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# SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.Tech-All Branches Except Bio Groups

Title of the Paper: Engineering Mathematics - II Max. Marks: 80

Sub. Code: 6C0016 (2006/07/08/09)

Time: 3 Hours

Date: 04/12/2010

Session: AN

PART - A

(10 X 2 = 20)

Answer ALL the Questions

1. Expand  $\tan 6\theta$  in terms of powers of  $\tan \theta$ .
2. If  $x + iy = C \cos(A - iB)$  show that  $\frac{x^2}{C^2 \cosh^2 B} + \frac{y^2}{C^2 \sinh^2 B} = 1$ .
3. Find the direction cosines of the line joining P(2,3,5) and Q(-1,3,2).
4. Prove that the two spheres  $x^2 + y^2 + z^2 - 2x + 4y - 4z = 0$  and  $x^2 + y^2 + z^2 + 10x + 2z + 10 = 0$  touch each other.
5. Define Beta and Gamma functions.
6. Prove that  $\frac{\beta(m+1, n)}{\beta(m, n+1)} = \frac{m}{n}$ .
7. Show that  $F = (y^2 - z^2 + 3yz - 2x)\mathbf{i} + (3xz + 2xy)\mathbf{j} + (3xy - 2xz + 2z)\mathbf{k}$  is irrotational.
8. State Green's theorem in a plane.

9. Prove that  $\int_{-a}^a f(x)dx = \begin{cases} 2\int_0^a f(x)dx, & \text{if } f(x) \text{ is even} \\ 0, & \text{if } f(x) \text{ is odd} \end{cases}$

10. Evaluate  $\int_0^1 \int_0^z \int_0^{y+z} dz dy dx$ .

PART – B

(5 x 12 = 60)

Answer All the Questions

11. Expand  $\cos 8\theta$  in a series of powers of (i)  $\sin\theta$  only and  
(ii)  $\cos\theta$  only.

(or)

12. If  $\tanh(x/2) = \tan(\theta/2)$ , show that  $x = \log \tan(\pi/4 + \theta/2)$  and conversely.

13. Show that the lines  $\frac{x-4}{2} = \frac{y-5}{3} = \frac{z-6}{4}$  and  $\frac{x-2}{3} = \frac{y-3}{4} = \frac{z-4}{5}$  are coplanar and find the equation of the plane in which they lie.

(or)

14. Find the equation of the sphere passing through the points (1,1,-2) and (-1,1,2) and having its centre on the line  $x + y - z - 1 = 0 = 2x - y + z - 2$ .

15. Prove that  $\beta(m,n) = \int_0^{\infty} \frac{x^{m-1}}{(1+x)^{m+n}} dx$ . Hence deduce that

$$\beta(m,n) = \int_0^1 \frac{x^{m-1} + x^{n-1}}{(1+x)^{m+n}} dx.$$

(or)

16. Express  $\int_0^1 x^p (1-x^q)^m dx$  in terms of Gamma functions.

Hence Evaluate  $\int_0^1 x^3(1-\sqrt{x}) dx$

17. Verify Stokes theorem when  $F = (2xy - x^2)\mathbf{i} - (x^2 - y^2)\mathbf{j}$  and  $C$  is the boundary of the region enclosed by the parabolas  $y^2 = x$  and  $x^2 = y$ .

(or)

18. Verify Gauss divergence theorem for  $F = x^2\mathbf{i} + y^2\mathbf{j} + z^2\mathbf{k}$  where  $S$  is the surface of the cuboid formed by the planes  $x = 0$ ,  $x = a$ ,  $y = 0$ ,  $y = b$ ,  $z = 0$  and  $z = c$ .

19. Find a reduction formula for  $\int \sin^n x dx$  ( $n$  is positive integer)

(or)

20. Change the order of integration in  $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy dx$

and then evaluate it.

