AMIETE – ET (OLD SCHEME)

Code: AE07 **Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING**
Time: 3 Hours **Max. Marks: 100**

DECEMBER 2009

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2×10)**

a. The value of π is approximated by 22/7. If seven significant digits are used, the percentage relative error in this approximation is

(A) 0.4. **(B)** 0.04.
(C) 2.2. **(D)** 4.

b. A root of the equation $x^2 + x - \cos x = 0$ is near $-1.25.$ Using one iteration of the Newton-Raphson method, the next approximation to the root is obtained as

(A) –1.25115. **(B)** –1.29856.
(C) –1.70161. **(D)** 1.26211.

c. The data P(1) = 14, P(2) =35, P(3) = 72 is given. An approximation to P(2.5) using all the data values is given by

(A) 61.5 **(B)** 53.5
(C) 56.0 **(D)** 51.5

d. Let $p(x) = ax^3 + bx^2 + cx + d,$ be a third degree polynomial. Then, the forward difference $\Delta^4 p(x)$ is given by

(A) 6*a*. **(B)** 0.
(C) 3*a*. **(D)** 6*a* + *b*.

e. The value of the integral $\int_1^4 \frac{(3x+5)}{x}dx$ evaluated by the trapezoidal rule with *h* = 1, is obtained as

(A) 16.292 **(B)** 18.292
(C) 15.322 **(D)** 16.992

f. The initial value problem $y' = -2ty^2$, y(0) = 1 is given. The approximation to $y(0.4)$ obtained by the Euler method with $h = 0.2$ is

(A) 1.0. **(B)** 0.62.
(C) 0.82. **(D)** 0.92.

g. If λ is an eigen value of A, then eigen value of A^{-1} is

(A) $1/\lambda$ **(B)** $-1/\lambda$
(C) $-\lambda$ **(D)** λ^2

h. If f(x) = $\frac{1}{x}$, then the value of f[a,b] will be

(A) $\frac{1}{ab}$ **(B)** $-\frac{1}{ab}$
(C) $\frac{a}{b}$ **(D)** $-\frac{a}{b}$

i. For the Simpson's $\frac{1}{3}$ rd rule, the interpolating polynomial is a
(A) Straight-line **(B)** Parabola
(C) Cubic curve **(D)** None

j. The value of the integral $I = \int_{-1}^{+1} e^{x^2} dx$ using Gaussian integration formula for n = 2 is
(A) $e^{1/2}$ **(B)** $2e^{1/3}$
(C) $2e^{-1/3}$ **(D)** None

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2a. A root of the equation $x^2 + x - 2e^{-x} = 0$ is to be determined. Obtain an interval of length 1 unit, in which a positive root lies. Taking the end points of this interval as initial approximations, perform two iterations of the secant method to find approximations to the root **(7)**

b. Write a *C* program to compute x^n using **while** loop. Output *x*, *n* and x^n. **(6)**

c. If $\Delta(f_i / g_i) = (a\Delta f_i + b\Delta g_i)/(g_i g_{i+1})$, then find the values of a and b. **(3)**

Q.3a. Perform four iterations of the Newton-Raphson method to find the smallest positive root of the equation

$$f(x)=x^3-5x+1=0$$ **(8)**

b. Using Lagrange interpolation, find $f(2)$, from the table of values

x	0	1	3	4
$f(x)$	1	4	34	73

(8)

Q.4a. Derive the least squares straight line approximation $f(x) = a + bx$ for a data of N values (x_i, f_i). Hence, obtain the least squares straight line approximation to the data

x	0.1	0.2	0.3	0.4	0.5
$f(x)$	1.43	1.92	2.47	3.08	3.75

Also, find the least squares error. **(8)**

b. Write a *C* program to find a simple root of $f(x) = 0$ by the regula-falsi method. Input (i) a, b (two initial approximations between which the root lies), (ii) n (maximum number of iterations) and (iii) error tolerance "eps". Output (i) approximate root, (ii) number of iterations taken. If the inputted value of n is not sufficient, the program should write "Iterations are not sufficient". Write the subprogram for $f(x)$ as $f(x) = x^4 - x - 10$. **(8)**

Q.5a. The following data represents the function $f(x) = \cos(x+1)$.

x	0.0	0.2	0.4	0.6
$f(x)$	0.540302	0.362358	0.169967	-0.029200

Estimate $f(0.5)$ using the Newton's backward difference interpolation. Find the magnitude of the actual error. **(8)**

b. The system of equations $x^2 y + y^3 + x = 1.0,\ 5xy^2 - y^3 = 0.4,$ has a solution near x = 0.7, y = 0.3. Perform two iterations of the Newton's method to obtain the root.
(8)

Q.6a. Find the inverse of the coefficient matrix of the system of equations

$$\begin{bmatrix} 2 & 2 & 1 \\ 4 & 2 & 3 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

by the Gauss-Jordan method with partial pivoting and hence solve the system. **(8)**

b. The system of equations

$$\begin{aligned} 4x - y \quad\quad &= 5 \\ -x + 4y - z &= 0.5 \\ -y + 4z &= 5 \end{aligned}$$

is given. Using the Gauss-Seidel iteration scheme in matrix form for its solution, find whether the scheme converges. If it converges, find the rate of convergence. **(8)**

Q.7a. The following data is given

x	1.0	1.5	2.0	2.5	3.0
$f(x)$	3.0	6.625	13.0	22.875	32.5

From this data, evaluate $\int_1^3 f(x)dx,$ using the Simpson rule with three and five points. **(7)**

b. Write a *C* program to evaluate $\int_a^b f(x)dx$, by trapezium rule of integration based on $n + 1$ points. Input the values of *n*, *a*, *b*. Write $f(x) = 1/(3+5x)$ as a function sub- program. Output all the data and the computed value. **(9)**

Q.8 a. The formula $f'(a) = [3f(a) - 4f(a-h) + f(a-2h)]/(2h),$ is suitable for approximating $f'(a),$ where *a* is the last value in the data. Calculate $f'(2)$ from the table of values, using all possible step lengths.

x	1.6	1.8	1.9	1.95	2.0
$f(x)$	7.5530	8.8497	9.5859	9.9787	10.3891

(8)

b. The formula $f'(a) = [f(a+h) - f(a-h)]/(2h),$ is being used to compute $f'(1)$ from a table of values. Using Taylor series, find the leading term of the error. Derive the formula for obtaining the improved (Richardson's) extrapolated value of $f'(a)$. In a particular problem, the following results were obtained with different step lengths, using the above formula.

h	0.4	0.2

$f'(1)$	0.52601	0.53671

Compute the improved (Richardson's) extrapolated value of $f'(1)$. **(8)**

Q.9a. Gauss-Legendre two point integration formula can be written as

$$\int_{-1}^{1} f(x)dx = bf(-p) + df(p). \qquad p \neq 0.$$

Determine the values of *b, d, p.* **(7)**

b. Using the Gauss-Laguerre two point formula, evaluate the integral $\int_{0}^{\infty} \frac{e^{-x}}{1+x} dx.$ **(4)**

c. Use Runge-Kutta method of fourth order to determine $y(0.2)$ with $h = 0.2$, for the initial value problem $y' = (y^2 - x^2)/(y^2 + x^2),\ y(0) = 1.$ **(5)**