AMIETE – ET (OLD SCHEME)

Code: AE07 Subject: NUMERICAL ANALYSIS & COMPUTER PROGRAMMING
Time: 3 Hours Max. Marks: 100

JUNE 2010

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2×10)

a. If we take $x = 0.178693\times10^1$, $y = 0.178439\times10^1$ each correct to six digits in decimal system then the value of $x - y = 0.000254\times10^1$ is correct to

(A) 3 digits **(B)** 4 digits
(C) 5 digits **(D)** 6 digits

b. Identify the number of True statements among the following:

(i) An iterative method is said to be of order p, if there exists a non-zero constant C and p is the largest positive real number such that

$$|\varepsilon_{k+1}| \le C\,|\varepsilon_k|^p$$

is satisfied where ε_k is the error at the k-th iteration

(ii) The rate of convergence of Secant method is p=1

(iii) The Regula-Falsi method has linear rate of convergence

(A) 1 **(B)** 2
(C) 3 **(D)** None of the above

c. Suppose the coefficient matrix A of a given system of equations is decomposed in to

A=LU

where L and U are the lower and upper triangular matrices respectively. If we choose the diagonal elements of L to be equal to the value 1 then the method is called

(A) Gauss-Jordan method **(B)** Doolittle's method
(C) Crout's method **(D)** None of the above

d. For the following values given

x (in degrees)	10	20	30
$f(x)$	1.1585	1.2817	1.3660

using quadratic interpolating polynomial $f(.)$ that fits the data, find $f(\pi/12)$?

(A) 1.0729 **(B)** 1.1925
(C) 1.2246 **(D)** None of the above

e. The following table of values is given:

x	-1	1	2	3	4	5	7
$f(x)$	1	1	16	81	256	625	2401

Using the formula $f'(x_1) = (f(x_2) - f(x_0)/(2h)$ and the Richardson extrapolation, find $f'(3)$?

(A) 108 **(B)** 115
(C) 127 **(D)** None of the above

f. Identify the correct statements from the following
(i) The problem of Least Squares approximation is a minimization problem

(ii) The Legendre polynomials $P_n(x)$ defined on [-1,1] are orthogonal polynomials

(iii) The Chebyshev polynomials $T_n(x)$ are defined on [-1,1] by,
$T_n(x) = \cos^{-1}(n \cos x)$

(A) (i) & (ii) **(B)** (i) & (iii)
(C) (ii) & (iii) **(D)** (i), (ii) & (iii)

g. Simpson's three-eighth rule of numerical integration is exact for polynomials of degree up to

(A) 1 **(B)** 2
(C) 3 **(D)** any finite degree

h. The value of the integral

$$I = \int_{-1}^{1} (1 - x^2)^{3/2} \cos x dx$$

using 1-point Gauss-Chebyshev formula will be

(A) 2.1276 **(B)** 2.5672
(C) 2.9831 **(D)** None of the above

i. The order of convergence of Newton-Raphson method is

(A) 1 **(B)** 2
(C) 3 **(D)** 4

j. The value of y corresponding to x=0.1 for the differential equation
$\frac{dy}{dx} = x + y\,;\; y(0) = 1.$
Using Euler's method.

(A) 1.10 **(B)** 1.36
(C) 1.94 **(D)** 2.19

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Show that the Newton-Raphson method for finding the root of the equation $f(x)=0$ has second order convergence. **(8)**

b. Write a C program to find a simple root of the equation of $f(x)=0$ by the Regula-Falsi method. The inputs are : (i) $x0$, $x1$ (the initial interval in which the root lies), (ii) maximum number of iterations, (iii) the error tolerance 'tol'. The outputs are: (i) approximate root (ii) number of iterations taken. If the input value of 'n' is not sufficient then your program should give an error message: "Iterations not sufficient". Also write a function to evaluate $f(x)$ where $f(x) = x^3 - 5x + 1$. **(8)**

Q.3 a. Obtain a second degree polynomial approximation to

$$f(x) = (1+x)^{1/2}, x \in [0,0.1]$$

using the Taylor series expansion about $x=0$. Use the expansion to approximate $f(0.05)$ and find a bound of the truncation error **(8)**

b. Perform three iterations of the Newton-Raphson method to solve the system of equations

$$x^2 + xy + y^2 = 7$$

$$x^3 + y^3 = 9$$

by taking the initial approximation as $x_0 = 1.5, y_0 = 0.5.$ **(8)**

Q.4 a. Solve the following system of equations using Gauss elimination with partial pivoting **(8)**

$$2x_1 + 2x_2 + x_3 = 6$$

$$4x_1 + 2x_2 + 3x_3 = 4$$

$$x_1 + x_2 + x_3 = 0$$

b. Using the Gauss-Seidel method, solve the system of equations

$$20x_1 + 2x_2 + 6x_3 = 28$$

$$x_1 + 20x_2 + 9x_3 = -23$$

$$2x_1 - 7x_2 - 20x_3 = -57$$

starting from (0,0,0) up to 5 iterations. **(8)**

Q.5 a. Differentiate the following:

(i) Call by values and Call by reference in C program

(ii) Structures & Unions **(4+4)**

b. A polynomial fits the points (1,4), (3,7), (4,8) and (6,11). Using Newton's divided difference formula interpolate the value of y at x=2. **(8)**

Q.6 a. Find the least-squares approximation of second degree for the discrete data

x	-2	-1	0	1	2
f(x)	15 19	1		1	3

(8)

b. Determine the step size that can be used in the tabulation of $f(x)$=sin x in the interval $\left[0, \frac{\pi}{4}\right]$ at equally spaced nodal points so that the truncation error of the quadratic interpolation is less than 5×10^{-8}. **(8)**

Q.7 a. A differentiation rule of the form

$hf'(x_2) = \alpha_0 f(x_0) + \alpha_1 f(x_1) + \alpha_2 f(x_3) + \alpha_3 f(x_4)$

where $x_j = x_0 + jh,\ j = 0,1,2,3,4$ is given.

(i) Determine the values of $\alpha_0, \alpha_1, \alpha_2$ and α_3 so that the rule is exact for a polynomial of degree 4.

(ii) Find the error term.

(iii) Calculate $f'(0.3)$ using five places of $f(x)$=sin x with h=0.1. **(4+3+3)**

b. Construct the divided difference table for the data:

(0.5, 1.625), (1.5, 5.875), (3.0, 31.0), (5.0, 131.0) (6.5, 282.125), (8.0, 521.0) **(6)**

Q.8 a. By applying composite Simpson's rule with 4 equal sub-intervals, compute the integral

$$I = \int_0^{\frac{\pi}{2}} \sqrt{\sin x}\, dx$$

(6)

b. Evaluate the integral

$$\mathrm{I} = \int_1^2 \frac{2\mathrm{x}}{1+\mathrm{x}^4}\,\mathrm{dx}$$

using Gauss-Legendre 2-point and 3-point quadrature rules **(5+5)**

Q.9 a. Find the Cholesky decomposition of the following matrix

$$\begin{bmatrix} 1 & 2 & 3 \\ 2 & 8 & 22 \\ 3 & 22 & 82 \end{bmatrix}$$

(8)

b. For the given initial value problem

$$y'(x) = x^2 + y^2, \qquad y(0) = 0$$

with h=0.2, estimate y(0.4) using the fourth order classical Runge-Kutta method.

(8)