SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act,1956)

Course & Branch :B.E/B.Tech – Common to ALL Branches (Excepts to Bio Groups)
Title of the Paper :Engineering Mathematics – II Max. Marks :80
Sub. Code :6C0016 Time : 3 Hours
Date :03/12/2009 Session :AN

PART - A (10 x 2 = 20)
Answer ALL the Questions

1. Prove that sin(ix) = I sinhx.

2. Prove that cosh (A + B) = cosh A cosh B + sinh A sinhB.

3. Find the distance between the planes
$x - 2y + 2z - 8 = 0$ *and* $6y - 3x - 6z = 57$.

4. Find the tangent plane to the sphere
$x^2 + y^2 + z^2 + 6x - 2y - 4z = 35$ at (3, 4, 4).

5. Prove that $\Gamma(\alpha + 1) = \alpha\, \Gamma\, \alpha$.

6. Define $\beta(m, n)$ and prove $\beta(m, n) = \beta(n, m)$.

7. Find a unit normal vector ‘n’ of the cone of revolution
$z^2 = 4(x^2 + y^2)$ at the point (1, 0, 2).

8. Is the flow of a fluid whose velocity vector v = [secx, cosecx, 0] is irrotational?

9. Evaluate $\int_0^{\frac{\pi}{2}} \sin^7 x dx$.

10. Evaluate $\int_0^a \int_0^b (x+y) dxdy$.

PART – B (5 x 12 = 60)

Answer All the Questions

11. Express cos6θ and $\frac{\sin 6\theta}{\sin\theta}$ in series of powers of cosθ. Hence obtain tan6θ in terms of tanθ.

(or)

12. (a) Find real and imaginary parts of sin(x + iy) and tan(u + iv). If sin(x + iy) = tan (u + iv), prove tanx sinh2v = tanhy sin2u.

(b) Show that $\left(\frac{1+\sin\theta+i\cos\theta}{1+\sin\theta-i\cos\theta}\right)^n = \cos\left(n\frac{\pi}{2}-n\theta\right)+i\sin\left(n\frac{\pi}{2}-n\theta\right)$.

13. (a) Find the equation of the plane through (1, 2, 3) and perpendicular to $x - y + z = 2$ and $2x + y - 3z = 5$.

(b) Find the ratio in which the sphere $x^2 + y^2 + z^2 - 2x + 6y + 14z + 3 = 0$ divides the line joining points P(2, -1, -4) and Q(5,5,5).

(or)

14. (a) Find the equation of the sphere which pass through the circle $x + 2y + 3z = 8$. $x^2 + y^2 + z^2 - 2x - 4y = 0$ and touches the plane $4x + 3y = 25$.

(b) Find the equation of the plane which bisects perpendicularly the join of (2, 3, 5) and (5, -2, 7)

15. Using Beta and Gamma function, show that for any positive integer m

(a) $\int_0^{\frac{\pi}{2}} \sin^{2m-1}(\theta)d\theta = \frac{(2m-2)(2m-4)...2}{(2m-2)(2m-3)...3}$.

(b) $\int_0^{\frac{\pi}{2}} \sin^{2m}(\theta)d\theta = \frac{(2m-1)(2m-3)...1\pi}{2m(2m-2)..........22}$.

(or)

16. (a) Explain $\int_0^1 x^m(1-x^p)dx$ in terms of Beta function and hence evaluate $\int_0^1 x^{\frac{3}{2}}\left(1-\sqrt{x}\right)^{\frac{1}{2}} dx$.

(b) Evaluate $\int_0^\infty \sqrt{x}\, e^{-x^2}\, dx$ in terms of Gamma function.

17. Verify Green's theorem for $\int_C$ [(xy + y^2) dx + x^2 dy], where C is bounded by y = x; and y = x^2.

(or)

18. Using Stoke's theorem evaluate $\int_C$ [(x + y) dx + (2x – z) dy + (y + z) dz] where C is the boundary of the triangle with vertices (2, 0, 0), (0, 3, 0) and (0, 0, 6).

19. (a) Change the order of integration in I = $\int_0^1 \int_{x^2}^{2-x}$ xy dx dy and hence evaluate the same.

(b) Obtain a reduction formula for $V_n = \int_0^{\frac{\pi}{2}} x^n \cos 3x \, dx$ and hence evaluate V_2.

(or)

20. (a) Evaluate $\int_{-1}^{1} \int_{0}^{z} \int_{x-z}^{x+z} (x + y + z) \, dx \, dy \, dz$.

(b) Prove that $\int_0^{\frac{\pi}{2}} \log \sin x \, dx = -\frac{\pi}{2} \log 2$.