Total No. of Questions—12] [Total No. of Printed Pages—8

[3762]-115

S.E. (Mechanical, Production and S/W) (I Sem.)

EXAMINATION, 2010

ENGINEERING MATHEMATICS—III

(2008 COURSE)

Time : Three Hours **Maximum Marks : 100**

N.B. :— (*i*) Answers to the two Sections should be written in separate answer-books.

(*ii*) In Section **I**, attempt Q. No. **1** or Q. No. **2**, Q. No. **3** or Q. No. **4**, Q. No. **5** or Q. No. **6**.

(*iii*) In Section **II**, attempt Q. No. **7** or Q. No. **8**, Q. No. **9** or Q. No. **10**, Q. No. **11** or Q. No. **12**.

(*iv*) Neat diagrams must be drawn wherever necessary.

(*v*) Figures to the right indicate full marks.

(*vi*) Use of non-programmable electronic pocket calculator is allowed.

(*vii*) Assume suitable data, if necessary.

SECTION I

1. (*a*) Solve the following Differential Equations (any *three*) : [12]

(*i*) $(D^4 + 5D^2 + 4)y = \cos\dfrac{x}{2}\cos\dfrac{3x}{2}$

(*ii*) $(D^2 - 2D - 3)y = 3e^{-3x}\sin(e^{-3x}) + \cos(e^{-3x})$

(*iii*) $(5+2x)\dfrac{d^2y}{dx^2} - 6(5+2x)\dfrac{dy}{dx} + 8y = 5\log(5+2x)$

(*iv*) $(D^3 - 5D^2 + 8D - 4)y = 2e^x + e^{2x}$

(*v*) $\dfrac{dx}{y} = \dfrac{dy}{-x} = \dfrac{dz}{xe^{x^2+y^2}}$.

P.T.O.

(*b*) Solve the simultaneous differential equation :

$$\frac{dx}{dt} + y = \sin t \, ; \; \frac{dy}{dt} + 4x = \cos t$$

given $x = 0$, $y = 1$ when $t = 0$. [5]

Or

2\. (*a*) Solve the following differential equations (any *three*) : [12]

(*i*) $(D^3 - 2D + 4)y = 3x^2 - 5x + 2$

(*ii*) $\dfrac{d^2y}{dx^2} + \dfrac{1}{x}\dfrac{dy}{dx} = A + B \log x$

(*iii*) $(D^2 + 1)y = \dfrac{1}{1 + \sin x}$ by variation of parameters

(*iv*) $(D^2 + D - 6)y = e^{-2x} \sin 3x$

(*v*) $(D^2 + 13D + 36)y = e^{-4x} + \sinh x$

(*b*) A spring stretches 1 cm under tension of 2 kgs and has negligible weight. It is fixed at one end and is attached to a weight W kgs at the other. It is found resonance occurs when an axial periodic force 2 cos 2t kgs acts on the weight. Show that when the free vibrations have died out, the forced vibrations are given by $x = ct \sin 2t$, and find values of W and C. [5]

3\. (*a*) Find Laplace transform of (any *two*) : [6]

(*i*) $\dfrac{\cos\sqrt{t}}{\sqrt{t}}$

(*ii*) $e^{-3t}\displaystyle\int_0^t t \sin 2t \, dt$

(*iii*) $\dfrac{d}{dt}\left(\dfrac{\sin t}{t}\right)$.

(*b*) Solve using Laplace transform method :

$$y'' + y = t, \; y(0) = 1, \quad y'(0) = -2.$$ [5]

(c) Find Fourier sine transform of : [6]

$$f(x) = \begin{cases} x & ; \quad 0 \le x \le 1 \\ 2 - x & ; \quad 1 \le x \le 2 \\ 0 & ; \quad x > 2 \end{cases}.$$

Or

4. (a) Find Inverse Laplace transform of (any *two*) : [8]

(i) $\frac{1}{s} \log\left(\frac{s+3}{s+2}\right)$

(ii) $\frac{1}{(s+4)^{3/2}}$

(iii) $\frac{1}{(s+1)(s^2+1)}$ by convolution theorem.

(b) Evaluate :

$$\int_0^\infty te^{-2t} \cos t \; dt.$$ [4]

(c) Solve the integral equation :

$$\int_0^\infty f(x) \cos \lambda x \, dx = e^{-\lambda}, \; \lambda > 0.$$ [5]

5. (a) Solve :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}, \text{ if :}$$ [8]

(i) u is finite for all t

(ii) $u = 0$ when $x = 0, \pi$ for all t

(iii) $u = \pi x - x^2$ when $t = 0$, $0 \le x \le \pi$.

(b) An infinitely long uniform metal plate is enclosed between the lines $y = 0$ and $y = l$ for $x > 0$. The temperature is zero along the edges $y = 0$, $y = l$ and at infinity. If the edge $x = 0$ is kept at constant temperature u_0, find the temperature distribution $u(x, y)$. [8]

Or

6. (*a*) The initial temperature along the length of an infinite bar is given by :

$$u(x,0) = \begin{cases} 2 & ; \ |x| < 1 \\ 0 & ; \ |x| > 1 \end{cases}$$

If the temperature $u(x, t)$ satisfies the equation :

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} \quad -\infty < x < \infty, \quad t > 0$$

find the temperature at any point of the bar at any time t using Fourier transform. [8]

(*b*) A string is stretched and fastened to two points l apart. Motion is started by displacing the string in the form,

$$u = a \sin \frac{\pi x}{l}$$

from which it is released at time $t = 0$. Find the displacement $u(x, t)$ from one end. [8]

SECTION II

7. (*a*) Calculate the first four moments about the mean of the following distribution : [6]

Marks	Number of students
0—10	6
10—20	26
20—30	47
30—40	15
40—50	6

(*b*) A group of 20 aeroplanes are sent on an operational flight. The chances that an aeroplane fails to return from the flight is 5 percent. Determine the probability that : [5]

(*i*) No plane returns

(*ii*) At most 3 planes do not return.

(c) The two lines of regression are $9x + y - \lambda = 0$ and $4x + y = \mu$ and the means of x and y are 2 and -3 respectively. Find the values of λ and μ. Also find the regression coefficients b_{xy} and b_{yx}. [5]

Or

8. (a) The following marks have been obtained by a class of students in two papers of Mathematics :

Paper I	Paper II
45	56
55	50
56	48
58	60
60	62
65	64
68	65
70	70
75	74
80	82
85	90

Calculate the coefficient of correlation for the above data. [6]

(b) 5,000 candidates appeared in a certain paper carrying a maximum of 100 marks. It was found that marks were normally distributed with mean 39.5 and standard deviation 12.5. Determine approximately the number of candidates who secured a first class for which minimum of 60 marks is necessary.

Given : $z = 1.64$

Area $= 0.4495$. [5]

(c) The demand for a particular spare part in a factory was found to vary from day to day. In a sample study, the following information was obtained :

Days	Number of parts demanded
Mon.	1124
Tues.	1125
Wed.	1110
Thurs.	1120
Fri.	1126
Sat.	1115

Test the hypothesis that the number of parts demanded does not depend on the day of the week. Given $\chi^2_{5,0.05} = 11.07$. [5]

9. (a) If

$$\bar{r} = \bar{a} \sinh t + \bar{b} \cosh t$$

where $\bar{a}$ and $\bar{b}$ are constant vectors, prove that :

$$\frac{d\bar{r}}{dt} \times \frac{d^2\bar{r}}{dt^2} = \text{constant}.$$ [4]

(b) Prove the following vector identities (any *two*) : [8]

(i) $\nabla^2 \left[\nabla \cdot \left(\frac{\bar{r}}{r^2} \right) \right] = \frac{2}{r^4}$

(ii) $\nabla \left(\frac{\bar{a} \cdot \bar{r}}{r^n} \right) = \frac{\bar{a}}{r^n} - \frac{n(\bar{a} \cdot \bar{r})}{r^{n+2}} \bar{r}$

(iii) $\nabla \times \left[\bar{a} \times \left(\bar{b} \times \bar{r} \right) \right] = \bar{a} \times \bar{b}$.

(c) If directional derivative of

$$\phi = ax^2y + by^2z + cz^2x \quad \text{at } (1, 1, 1)$$

has maximum magnitude 15 in the direction parallel to :

$$\frac{x-1}{2} = \frac{y-3}{-2} = \frac{z}{1}$$

find the values of a, b, c. [5]

Or

10. (*a*) Show that :

$$\overline{F} = (2xz^3 + 6y)\,\hat{i} + (6x - 2yz)\,\hat{j} + (3x^2z^2 - y^2)\,\hat{k}$$

is irrotational. Find scalar potential ϕ such that :

$$\overline{F} = \nabla\phi.$$ [6]

(*b*) Find the directional derivative of

$$\phi = 4xz^3 - 3x^2y^2z \text{ at } (2, -1, 2)$$

along tangent to the curve

$$x = e^t \cos t,\ y = e^t \sin t,\ z = e^t \text{ at } t = 0.$$ [6]

(*c*) For a solenoidal vector field $\overline{E}$, show that curl curl curl curl

$$\overline{E} = -\nabla^4\,\overline{E}.$$ [5]

11. (*a*) If

$$\overline{F} = (2xy + 3z^2)\,\hat{i} + (x^2 + 4yz)\,\hat{j} + (2y^2 + 6xz)\,\hat{k},$$

evaluate :

$$\int_c \overline{F}\,.\,d\overline{r}$$

where c is the curve :

$$x = t,\ y = t^2,\ z = t^3$$

joining the points (0, 0, 0) and (1, 1, 1). [5]

(*b*) Evaluate :

$$\iint_s \left(x^3\,\hat{i} + y^3\,\hat{j} + z^3\,\hat{k}\right).\,d\overline{s}$$

where s is the surface of the sphere :

$$x^2 + y^2 + z^2 = 9$$ [6]

(c) Verify Stokes theorem for

$$\overline{\mathrm{F}} = xy^2\hat{i} + y\hat{j} + xz^2\hat{k}$$

for the surface of rectangular lamina bounded by :

$x = 0,\ y = 0,\ x = 1,\ y = 2,\ z = 0.$ [6]

Or

12. (a) Find the work done in moving a particle once round the ellipse :

$$\frac{x^2}{25} + \frac{y^2}{16} = 1,\ \ z = 0$$

under the field of force given by :

$$\overline{\mathrm{F}} = (2x - y + z)\hat{i} + (x + y - z^2)\hat{j} + (3x - 2y + 4z)\hat{k}.$$ [5]

(b) Evaluate :

$$\iint_s (\nabla \times \overline{\mathrm{F}}) \,.\, d\overline{s}$$

where

$$\overline{\mathrm{F}} = (x^3 - y^3)\hat{i} - xyz\hat{j} + y^3\hat{k}$$

and s is the surface $x^2 + 4y^2 + z^2 - 2x = 4$ above the plane $x = 0$. [6]

(c) Use divergence theorem to evaluate :

$$\iint_s \left(y^2z^2\hat{i} + z^2x^2\hat{j} + x^2y^2\hat{k} \right) . d\overline{s}$$

where s is the upper part of the sphere $x^2 + y^2 + z^2 = 16$ above xoy plane. [6]