

ANSWER ALL QUESTIONS
SECTION A-(10 X 1=10 MARKS)
CHOOSE THE BEST ANSWER FROM THE FOUR ALTERNATIVES GIIVEN BELOW EACH OF THE FOLLOWING QUESTIONS

1. A simple graph
 (a) can have self loops and parallel edges
 (b) can have self loops but not parallel edges
 (c) can have only parallel edges
 (d) can have neither self loops nor parallel edges

2. The incidence degree of the vertex v of the following graph is

 (a) 2 (b) 4 (c) 3 (d)0.

3. A graph in which all the vertices have the same degree is called
 (a)planar graph (b)regular graph (c) walk (d) circuit

4. A graph G is said to be disconnected if
 (a)there is exactly one path between any two vertices
 (b)there is atleast one path between any two vertices
 (c) there is no path between any two vertices
 (d)there are two vertices so that there is no path between them.

5. An Euler graph has
 (a) an odd number of vertices
 (b)odd number of vertices of even degree
 (c)every vertex is of even degree
 (d)there is no vertex of even degree.

6.The number of edge disjoint Hamiltonian circuits in a complete graph of n vertices where $n \geq 3$ is
 (a)n/2 (b)n(n-1)/2(c)(n-1)/4 (d)n-1/2.

7.the eccentricity of a vertex v in a graph G is defined as
(a)degree of v.
(b) degree of v-1
(c)min d(v,vi)
v_i ε d(v,v_i)
(e) max d(v,vi)
viεG
8.Every connected graph has
(a)exactly one spanning tree
(b)no spanning tree
(c)every tree in the graph is a spanning tree
(d)atleast one spanning
9.The vertex connectivity of any graph is
(a) always 1
(b)always equal to edge connectivity
(c) the number of vertices in the graph
(d)always less than or equal to edge connectivity.
10.A connected planar graph with n vertices and e edges has
(a)(e-n+1) regions (b)e regions(c)(n-1)regions(d)e-n+2 regions.

SECTION-B (5X5=25 MARKS)

11. (a) Show that in any graph , the number of vertices of odd degree is alwaya even
(or)
(b) Show that an edge e of a graph Gis a cut edge iff e is contained in no cycle of G.
12 (a) If G is a block with $\upsilon \geq 3$, show that any two edges of G lie on a common cycle
(or)
(b) If G is a Hamiltonian show that for every nonempty proper subsets
S of V, $w(G-S) \leq |S|$
13. (a) If a matching M in G is a maximum matching , show that G contains no
M-augumenting path
(or)
(b) Show that every 3-regular graph without cut edges has a perfect matching.
14. (a) With usual notations , show that $\alpha + \beta = \upsilon$.
(or)
(b) Show that in a critical graph no vertex cut is a clique
15. (a) Show that K_5 is non-planar.
(or)
(b) Show that a loopless digraph D has an independent set S every vertex of D not in S reachable from a vertex in S by a directed path of length almost 2.
16. (a) (i) Define a component and give an example
(ii) Prove that $\sum_{v\in V} d(v) = 2e$.
(or)
(b) (i) Show that in a tree show that $e = \upsilon - 1$.

17. (a) Show that a graph G with $\upsilon \geq 3$ is 2-connected iff any two vetices of G are connected by atleast two internally disjoint paths.

(or)

(b) (i) If G is a simple graph with $\upsilon \geq 3$ and $\delta \geq 3/2$, show that G is Hamiltonian

(ii) A connected graph has an Euler trail iff it has almost two vertices of odd degree. Prove

18. (a) State and Prove Vizing's theorem

(or)

(b) Show that G has a perfect matching iff $o(G-S) \leq |S|$ for all $S \subset V$ with usual notations.

19. (a) (i) If $\delta > 0$, show that $\alpha' + \beta' = \upsilon$

(ii) Prove Brook's theorem

(or)

(b) State and Prove Erdo's theorem

20. (a) (i) Derive Euler's formula

(ii) Show that all planar embeddings of a given connected planar graph have the same number of all faces

(or)

(b) Show that a digraph D contains a directed path of length X-1