# SATHYABAMA UNIVERSITY

**(Established under section 3 of UGC Act, 1956)**

Course & Branch: B.E/B.Tech – (Common to ALL Branches)
Except Bio Groups
Title of the paper: Engineering Mathematics - II
Semester: II                Max.Marks: 80
Sub.Code: 6C0016(2006-07-08)                Time: 3 Hours
Date: 12-05-2009                Session: FN

PART - A        (10 X 2 = 20)

Answer ALL the Questions

1. Expand Cos4θ in a series of Powers of Cos θ.

2. Prove that Cosh (A + B) = Cosh A Cosh B + Sinh A Sinh B.

3. Using direction cosines, prove that the points A (3, 1, 3), B(1, -2, 1) and C(-1, -5, -5) are Collinear.

4. Find the intercepts made by the plane ax + by + cz + d = 0 on the co-ordinate axes.

5. Define Gamma and Beta function.

6. Prove that $\int_0^\infty x^4 e^{-x^2} dx = \frac{3}{8}\sqrt{\pi}.$

7. Find grad φ at the point (1, -2, -1) when $\varphi = 3x^2y - y^3z^2$.

8. Show that $\overline{F} = (y^2 - z^2 + 3yz - 2x)\ \overline{i} + (3xy + 2xy)\ \overline{J} + (3xy - 2xz + 2z)\ \overline{K}$ is solenoidal.

9. Prove that $\int_0^a f(x)dx = \int_0^a f(a-y)dy$.

10. Write down the reduction formula for $\int Sin^n xdx$.

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Prove that $\dfrac{Sin 7\theta}{Sin\theta} = 7 - 56 Sin^2\theta + 112 Sin^4\theta - 64 Sin^6\theta.$

(b) If $\dfrac{Sin\ \theta}{\theta} = \dfrac{19493}{19494}$, Prove that θ is equal to 1° nearly.

(or)

12. (a) If Sin (θ + φ) Cos α + isin α, prove that $Cos^2\theta = \pm \sin\alpha$.

(b) If tan h $\dfrac{x}{2} = \tan\ \theta/_2$, *Show that* $x = \log\ \tan\left(\pi/_4 + \theta/_2\right)$

13. (a) Find the equation of the plane which passes through the points (6, 2, -4) and (3, -4, 1) and is parallel to the line joining the points (1, 0, 3) and (-1, 2, 4).

(b) Prove that the lines $\dfrac{x-4}{2} = \dfrac{y-5}{3} = \dfrac{z-6}{4}$ *and* $\dfrac{x-2}{3} = \dfrac{y-3}{4} = \dfrac{z-4}{5}$ are coplanar and find the equation the plane in which they lie.

(or)

14. (a) Find the length and equations of the shortest distance between the lines $\dfrac{x-3}{3} = \dfrac{y-8}{-1} = \dfrac{z-3}{1}$ *and* $\dfrac{x+3}{-3} = \dfrac{y+7}{2} = \dfrac{z-6}{4}$.

(b) Find the equation of the sphere passing through the circle given by $x^2 + y^2 + z^2 + 3x + y + 4z - 3 = 0$ and $x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$ and the point (1, -2, 3).

15. (a) Evaluate $\int_0^1 x^m (1 - x^n)^p \, dx$ in terms of Gamma functions and hence find $\int_0^1 \frac{dx}{\sqrt{1 - x^n}}$.

(b) Show the volume of the region of space bounded by the co-ordinate planes and the surface $\sqrt{x/a} + \sqrt{y/b} + \sqrt{z/c} = 1 is \frac{abc}{90}$.

(or)

16. (a) Prove that $\beta(n, n) = \frac{\sqrt{\pi}\,\Gamma(n)}{2^{2n-1}\,\Gamma(n + 1/2)}$

(b) Evaluate $\iiint \frac{dx \quad dy \quad dz}{\sqrt{1-x^2-y^2-z^2}}$, taken over the region of space in the positive octant bounded by the sphere $x^2 + y^2 + z^2 = 1$.

17. Verify Stoke's theorem for $\vec{F} = y^2 z \bar{i} + z^2 x \bar{J} + x^2 y \bar{K}$ where S is the open surface of the cube formed by the planes x = ± a, y = ± a, and z = ± a in which the plane z = -a is cut.

(or)

18. Verify Gauss divergence theorem for $\vec{F} = x^2 \bar{I} + y^2 \bar{J} + z^2 \bar{K}$ where S is the surface of the cubold formed by the planes x = 0, x = a, y = 0, y = b, z = 0 and z = c.

19. (a) Prove that $\int_0^{\frac{\pi}{4}} \log(1 + \tan\theta)\, d\theta = \frac{\pi}{8} \log 2$.

(b) Evaluate $\int_0^{\frac{\pi}{2}} Sin^9 x \, Cos^5 x \; dx$.

(or)

20. (a) Evaluate $\int_0^3 \int_1^{\sqrt{4-y}} (x + y)\, dxdy$ , by changing the order of Integration.

(b) Evaluate $\int_0^{2\pi} \int_0^{\pi} \int_0^{a} r^4 \, Sin\, \phi drd\phi d\theta$ .