# AMIETE – ET/CS/IT (OLD SCHEME)

**Code: AE06/ AC04/ AT04** **JUNE 2009** **Subject: SIGNALS & SYSTEMS**
**Time: 3 Hours** **Max. Marks: 100**

**NOTE: There are 9 Questions in all.**

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

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**Q.1 Choose the correct or the best alternative in the following: (2×10)**

a. The Fourier transform of $x(t) = t \cdot e^{-at}u(t); (a > 0)$

**(A)** $\frac{1}{(a + j\omega)}$ **(B)** $\frac{1}{(a - j\omega)}$

**(C)** $\frac{1}{(a + j\omega)^2}$ **(D)** $\frac{1}{(a - j\omega)^2}$

b. Consider a continuous-time system with input x(t) and output y(t) related by $y(t) = x(t) \cdot \sin(t)$. The system is

**(A)** Linear **(B)** Causal

**(C)** Non-linear **(D)** Non-causal

c. The equation $y(t) = \int_{-\infty}^{\infty} x(\tau) \cdot h(t - \tau)\, d\tau$ is also called as

**(A)** Superposition integral **(B)** Continuous integral

**(C)** Time integral **(D)** None of the above

d. According to time shifting property of Fourier transform, $x(t - t_0)$ is equal to

**(A)** $X(\omega) \cdot e^{j\omega t_0}$ **(B)** $X(\omega) \cdot e^{-j\omega t_0}$

**(C)** $X(j\omega) \cdot e^{\omega t_0}$ **(D)** $X(j\omega) \cdot e^{-\omega t_0}$

e. Parseval's relation for a periodic signals $\int_{-\infty}^{\infty} \left|x(t)^2\right| dt$ equal to

**(A)** $\frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega$ **(B)** $\int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega$

**(C)** $\frac{1}{\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2\, d\omega$ **(D)** None of the above

f. The discrete fourier transform of $(n + 1)a^n u[n]; |a| < 1$ is equal to

(A) $\frac{1}{\left(1+ae^{-j\omega}\right)^2}$ (B) $\frac{1}{\left(1+ae^{-j\omega}\right)}$

(C) $\frac{1}{\left(1-ae^{-j\omega}\right)}$ (D) $\frac{1}{\left(1-ae^{-j\omega}\right)^2}$

g. An ideal low pass filter is

**(A)** Causal **(B)** Non-causal

**(C)** Inverse causal **(D)** None

h. If $x(t) \overset{\mathcal{L}}{\leftrightarrow} X(S)$ with ROC=R, then Laplace transform of $x(t-t_0)$ is equal to

**(A)** $e^{-st_0} \cdot X(s)$ **(B)** $e^{st_0} \cdot X(s)$

**(C)** $e^{-t_0} \cdot X(s)$ **(D)** $e^{t_0} \cdot X(s)$

i. For a causal linear time invariant system, the impulse response for t < 0 is equal to

**(A)** Unity **(B)** Infinity

**(C)** Zero **(D)** None

j. The ROC associated with the system function for a causal system is a

**(A)** Right half plane **(B)** Left half plane

**(C)** Middle of plane **(D)** None of the above

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**Answer any FIVE Questions out of EIGHT Questions.**
**Each question carries 16 marks.**

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**Q.2** a. Find out whether signal $s(t) = A \cdot \cos(w_0 t + \theta)$ is an energy signal or a power signal. Also find if $s(t) = A \cdot u(t)$ is an energy signal or a power signal. **(8)**

b. Find out the response of a continuous –time system to unit step input given the impulse response, $h(t) = \frac{1}{RC} \cdot e^{-t/RC} \cdot u(t)$. **(8)**

**Q.3** a. Find the trigonometric Fourier series for the waveform shown in Fig.1. **(8)**

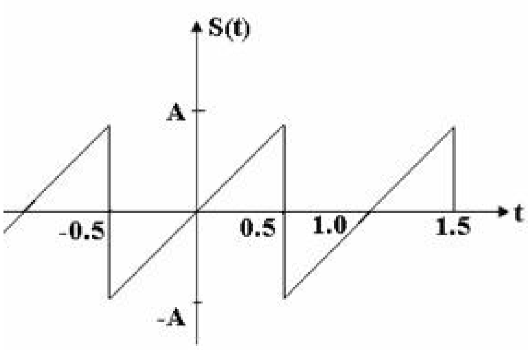


**Fig.1**

b. Determine the fourier series coefficients $A_k$ for a discrete-time periodic signal given as $s(n) = \cos \omega_0 n$, where $\omega_0 = 2\pi/N_0$. **(8)**

**Q.4** a. Determine the continuous-time fourier transform of a continuous –time signal, $s(t) = e^{-A|t|}; A > 0$. **(8)**

b. Find the fourier transform of the pulse shown in Fig.2 below **(8)**

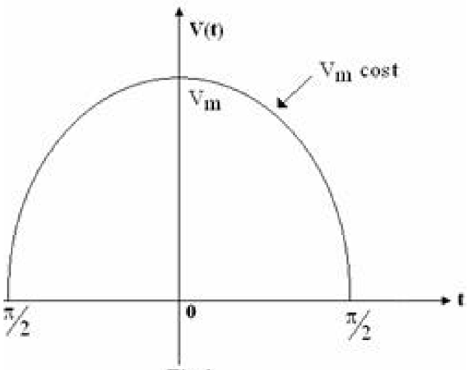


Fig.2

**Q.5** a. Using discrete time fourier transform, determine the frequency response and impulse response of a causal discrete-time LTI system that is characterised by, $y(n) - A \cdot y(n-1) = s(n); |A| < 1$. **(8)**

b. Determine the discrete-time frequency transform of the discrete-time periodic signal, $s(n) = \cos \omega_0 n$, with fundamental frequency $\omega_0 = {}^{2\pi}\!/_5$. **(8)**

**Q.6** a. The frequency response for a causal and stable continuous time LTI system is given by, $H(j\omega) = \dfrac{1 - j\omega}{1 + j\omega}$.

(i) Find the magnitude of $H(j\omega)$.

(ii) Determine which of the following statement is true about $\tau(\omega)$, the group delay of the system

1. $\tau(\omega) = 0; \text{for } \omega > 0$
2. $\tau(\omega) > 0; \text{for } \omega > 0$
3. $\tau(\omega) < 0; \text{for } \omega > 0$ **(8)**

b. Find the Nyquist rate and Nyquist interval for the continuous-time signal, $s(t) = \dfrac{1}{2\pi}\cos(4000\pi t)\cdot\cos(1000\pi t)$. **(8)**

**Q.7** a. Find the z-transform and ROC for the signal sequence $s(n) = \left[4\left(2^n\right) - 5\left(3^n\right)\right]u(n)$. **(6)**

b. State initial value theorem for z-domain transfer function. Find the initial value of the corresponding sequence, s(n) having a z-transform $s(z) = 2 + 3z^{-1} + 4z^{-2}$. **(6)**

c. Determine the sequence s(n) whose z-transform $s(z) = \dfrac{1}{1 - \frac{1}{4}z^{-1}}, |z| < \dfrac{1}{4}$. **(4)**

**Q.8** a. A binary source generates digits 1 and 0 randomly with probabilities 0.6 and 0.4 respectively.
(i) What is the probability that two 1's and three 0's will occur in a five-digit sequence?
(ii) What is the probability that atleast three 1's will occur in a five-digit sequence? **(6)**

b. The pdf of a random variable X is given by, $f(x) = \begin{cases} K; & a \le x \le b \\ 0; & \text{otherwise} \end{cases}$
where, K is a constant
(i) Determine the value of K.
(ii) Let a = -1 and b=2. Calculate $P[(|X|) \le c]$ for $c = \frac{1}{2}$. **(6)**

c. Consider a random process X(t) given by, $X(t) = A \cdot \cos(\omega t + \theta)$; where $\theta$ is a random variable uniformly distributed in the range $(0, 2\pi)$. Show that the process is ergodic in the mean. **(4)**

**Q.9** Write short notes on:

(i) Joint probability.
(ii) Conditional probability.
(iii) Cross spectral density.
(iv) White noise. **(4×4 = 16)**