June 2006

Code: A-07
Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING
Time: 3 Hours
 Max. Marks: 100

NOTE: There are 9 Questions in all.
- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following:
      (2x10)

a. An integral is being evaluated by the three point trapezoidal rule as

$$\int_{a}^{b} f(x)dx = \frac{h}{2}\left[f(a) + 2f\left(\frac{a+b}{2}\right) + f(b) \right]$$

where $h = (b-a)/2.$ The round off errors in
$f(a), f[(a+b)/2], f(b)$ are $\varepsilon_1, \varepsilon_2, \varepsilon_3$ and $\varepsilon = \max(|\varepsilon_1|, |\varepsilon_2|, |\varepsilon_3|).$ Then, the round off error in evaluating the integral is bounded by

(A) $2h\varepsilon$. (B) $h/2$.

(C) $h\varepsilon$. (D) $4h\varepsilon$.

b. A negative root of the equation $2x^5 + 5x^4 + 2x^2 + 7x + 5 = 0$ lies in the interval
(A) $(-1, 0)$. (B) $(-2, -1)$.
(C) $(-3, -2)$. (D) $(-4, -3)$.

c. We want to use the Gauss - Jacobi iteration method to solve the system
$$\begin{bmatrix} 2 & 3 \\ 4 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

Let $x^{(0)}$ be the approximate initial solution vector. Then, the iteration

(A) converges for all $x^{(0)}$. (B) diverges for any $x^{(0)}$.
(C) has rate of convergence 1.35. (D) has rate of convergence 1.

d. The backward difference $\nabla^2 f(x)$ is approximately equal to

(A) $h^2 f''(x)/2$.

(B) $h^3 f'''(x)$.

(C) $h^3 f'''(x)/6$.

(D) $h^2 f''(x)$.

e. The polynomial that fits the data

x	0.1	0.2	0.3	0.4
$f(x)$	1.11	1.24	1.39	1.56

is

(A) $x^2 - x + 1$.

(B) $x^3 + x + 1$.

(C) $x^2 + x + 1$.

(D) $x^2 - 5x + 1$.

f. The following data for a function $f(x)$ is given.

x	1	2	3	4
$f(x)$	9	23	43	69

Then, applying the trapezoidal rule with suitable step length, we obtain the value of

the integral $\int_1^4 f(x)dx$ as

(A) 105.

(B) 115.

(C) 75.

(D) 145.

g. The numerical differentiation formula

$$f''(x) \approx \frac{1}{h^2}[f(x+h) - 2f(x) + f(x-h)]$$

is given. Then, the order of the formula is

(A) 1.

(B) 2.

(C) 3.

(D) 4.

h. Taylor series method of second order with step length $h = 0.1$ is used to approximate $y(1.1)$ for the initial value problem $y' = t + y, y(1) = 2$. The approximate value of $y(1.1)$ is

(A) 2.25.

(B) 2.65.

(C) 3.2.

(D) 2.32.

i. Find the output for the following C program

```
#include<stdio.h>
main()
{
char S1[]="Butter";
char S2[]="Milk";
S1=S2;
```

```
printf("%s",S1);
}
```
(A) Milk.   **(B)** Butter.

(C) Butter Milk.   **(D)** Compilation error saying 'lvalue required'.

j. Find the output for the following C program
```
main()
{
int x = 5;
printf("%d %d %d\n", x, x<<2, x>>2);
}
```
(A) 5 5 5.   **(B)** 5 20 1.

(C) 5 3 7.   **(D)** Compilation error.

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Locate a root of the equation $x^2 + x + 0.25 - e^{-x} = 0$ in an interval of length one unit.
(i) Perform two iterations of the bisection method to improve the approximation to the root. (ii) Taking the mid point of the last interval obtained in (i) as initial approximation perform two iterations of the Newton-Raphson method. **(8)**

b. Define the order and error constant of an iterative method for finding a root of the equation $f(x) = 0$. An approximation for finding $N^{1/3}, N > 0$ is being computed by the iteration method

$$x_{n+1} = \frac{1}{3} x_n \left(2 + \frac{N}{x_n^3} \right)$$

Find the order of the method and the error constant. **(8)**

Q.3 a. The system of equations $x^2 + y^2 + xy = 9,\ x^3 + y^3 = 11.7$ has a solution near $x = 1.3,\ y = 2$. Perform two iterations of the Newton's method to improve the solution. **(8)**

b. Solve the system of equations

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 28 \\ 96 \end{bmatrix}$$

by the Choleski method.
(8)

Q.4 a. Using Gauss elimination, find the conditions under which the system of equations

$$\begin{bmatrix} 5 & 3 & 1 \\ 2 & 4 & a \\ 1 & 3 & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}$$

has a unique solution or no solution. **(6)**

b. The system of equations

$$\begin{bmatrix} 3 & 1 & 1 \\ -1 & 4 & 1 \\ 1 & -2 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 3 \\ -4 \\ 0 \end{bmatrix}$$

is to be solved by the Gauss-Jacobi iteration method. Obtain the iteration matrix of the method. Find the rate of convergence of the method. (Use the Newton-Raphson method to find a root of the characteristic equation of the iteration matrix). **(10)**

Q.5 a. Fit a polynomial for the following data

x	-2	-1	1	2	4	5
$f(x)$	-19	-9	-1	9	71	135

(5)

b. If $f(x) = e^{ax}$, evaluate $\nabla^n f(x)$, where ∇ is the backward difference operator. **(4)**

c. Write a C – program for evaluating the Lagrange polynomial of degree N based on N + 1 points $(x_k, y_k), k = 0, 1, 2, \cdots, N$. Input maximum degree as 10. **(7)**

Q.6 a. Use Newton's backward difference polynomial to interpolate at $x = 1.5$, from the following data

x	-2	-1	0	1	2
$f(x)$	20	12	6	2	0

(7)

b. A physicist studying a decaying process decides to fit an approximation of the form $f(x) = ae^{-t} + be^{-2t}$ to a data. Derive the normal equations to determine a and b using the least squares approximation. Fit the above approximation to the following data

t	0.1	0.2	0.3	0.4	0.5
$f(t)$	0.539	0.558	0.562	0.556	0.542

(9)

Q.7 a. A numerical differentiation formula is defined as

$$f'(x) = af(x) + bf(x - h) + cf(x - 2h)$$

Find the values of a, b, c such that the formula is of as high order as possible. Find the leading term of the truncation error. **(8)**

b. Evaluate $\displaystyle\int_0^1 \frac{dx}{x^2+6x+10}$ using Simpson's rule with 2 and 4 subintervals. Compare these values with the exact solution. **(8)**

Q.8 a. Consider the numerical integration formula

$$\int_{-1}^{1} f(x)dx = af(-1)+bf(x_1)$$

Find the values of the parameters a, b, x_1 such that the formula is exact for polynomials of order as high as possible. Find the error term. **(8)**

b. Write a C – program to evaluate the integral $\displaystyle\int_a^b f(x)dx$ by trapezoidal rule with N equal subintervals, using the function as $f(x)=x/(x^2+3x+1)$. **(8)**

Q.9 a. Evaluate the integral $\displaystyle\int_0^1 \frac{\cos x}{1+\sin 2x}dx$ using the Gauss-Legendre three point formula. **(6)**

b. Derive the truncation error term of the Euler method for solving the initial value problem $y'=f(x,y), y(x_0)=y_0$. **(4)**

c. Use the classical Runge-Kutta method of fourth order to find the numerical solution at $x=0.4$ for the initial value problem
$$y'=\sqrt{3x+2y}, \quad y(0.2)=0.2$$
with $h=0.2$. **(6)**