SECTION - A

10 × 2 = 20

VERY SHORT ANSWER TYPE QUESTIONS

Attempt ALL questions. Each question carries 2 marks.

1. Find the power of the point $(5, -6)$ with respect to $x^2 + y^2 + 8x + 12y + 15 = 0$.

2. A plane passes through a fixed point (a, b, c). Show that the foot of the perpendicular from the origin to the plane lies on the sphere $x^2 + y^2 + z^2 - ax - by - cz = 0$.

3. Find the value of k if the line $2y = 5x + k$ touches the parabola $y^2 = 6x$.

4. Find the equation of the ellipse whose axes are the coordinate axes and passes through the points $(-2, 2)$ and $(3, -1)$.

5. If $y = e^{-x} \cos x$ then show that $y_4 + 4y = 0$.

6. Evaluate $\int \dfrac{e^{\log x}}{x} \, dx$.

7. Evaluate $\int x^2 \cos x \, dx$.

8. Evaluate $\int_0^{\pi/2} \sin^7 x \, dx$.

9. Find the area cut off between $x = 0$, $2x = y^2 - 1$.

10. Solve $y(1 + x)dx + x(1 + y) \, dy = 0$.

SECTION - B

5 × 4 = 20

SHORT ANSWER TYPE QUESTIONS

Attempt any 5 questions. Each question carries 4 marks.

11. Find the pole of the line $x + y + 2 = 0$ w.r.t. the circle $x^2 + y^2 - 4x + 6y - 12 = 0$.

12. Find the vertex, focus, latus rectum and the equations of axis and directrix of the parabola $y^2 - x + 4y + 5 = 0$.

13. If the lines $3x - 4y = 12$ and $3x + 4y = 12$ meets on a hyperbolas $S = 0$ of conjugate axis is 6 then find the eccentricity of the hyperbola $S = 0$.

14. Find the area of the triangle formed by the points $(1, \pi/6)$, $(2, \pi/3)$, $(3, \pi/2)$.

15. Evaluate $\int \dfrac{2x+3}{(x+2)(x^2+4)}\,dx$.

16. Solve $\dfrac{dy}{dx} = \sin(x+y) + \cos(x+y)$.

17. Solve $(1+y^2)\,dx = (Tan^{-1}\,y - x)\,dy$.

SECTION - C

$5 \times 7 = 35$

LONG ANSWER TYPE QUESTIONS

Attempt **any 5** questions. Each question carries 7 marks.

18. Find the equation to the circle circumscribing the triangle formed by the lines $x+y=6$, $2x+y=4$, $x+2y=5$.

19. Find the equation of the circle coaxal with the circles $x^2+y^2-7x+12=0$, $x^2+y^2+8x+12=0$ and through the point $(-2, 3)$.

20. Show that the locus of the poles of chords of the ellipse $\dfrac{x^2}{a^2}+\dfrac{y^2}{b^2}=1$ which touch the parabola $y^2 = 4px$ is $pa^2 y^2 + b^4 x = 0$.

21. If $y = e^{-x^2}$ then show that $y_{n+2} + 2x\,y_{n+1} + 2(n+1)\,y_n = 0$.

22. If $I_n = \int \cos^n x\,dx$ then show that $I_n = \dfrac{\cos^{n-1} x \sin x}{n} + \dfrac{n-1}{n}\,I_{n-2}$. Hence find $\int \cos^5 x\,dx$.

23. Evaluate $\displaystyle\int_0^{\pi} \dfrac{x}{1+\sin x}\,dx$.

24. Find the approximate value of $\displaystyle\int_0^1 \dfrac{dx}{1+x^2}$, using Simpson's rule with $n=4$.