SATHYABAMA UNIVERSITY
(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.Tech – Common to ALL Branches
(Excepts to Bio Groups)
Title of the paper: Engineering Mathematics - II
Semester: II — Max. Marks: 80
Sub.Code: 6C0016 (2006/2007) — Time: 3 Hours
Date: 04-12-2008 — Session: AN

PART – A (10 x 2 = 20)
Answer All the Questions

1. If $2 \cos\theta = x + \frac{1}{x}$, Prove that $2 \cos r\theta = x^r + \frac{1}{x^r}$.

2. Separate the real of imaginary part of Sinh(x+iy)

3. Prove that the points A (3,2,4), B (4,5,2), C(5,8,0) are collinear.

4. Prove that the planes $5x - 3y + 4z = 1$, $8y + 3y + 5z = 4$ and $18x - 3y + 13z = 6$ contain a common line.

5. What is the reduction formula for $\Gamma(n)$?

6. Define Beta function.

7. Prove that div grad f = $\nabla^2 f$.

8. State Stoke's theorem.

9. Evaluate $\int_0^{\pi/2} \sin^6 x\, dx$.

10. Find the area of a circle $x^2 + y^2 = 1$, which lies in the positive quadrant.

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) If tan (θ + iφ) = $e^{i\alpha}$, then show that

(i) $\theta = (n+\frac{1}{2})\ \pi/2$

(ii) φ = ½ log tan (π/4 + ∝/2).

(b) Given $\frac{1}{\int} = \frac{1}{LPi} + CPi + \frac{1}{R}$, where L,P,R are real, express ∫ in the form A $e^{i\theta}$ giving the values of A and θ.

(or)

12. (a) Expand Sin ⌉θ $\cos^3\theta$ in a series of sines of multiples of θ.

(b) If $\cos^{-1}(x+iy)$ = ∝ + iβ, then Prove that x^2 $\text{sech}^2\beta$ + y^2 $\text{cosech}^2\beta$ = 1.

13. (a) Find the equation of the square through the points (0,0,0), (0,1,-1), (-1,2,0) and (1,2,3). Locate its centre of find the radius.

(b) Find the equation in the symmetrical of the projection of the line $\frac{x-1}{2} = -(y+1) = \frac{z-3}{4}$ on the plane $x + 2y + z = 12$.

(or)

14. (a) Find the angle between the line $\frac{x-x^1}{l} = \frac{y-y^1}{m} = \frac{z-z^1}{n}$ of the plane $ax+by+cz+d=0$.

(b) Show that the shortest distance between z-axis of the line $ax+by+cz+d=0=a^1x+b^1y+c^1z+d^1$ is

$$\frac{dc^1 - d^1c}{\sqrt{(ac^1-a^1c)^2+(bc^1-b^1c)^2}}$$

15. (a) Prove that $\beta(m,n) = \frac{\Gamma(m).\Gamma(n)}{\Gamma(m+n)}$.

(b) Evaluate $\int_{o}^{\pi/2} \cos^m x \sin^n x \, dx$, where *m* of *n* is even integers.

(or)

16. (a) Prove that $\Gamma(n+ ½)=\frac{\Gamma(2n+1).(\Pi)}{2^{2n-1}.\Gamma(n+1)}$

(b) Prove that $\beta(n,n) = \frac{\sqrt{\Pi}\,\Gamma(n)}{2^{2n-1}\Gamma(n+½)}$.

17. (a) Find the workdone in moving a particle in the force field $F = 3x^2i + (2xz-y)j + zk$ along (i) the straight line from (0,0,0) to (2,1,3). (ii) the curve defined by $x^2= 4y$, $3x^3= 8z$ from $x=0$ to $x = 2$.

(b) Prove that $\nabla r^n = nr^{n-2}\vec{r}$, where $\vec{r} = xi + yj + zk$.

(or)

18. (a) Prove that $\nabla \times (\nabla \times V) = \nabla(\nabla.V) - \nabla^2 V$.

(b) Verify Gauss divergence theorem, for $f = 4xzi - y^2j + yzk$ taken over the cube bounded by $x = y = z = 0$ of $x =y=z= 1$.

19. (a) Calculate $\iint r^3\, dr d\theta$, over the area included between the circles $r = 2 \sin\theta$ and $r = 4 \sin \theta$.

(b) Show that the area between the Parabolas $y = 4\ ax$ and $x^2 = 4\ ay$ is $\frac{16}{3}a^2$.

(or)

20. (a) Find the volume of the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1.$

(b) Show that $u_n - (n+a)u_{n-1} + a(n-1)u_{n-2} = 0,$ if $un = \int_o^a x^n e^{-x} dx.$