SATHYABAMA UNIVERSITY

(Established under section 3 of UGC Act, 1956)

Course & Branch: B.E/B.Tech
Title of the paper: Engineering Mathematics – II
(Common to all branches except Bio groups)

Semester : II — Max. Marks: 80
Sub.Code: 6C0016 — Time: 3 Hours
Date: 21-05-2007 — Session: FN

PART – A (10 x 2 = 20)

Answer ALL the Questions

1. Separate sin (x + iy) in to real and imaginary parts.

2. State Demoivre's Theorem.

3. Find the equation of the plane passing through (1, 2, 3) parallel to $4x + 5y - 3z = 7$.

4. Find the equation of the sphere whose centre is (2, -3, 1) and radius is 5 units.

5. Prove that $\int x^4 e^{-x^2} dx = \frac{3}{8}\sqrt{\pi}$

6. Find the value of $\int_0^{\frac{\pi}{2}} \sin^5\theta \cos^7\theta \quad d\theta$.

7. Find the values of the constants a, b, c, so that $\vec{F} = (axy + bz^3)\vec{i} + (3x^2 - cz)\vec{j} + (3xz^2 - y)\vec{k}$ may be irrotational.

8. Prove that curl (grad ϕ) = 0.

9. Evaluate $\int_0^2 \int_0^1 \int_0^3 dzdydx.$

10. Evaluate $\int_0^1 \int_0^2 xydydx$

PART – B (5 x 12 = 60)

Answer All the Questions

11. (a) Expand Cos 7θ in descending powers of cosθ.

(b) If u = log $\tan\left(\frac{\pi}{4}+\frac{\theta}{2}\right)$ then prove that $\tanh\left(\frac{u}{2}\right)=\tan\left(\frac{\theta}{2}\right)$

(or)

12. (a) Separate $\tan^{-1}(x + iy)$ in to real and imaginary parts.

(b) Prove that $\sinh^{-1} x = \log\left(x+\sqrt{x^2+1}\right)$

13. Find the shortest distance between the lines

$$\frac{x+1}{-3}=\frac{y-3}{2}=\frac{z+2}{1} \text{ and } \frac{x}{1}=\frac{y-7}{-3}=\frac{z+7}{2}$$

(or)

14. Show that the lines

$\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{x+7}{2}=\frac{z-6}{4}$ intersect. Find the coordinates of the point of intersection and the equation to the plane containing the,

15. (a) Prove that $\beta(m,n)=\frac{\Gamma m\,\Gamma n}{\Gamma m+n}$

(b) Find the values of $\int_0^{\frac{\pi}{2}} \sqrt{\sin\theta}\, d\theta \int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{\sin\theta}}$

(or)

16. (a) Find the value of $\iint x^m y^n \, dxdy$ taken over the area $x \geq 0, y \geq 0, x + y \leq 1$ in terms of gamma functions.
(b) Prove that $\beta(m, n + 1) + \beta(m + 1, n) = \beta(m, n)$.

17. Verify Green's theorem in a plane for $\int_c \left[\left(3x^2 - 8y^2\right)dx + \left(4y - 6xy\right)dy\right]$ where c is the boundary of the region defined by the lines $x = 0, y = 0$ and $x + y = 1$.

(or)

18. Verify gauss-Divergence Theorem for $\vec{F} = x^2 \vec{i} + z \vec{j} + yz \vec{k}$ over the cube formed by $x = \pm 1, y = \pm 1, z = \pm 1$

19. Change the order of integration $\int_0^4 \int_{\frac{x^2}{4}}^{2\sqrt{x}} dy\, dx$ and then evaluate it.

(or)

20. Establish the reduction formula for $e^{ax} x^n$.