December 2005

Code: A-07
Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING
Time: 3 Hours
Max. Marks: 100

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: (2x10)

a. A second degree polynomial approximation to $f(x) = (1+2x)^{1/3},\ x \in [0, 0.1]$ using Taylor Series expansion is given by

(A) $1+\frac{2x}{3}+\frac{4x^2}{9}$. **(B)** $1+\frac{2x}{3}-\frac{4x^2}{9}$.

(C) $1-\frac{2x}{3}-\frac{4x^2}{9}$. **(D)** None of above.

b. The order of Newton-Raphson method $x_{k+1} = x_k - \frac{f(x_k)}{f'(x_k)}$ for finding a multiple root of multiplicity 3 of the equation $f(x) = 0$, is

(A) 1 **(B)** 2

(C) 3 **(D)** 4

c. The combination "\a" in C gives a

(A) vertical tab. **(B)** form feed.

(C) system beep. **(D)** carriage return.

d. For a two point Gauss-Hermite integration rule $\int_{-\infty}^{\infty} e^{-x^2} f(x)\,dx = \lambda_0 f(x_0) + \lambda_1 f(x_1)$, the values of $\lambda_0,\ \lambda_1,\ x_0$ and x_1 are given by

(A) $x_0 = -\frac{1}{\sqrt{2}},\ x_1 = -\frac{1}{\sqrt{2}},\ \lambda_0 = \frac{\sqrt{\pi}}{2} = \lambda_1$.

(B) $x_0 = -\frac{1}{\sqrt{2}},\ x_1 = \frac{1}{\sqrt{2}},\ \lambda_0 = \frac{\sqrt{\pi}}{2} = \lambda_1$.

(C) $x_0 = \frac{1}{\sqrt{2}}, x_1 = -\frac{1}{\sqrt{2}}, \lambda_0 = \frac{\sqrt{\pi}}{2} = \lambda_1$.

(D) $x_0 = \frac{1}{\sqrt{2}}, x_1 = \frac{1}{\sqrt{2}}, \lambda_0 = \frac{\sqrt{\pi}}{2} = \lambda_1$.

e. If $x = 5$, the output of the statement sign = (x < 0) ? -1 : ((x == 0) ? 0 : 1); would be

(A) −1 **(B)** 0
(C) 1 **(D)** error

f. If a general iteration method $x_{k+1} = \phi(x_k)$ is of order p then

$\phi'(x_k) = 0 = \phi''(x_k) = \phi'''(x_k) = \cdots\cdots = \phi^r(x_k)$, by definition. The value of r is

(A) p+1 **(B)** p
(C) p-1 **(D)** None of above

g. The left shift operator "<<" has the effect of

(A) multiplying the number by 2.
(B) dividing the number by 2.
(C) adding 2 to the number.
(D) subtracting 2 from the number.

h. The solution for the system of equations

$$4x_1 + 3x_2 + x_3 = 12$$
$$2x_1 - 2x_2 + 2x_3 = 4$$
$$x_1 + 6x_2 + 3x_3 = 11$$

is given by

(A) $x_1 = \frac{3}{2}, x_2 = 2, x_3 = 0$. **(B)** $x_1 = \frac{18}{7}, x_2 = \frac{4}{7}, x_3 = 0$.
(C) $x_1 = 1, x_2 = 1, x_3 = 1$. **(D)** $x_1 = 2, x_2 = 1, x_3 = 1$.

i. The format for unsigned integer in C printf statement is

(A) %i **(B)** %I
(C) %u **(D)** None of above

j. What should be the condition on α such that the method

$x^{(n+1)} = x^{(n)} + \alpha(Ax^{(n)} - y)$ where $A = \begin{bmatrix} 3 & 2 \\ 1 & 2 \end{bmatrix}$, converges?

(A) $\alpha < 0$. **(B)** $\alpha > 0$.
(C) $\alpha = 0$. **(D)** None of above.

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. The system of equations

$$2x^2 + xy = 1.6$$
$$xy^2 + y^3 = 0.192$$

is to be solved. Write the Newton's iterative procedure and iterate two times starting with $x_0 = 0.5$ and $y_0 = 0.5$. **(8)**

b. The equation $f(x) = 0$ has a simple root in the interval $(3,4)$. The function $f(x)$ is such that $|f'(x)| \geq 2$ and $|f''(x)| \leq 1$ for all values of x in this interval. Assuming that the Newton-Raphson method converges for all initial approximations in $(3,4)$, find the number of iterations required to obtain the root correct to $5 * 10^{-5}$. **(8)**

Q.3 a. Show that the matrix

$$A = \begin{bmatrix} 12 & 4 & -1 \\ 4 & 7 & 1 \\ -1 & 1 & 6 \end{bmatrix}$$

is positive definite and hence decompose it as $A = L\,L^T$ using Cholesky method. **(8)**

b. Consider the linear system of equations

$$\begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 2 & 2 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 \\ 3 \\ 5 \end{bmatrix}$$

Discuss the convergence of Gauss Jacobi and Gauss Seidel methods. **(8)**

Q.4 a. Construct the divided difference table for the following data:

x :	-3	-2	0	1	4	6
$f(x)$:	136	22	4	4	472	2494

Use Newton's divided difference formula to find the Interpolating Polynomial and hence find the value of $f(5)$. **(6)**

b. When does a function need an *include* directive? **(5)**

c. Write a programme using a function *comb()* to calculate *C(n,k)* to print Pascal's Triangle down to row number 12. Each number in Pascal's Triangle is one of the combinations $C(n, k) = \dfrac{n!}{k!\,(n-k)!}$. If we count the rows and the diagonal columns starting with 0, then the number *C(n,k)* is in row *n* and column *k*. Pascal's Triangle looks like

$$
\begin{array}{ccccccccccccc}
 & & & & & & 1 & & & & & & \\
 & & & & & 1 & & 1 & & & & & \\
 & & & & 1 & & 2 & & 1 & & & & \\
 & & & 1 & & 3 & & 3 & & 1 & & & \\
 & & 1 & & 4 & & 6 & & 4 & & 1 & & \\
 & 1 & & 5 & & 10 & & 10 & & 5 & & 1 & \\
1 & & 6 & & 15 & & 20 & & 15 & & 6 & & 1
\end{array}
$$

(5)

Q.5 a. Starting with the set $\{1, x, x^2\}$, generate a set of orthogonal polynomials on $[-1, 1]$ with the weight function $w(x) = 1$. Using these polynomials find a Least squares approximation of the form $a + bx^2$ for the function $f(x) = \cos x$ on $[-1, 1]$. **(8)**

b. For the method

$$f'(x_0) = \frac{-3f(x_0) + 4f(x_1) - f(x_2)}{2h} + \frac{h^2}{3} f'''(\xi), \qquad x_0 < \xi < x_2$$

determine the optimal value of h using the criteria

(i) $|RE| = |TE|$

(ii) $|RE| + |TE| =$ minimum **(8)**

Q.6 a. Find the error term of the method $f'(x) \approx \dfrac{f(x+h) - f(x)}{h}$ as a power series in h. Derive the corresponding Richardson's extrapolation scheme. Using this method and the Richardson's extrapolation, find the best value of $f'(1)$ when $f(x)$ is given in tabular form as

x :	1	2	3	4	5
f(x) :	1	32	243	1024	3125

(8)

b. Write a function that rotates a two-dimensional square array 90^0 "clock-wise". Consider an array of integers. For example, an array

$$\begin{matrix} 22 & 50 & 76 \\ 34 & 65 & 55 \\ 58 & 93 & 33 \end{matrix} \quad \text{will become} \quad \begin{matrix} 58 & 34 & 22 \\ 93 & 65 & 50 \\ 33 & 55 & 76 \end{matrix}$$

(8)

Q.7 a. Using the method of undetermined coefficients, find the nodes and weights of the quadrature formula

$$\int_0^\infty e^{-x} f(x)\,dx = \lambda_0 f(x_0) + \lambda_1 f(x_1)$$

(8)

b. Calculate $\int_0^1 \frac{x^2}{x^3+10}\,dx$ using Trapezoidal rule with number of points as 3, 5 and 9. Improve the results using Romberg Integration. **(8)**

Q.8 a. The following IVP is given $y' = 7x + \frac{3}{2}y, \quad y(0) = 1$

Use second order Taylor series method to get $y(0.4)$ with step length $h = 0.1$. **(8)**

b. Write a programme that plays the game of "rock, paper, scissors". In this game, two players will simultaneously say either "rock" or "paper" or "scissors". The winner is the one whose choice dominates the other. The rules are: paper dominates (wraps) rock, rock dominates (breaks) scissors and scissors dominates (cuts) paper. Use enumerated data type for the choices and the result. The result will be either a "tie" or "player 1 wins" or "player 2 wins". **(8)**

Q.9 a. Use the classical fourth order Runge-Kutta method to find the solution at $x = 0.8$ for $\frac{dy}{dx} = \sqrt{x+y}, \quad y(0.4) = 0.41$

Assume the step length $h = 0.2$. **(10)**

b. Construct a truth table for each of the following boolean expressions, showing its truth value (0 or 1) for all its combinations of truth values of its operands p, q and r.

i) !p || q
ii) p && q || !p && !q
iii) (p || q) && !(p && q)
iv) !(p && q)
v) p && (q && r)
vi) p || (q && r) **(6)**