JUNE 2008

Code: AE06/ AC04/ AT04 **Subject: SIGNALS & SYSTEMS**

Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: **(2x10)**

a. The period of the signal $x(t) = 10\sin 12\pi t + 4\cos 18\pi t$ is

(A) $\frac{\pi}{4}$ **(B)** $\frac{1}{6}$

(C) $\frac{1}{9}$ **(D)** $\frac{1}{3}$

b. The autocorrelation of a rectangular pulse is

(A) another rectangle pulse **(B)** Square pulse
(C) Triangular pulse **(D)** Sinc pulse

c. If the Fourier series coefficients of a signal are periodic then the signal must be

(A) continuous-time, periodic **(B)** discrete-time, periodic
(C) continuous-time, non periodic **(D)** discrete-time, non perodic

d. The area under the curve $\int_{-\infty}^{\infty} \delta(t)\,dt$ is

(A) ∞ **(B)** unity
(C) 0 **(D)** undefined

e. A transmission is said to be _____ if the response of the system is exact replica of the input signal.

(A) LTI **(B)** Distorted
(C) Distortionless **(D)** Causal

f. Laplace Transform of t^n is always equal to

(A) $\frac{n}{s^n}$ **(B)** $\frac{n!}{s^n}$

(C) $\frac{n!}{s^{n+1}}$ **(D)** All

g. For a stable system

(A) $|z|<1$ **(B)** $|z|=1$

(C) $|z|>1$ **(D)** $|z|\neq 1$

h. The region of convergence of a causal finite duration discrete time signal is

(A) The entire 'z' plane except z = 0
(B) The entire 'z' plane except z = ∞
(C) The entire 'z' plane
(D) A strip in z-plane

i. The CDF for a certain random variable is given as

$$F_X(x) = 0 \qquad -\infty < x \le 0$$
$$= kx^2 \qquad 0 < x \le 10$$
$$= 100k \qquad 10 < x < \infty$$

The value of k is

(A) 100 **(B)** 50

(C) 1/50 **(D)** 1/100

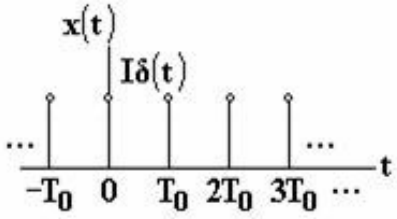
j. The group delay function $\tau(\omega)$ is related to phase function $\phi(\omega)$ as

(A) $\tau(\omega) = \frac{-d}{d\omega}\phi(\omega)$ **(B)** $\tau(\omega) = \frac{d}{d\omega^2}\phi(\omega)$

(C) $\tau(\omega) = \frac{d^2}{d\omega^2}\phi(\omega)$ **(D)** $\tau(\omega) = \frac{d^2}{d\omega}\phi(\omega)$

Answer any FIVE Questions out of EIGHT Questions.
Each question carries 16 marks.

Q.2 a. Find the Fourier Series of the following periodic impulse train? **(8)**



b. The Magnitude and phase of the Fourier Transform of a signal x(t) are shown in Fig 2. Find the signal x(t). **(8)**

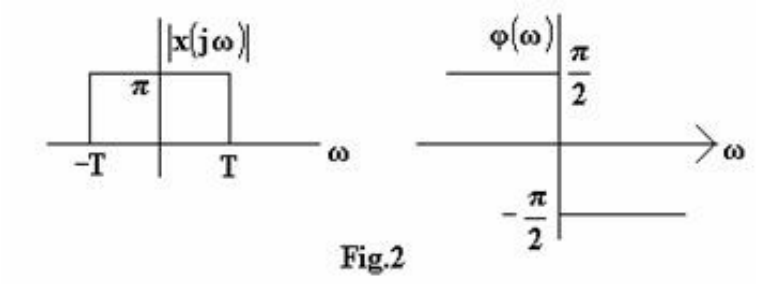


Fig.2

Q.3 a. Find the Discrete Time Fourier Transforms of the following signals and draw its spectra. **(8)**

(i) $x_1(n) = a^{|n|} \qquad |a| < 1$

(ii) $x_2(n) = \cos \omega_0 n$ with $\omega_0 = \frac{2\pi}{5}$

b. The frequency response for a causal and stable continuous time LTI system is expressed as

$$H(j\omega) = \frac{1 - j\omega}{1 + j\omega}$$

(8)

(i) Determine the magnitude of $H(j\omega)$

(ii) Find phase response of $H(j\omega)$

(iii) Find Group delay.

Q.4 a. Find the Nyquist rate and Nyquist interval for the continuous-time signal given below?

$$x(t) = \frac{1}{2\pi}\cos(4000\pi t) \cdot \cos(1000\pi t)$$ **(4)**

b. Consider a discrete-time LTI system with impulse response $h(n)$ given by

$h(n) = \alpha^n u(n)$

Determine whether the system is causal and condition for stability. **(4)**

c. Check for Causality, Linearity of the following signals? **(8)**

(i) $y(t) = x(\sqrt{t})$ (ii) $y(t) = x(t^2)$

(iii) $y(t) = 10x(t+2)+5$ (iv) $y[n] = \sum_{k=-\infty}^{n} x[n]$

Q.5 a. Determine the Laplace transform of the following given functions. **(6)**

(i) $x(t) = \cos^3(3t)$ (ii) $x(t) = t \sin at$

b. The transfer function of the system is given by $H(s) = \frac{2}{s+3} + \frac{1}{s-2}$

Determine the impulse response if the system is

(i) stable (ii) causal

State whether the system will be stable and causal simultaneously. **(10)**

Q.6 a. Determine the inverse Z Transform of the following X(Z) by the partial fraction expansion method. **(8)**

$$X(Z) = \frac{Z+2}{2Z^2 - 7Z + 2}$$

if the ROCs are (i) $|Z| > 3$ (ii) $Z < \frac{1}{2}$

(iii) $\frac{1}{2} < |Z| < 3$

b. A Causal discrete-time LTI system is described by

$$y(n) - \frac{3}{4}y(n-1) + \frac{1}{8}y(n-2) = x(n)$$

where $x(n)$ and $y(n)$ are the input and output of the system, respectively.

(i) Determine the H(z) for causal system function

(ii) Find the impulse response h(n) of the system

(iii) Find the step response of the system. **(8)**

Q.7 a. A random Variable X has the uniform distribution given by

$$f_X(x) = \frac{1}{2\pi} \quad \text{for } 0 \le x \le 2\pi$$

$$= 0 \quad \text{otherwise}$$

Determine mean, mean square, Variance. **(10)**

b. Discuss the Properties of Gaussian PDF. **(6)**

Q.8 a. A Stationary random Variable x(t) has the following autocorrelation function

$R_x(\tau) = \sigma^2 e^{-\mu|\tau|}$ where σ^2, μ are constants

$R_x(t)$ is passed through a filter whose impulse response is

$h(\tau) = \alpha e^{-\alpha\tau} u(\tau)$

where α is const, $u(\tau)$ is unit step function

(i) find power spectral density of random signal x(t)

(ii) find power spectral density of O/P signal y(t) **(8)**

b. Determine the convolution of the two continuous time functions given below:

$x(t) = e^{-at}u(t) \quad a > 0$

$h(t) = u(t)$ **(8)**

Q.9 a. Determine signal energy and power of the following signals

(i) $x(n) = u(n)$ (ii) $x(t) = e^{-3t}$ **(8)**

b. Find the DTFT of the following sequence

x(n)=u(n) **(4)**

c. Find the inverse Fourier Transform of $\delta(\omega)$ **(4)**