MECHANICAL SCIENCE—2009

Group — A
(Multiple Choice Questions)

1. Choose the correct alternatives of the following : 10×1

(i) A perpetual motion machine is

(a) a thermodynamic machine (b) a non-thermodynamic machine (c) a real machine (d) a hypothetical machine whose operation would violate the laws of thermodynamics

(ii) A thermodynamic system may be defined as a quantity of matter upon which attention is focused for study if

(a) it is only bounded by real surface (b) the boundary surface is constant in shape and volume (c) it is not bounded by an imaginary surface (d) it is bounded by either real surface or imaginary surface, irrespective of shape or voulme

(iii) The expression $\int Pdv$ may be applied for obtaining work os

(a) non-flow reversible process (b) steady flow reversible process (c) steady flow non-reversible process (d) steady flow adiabatie reversible process

(iv) The gas constant (R) is equal to the

(a) sum of two specific heats (b) difference of two specific heats
(c) product of two specific heats (d) none of these

(v) A carnot cycle operates between the temperature of 1000 K and 500 K. Then the efficiency of the cycle is

(a) 50% (b) more then 50% (c) less than 50% (d) none of these

(vi) In a reversible cycle, the entropy of the system

(a) increases (b) decreases
(c) does not change (d) depends on the properties of working substances

(vii) The latent heat of vaporization at critical point is

(a) less than zero (b) greater than zero (c) equal to zero (d) all of these

(viii) The work output of the theoretical Otto cycle

(a) increases with increase in compression ratio (b) increases with increase in pressure ratio (c) increases with increase in adiabatic index γ (d) follows all of these

(ix) Atmospheric pressure is

(a) Gauge pressure-Absolute pressure (b) Absolute pressure-Gauge pressue
(c) Absolute pressue-Vacuum pressure (d) Gauge pressure-Vacuum pressure

(x) A differential manometer is used for measuring the

(a) pressure at a point (b) velocity at a point
(c) difference of pressure at two points (d) discharge

(xi) Reynolds number is expressed as

(a) $\frac{\rho VD}{\mu}$ (b) $\frac{V^2D}{\rho}$ (c) $\frac{V\rho^2D}{\nu}$ (d) $\frac{V^2D^2}{\nu}$

(xii) During the throttling process

(a) internal energy does not change (b) pressure does not change

(c) entropy does not change (d) enthalpy does not change

Ans. (i) d ; (ii) d ; (iii) a ; (iv) b ; (v) a ; (vi) c ; (vii) c ; (viii) d ; (ix) b ; (x) c ; (xi) a ; (xii) d.

Group–B
(Short Answer Type Questions)
Answer *any three* Questions

2. (a) What is the quality of wet stream?

(b) What is the difference between a refrigerator and a heat pump? Establish the relation : $COP_{HP} = COP_R + 1$. 2+3

Ans. (a) Quality or dryness fraction is defined as the ratio of mass of saturated vapour to the total mass of the mixture. That is,

$$x = \frac{\text{mass of saturated vapour}}{\text{total mass of mixture}} = \frac{m_g}{m} = \frac{m_g}{m_f + m_g}$$

where m = total mass of liquid-vapour mixture

m_f = mass of saturated liquid

m_g = mass of saturated vapour

Ans. (b) The transfer of heat from a low temperature medium to a high temperature medium requires speial devices called **refrigerators** and **heat pumps.**

Refrigerators and heat pumps are simply heat engines operated in the reverse direction.

A heat pump is a device which works on a cycle, maintains temperature of a body which is more than the temperature of surroundings.

The objective of a refrigerator is to maintain the refrigerated space at a low temperature by reoving heat from it. A refrigerator operates between the ambient temperature and a low temperature.

The objective of a heat pump is to reject heat to a high temperature body. A heat pump operates between the ambient temperature and a high temperature.

3. Prove that entropy change for an ideal gas

$$\int ds = mC_v \ln\left(\frac{P_2}{P_1}\right) + mC_v \ln\left(\frac{V_2}{V_1}\right)$$

Ans. For an ideal gas, $dh = C_P\, dT$, $P_v = RT$ and $du = C_v\, dT$

The characteristic gas constant $R = C_P - C_v$

From the property of thermodynamic relation, we can write–

$$Tds = dh - vdP$$

$$\Rightarrow ds = \frac{dh}{T} - \frac{vdP}{T} = C_P \frac{dT}{T} - \frac{dP}{P} R$$

Integrating between any two states 1 and 2 for an ideal gas

$$\int_1^2 ds = \left[C_P \ln T - R \ln P\right]_1^2 = C_P \ln \frac{T_2}{T_1} - R \ln \frac{P_2}{P_1} = C_P \ln \frac{T_2}{T_1} - (C_P - C_v) \ln \frac{P_2}{P_1}$$

$$= C_P \ln \frac{T_2 P_1}{T_1 P_2} + C_v \ln \frac{P_2}{P_1} = C_P \ln \frac{V_2}{V_1} + C_v \ln \frac{P_2}{P_1}$$

4. A 0.025 m^3 vessel contains 0.3 kg of steam at 2 MPa. Determine the quality and enthalpy of steam. Given t_s = 212.2°C, v_f = 0.001177 m^3/kg, v_g = 0.0995 m^3/kg, h_f = 908.5 kJ/kg, h_{fg} = 1888.7 kJ/kg, s_f = 2.447 kJ/kg-K, s_{fg} = 3.590 kJ/kg-K. 5

Soln. : Specific volume of the stea

$$v = \frac{V}{m} = \frac{0\cdot025}{0\cdot3} = 0\cdot083 \text{ m}^3/\text{kg}$$

Let x be the quality of the steam.

$$v = v_f + x(v_g - v_f)$$

The specific enthalpy of the wet steam

$$h = hf + x(h_g - h_f) = 908\cdot5 + (0\cdot8322)(1888\cdot7) = 2480\cdot28 \text{ kJ/kg}$$

5. State and prove Pascal's law of pressure at a point of a fluid body.

Ans. Pascal's law states that pressure (or intensity of pressure) at a point in a static fluid is equal in magnitude in all directions.

It, let a small wedge shaped fluid element of unit length in equilibrium be considered. The mean pressures at the three surfaces are P_x, P_z and P_n. The forces acting on the element are pressure forces on the surfaces and the gravity forces. The force acting on the surface is the product of the mean pressure and the surface area.

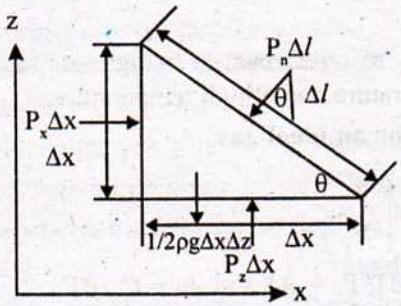


Figure. Static equilibrium of a fluid element.

From Nowton's second law, a corce balance in the x and z directions gives

$$\sum F_X = 0$$

$$P_x\Delta z - P_n l\sin\theta = 0$$

$$\sum F_X = 0$$

$$P_z\Delta z - P_n l\cos\theta - \frac{1}{2}\rho g\Delta x\Delta z = 0$$

where ρ is the density and $\rho g\frac{\Delta x\Delta z}{2}$ is the weight of the fluid element. From the Figure, (right angle triangle) $\Delta x = l\cos\theta$ and $\Delta z = l\sin\theta$.

Substituting these in equaitons.

$$P_x - P_n = 0$$
$$P_x = P_n$$

$$P_z - P_n - \frac{1}{2}\rho g\Delta z = 0$$

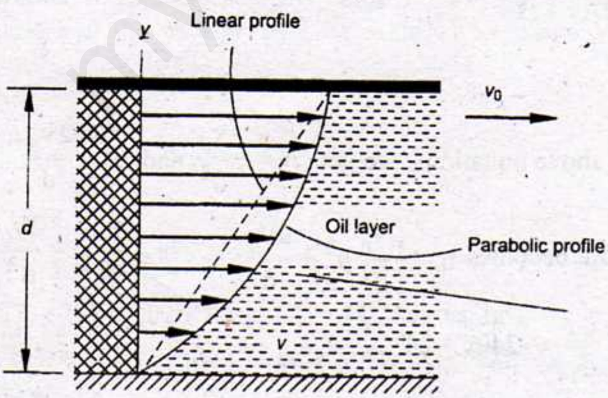
The last term of the equaiton drops out as $\Delta z \to 0$ and the wedge becomes infinitesimal and thus the fluid element shrinks to a point.

Then from equations, $P_x = P_z = P_n = P$

We can repeat the analysis for an element in the y – z plane and obtain a similar result. Thus, the pressure at a point in a fluid has the same magnitude in all directions.

6. (a) State Newton laws of viscosity.

(b) A large plate moves with a speed v_0 over a stationary plate on a layer of oil. If the velocity profile is that of a parabola (as shown in figure) with oil at the plate having the velocity as the plates, what is the shear stress on the moving plate from the oil? 4



Soln. : (a) Newton's law of viscosity states that for a well-ordered flow whereby fluid particles move in straight, parallel lines (parallel flow). Newton's law of viscosity states that for certain fluids, called Newtonian fluids, the shear stress (τ) on an interface tangent to the direction of flow is proportional to the distance rate of change of velocity $\left(\frac{du}{dy}\right)$, where the differentiation is taken in a direction normal dy to the interface.

Mathematically, Newton's law of viscosity can be expressed as

$$\tau \propto \frac{du}{dy}$$

$$\tau = \mu \frac{du}{dy}$$

where, the constant of proportionality μ is known as the viscosity coefficient or simply the viscosity which is the property of the fluid and depends on its state. Common fluids, such as water, air, mercury obey Newton's law of viscosity and are known as Newtonian fluids.

Soln. : (b) Let the equation of the velocity profile (parabolic) be

$$u = Ay^2 + By + C$$

where A, B and C are constants and their values are determined from boundary conditions.

Boundary conditions are

(i) $u = 0$ at $y = 0$

(ii) $u = v_0$ at $y = d$

(iii) $\frac{du}{dy} = 0$ at $y = d$

From (i), $0 = A(0) + B(0) + C$

$v_0 = Ad^2 + Bd$

From (iii), $\frac{du}{dy} = 2Ay + B$

$0 = 2Ad + B$

After solving the above equations, we get $A = -\frac{v_0}{d^2}$ and $B = \frac{2v_0}{d}$

The velocity profile becomes $u = -\frac{v_0}{d^2}y^2 + \frac{2v_0}{d}y$ or, $\frac{du}{dy} = -\frac{2v_0}{d^2}y + \frac{2v_0}{d}$

Velocity gradient $\frac{du}{dy} = -240y + 24$

Shear stress on the moving plate from the oil is given by

$$\tau = \mu\left(\frac{du}{dy}\right)_{y=d} = \mu\left(-\frac{2v_0}{d^2}d \cdot \frac{2v_0}{d}\right) = 0$$

7. (a) **Prove that where, $PV = mRT$ where P = pressure, V = volume of m mass of gas, T = temperature, R = characteristic gas constant.** 4

(b) **Write down the significance of the universal gas constant (R_m) and prove that $R = \frac{R_m}{M}$, where M = molecular weight of the substance.** 2+4

(c) **When 0.1421 kg of a gas is heated from 27°C to 27°C, it is observed that the gas requires 202 kJ of heat at constant pressure and 142 kJ of heat at constant volume. Find the adiabatic characteristic of gas constant and molecular weight of the gas.** 5

Ans. (a) For an ideal gas $P\bar{v} = R_m T$ ....(1)

When R_m is the universal gas constant, $\bar{v}$ is the volume per unit mole.

Since V is the total volume and n is the number of moles of the gas.

$$\therefore\ n\bar{v} = V \Rightarrow \bar{v} = \frac{V}{n}\ \text{.............(2)}$$

From (1), we can write–

$$\frac{PV}{n} = R_m T \Rightarrow PV = nR_m T\ \text{..................(3)}$$

If M is the molecular mass and m is the total mass of the gas then–

$$n = \frac{m}{M}\text{.............(5)}$$

From (3) $PV = \frac{m}{M} R_m T = m\frac{R_m}{M}T = mRT$ [$\because \frac{R_m}{M} = R$, R is characteristic gas constant].

Ans. (b) The universal gas constant is the same for all gases. 1 kg mole of all gases occupy the smae volume and the volume of 1 kg mole of all gases at NTP (273·115k and 1·0132 per) is 22·4146 m^3.

Ans. (c) Given

$m = 0{\cdot}1421$ kg

$T_1 = 27°C$

$T_2 = 127°C$

$mCp(T_2 - T_1) = 202$ kJ

or, $(0{\cdot}1421)\, C_P\, (127 - 27) = 202$ kJ or, $C_P = 14{\cdot}2153$ kJ/kg°C

Again, $mC_v\, (T_2 - T_1) = 142$ kJ (given) or, $(0{\cdot}1421)\, C_v\, (127 - 27) = 142$ kJ

or, $C_V = 9{\cdot}993$ kJ/kg°C

Characteristic gas constant, $R = C_P - C_V = 14{\cdot}2153 - 9{\cdot}993 = 4{\cdot}2223$ kJ/kg°C

Molecular weight, $M = \frac{\overline{R}}{R} = \frac{8 \cdot 3143}{4 \cdot 2223} = 1 \cdot 969$ kg / kgmol.

8. (a) What is steady flow process? Write the steady-flow energy equation for a single stream entering and a single stream leaving a control volume and explain the various terms in it. Calculate work done from SFEE for turbine.

(b) A turbine operates under steady flow condition and receiving at the following conditions :
Pressure = 1.2 MPa, Temperature = 188°C, Enthalpy = 2785 kJ/kg, velocity = 33.3 m/s and elevation = 3m.
Steam leaves the turbine at the following state :
P = 20 kPa, V = 20 kPa, V = 100 m/s, enthalpy = 2512 kJ/kg and heat lost to the surrounding at the rate 0.29 kJ/sec and the rate of steam flow through the turbine is 0.42 kg/sec. What is the power output of the turbine in kW? 8

Ans. (a) A steady flow is defined for a control volume as that type of flow in which the thermodynamic properties at a given position within or at the boundaries of the control volume are invariant with time. The properties include temperature, pressure, density, internal energy as well as velocity and acceleration of the flow stream. However, in a steady flow process, the state of the fluid can change as it passes through the control volume.

The steady flow energy equation for a single stream entering and a single stream leaving a control volume can be written as

$$m\left(u_1 + P_1 v_1 + \frac{V_1^2}{2} + gz_1\right) + Q = m\left(u_2 + P_2 v_2 + \frac{V_2^2}{2} + gz_2\right) + W$$

Where

$\dot{m}_1$ = mass flow rate at inlet

$\dot{m}_2$ = mass flow rate at outlet

u_1 = specific internal energy of fluid stream at inlet

u_2 = specific internal energy of fluid stream at outlet

$P_1 v_1$ = specific flow work done on the control volume by the entering fluid

$P_2 v_2$ = specific flow work done by the control volume on the leaving fluid

V_1 = velocity of fluid stream at inlet

V_2 = velocity of fluid stream at outlet

v_1 = specific volume of fluid at inlet

v_2 = specific volume of fluid at outlet

$\dot{Q}$ = rate of heat added to the control volume

$\dot{W}$ = rate of work done by the control volume during the same time.

Ans. (b) Mass flow rate = 0·42 kg/s

h_1 = 2785 kJ/kg

V_1 = 33·3 m/s

$Z_1 = 3\text{m}$
$h_2 = 2512 \text{ kJ/kg}$
$V_2 = 100 \text{ m/s}$
$Z_2 = 0 \text{ m}$

Rate of heat transfer $= -0{\cdot}42 \times 0{\cdot}29 \times 10^3$ J/s

$$\dot{m}\left(h_1 + \frac{V_1^{\,2}}{2} + 8Z_1\right) + \dot{Q} = \dot{m}\left(h_2 + \frac{V_2^{\,2}}{2} + gZ_2\right) + W_s$$

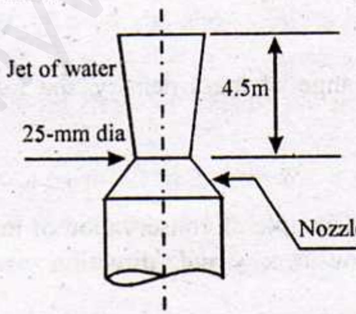
$$0{\cdot}42\left(2785 \times 10^3 + \frac{33{\cdot}3^2}{2} + 9{\cdot}81\right) - 0{\cdot}42 \times 0{\cdot}29 \times 10^3$$

$$= 0{\cdot}42\left(2412 \times 10^3 + \frac{100^2}{2} + 0\right) + \dot{W}_s$$

$$\dot{W}_s = 112683{\cdot}4 \text{ J/s} = 112{\cdot}68 \text{ kW}.$$

9. (a) **Derive an expression for continuity equation for a three-dimensional steady incompressible flow.**

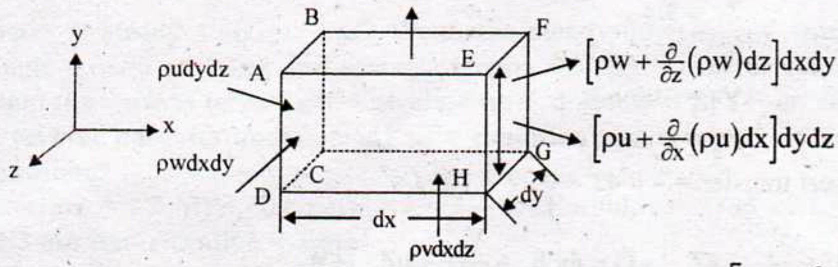
(b) **A jet of water from a 25-mm diameter nozzle is directed vertically upwards, assuming that jet remains steady and neglecting any los of energy. What will be the diameter at a point 4·5 m above the nozzle, if the velocity with which jet leaves the nozzle is 12 m/s?**



(c) **The velocity vector for a 2D incompressible flow field is given by $\vec{V} = \dfrac{x}{x^2 + y^2}\hat{i} + \dfrac{y}{x^2 + y^2}\hat{j}$. State whether the flow is continuous or discontinuous.** 5

Ans. (a) A rectangular parallelepiped with sides dx, dy and dz in the x, y and z directions, respectively, is considered as the control volume in three-dimensional Cartesian coordinates.

Let hte fluid enter through the surface ABCD (normal to the x-axis) with a velocity u and a density ρ. Rate of Rate of mass inflow through the surface ABCD (normal to x axis) = ρudydz



Rate of mass outflow through the surface EFGH (normal to x axis) = $\left[\rho u+\dfrac{\partial}{\partial x}(\rho u)dx\right]dydz$

Net rate of mass outflow in x direction $=\dfrac{\partial}{\partial x}(\rho u)$ dxdydz

Similarly,

Net rate of mass outflow in y direction $=\dfrac{\partial}{\partial y}(\rho v)$ dxdydz

Net rate of mass outflow in z direction $=\dfrac{\partial}{\partial z}(\rho w)$ dxdydz

Therefore, total net rate of mass outflow in x, y and z direction

$$=\left[\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)\right]dxdydz$$

The effect of mass loss in Eq (0.1) is to cause the time rate of decrease of mass encompasses by he volume.

Since $\dfrac{\partial \rho}{\partial t}$ is the rate of change of mass density, the rate of change of mass in control volume $=-\dfrac{\partial \rho}{\partial t}$ dxdydz.

Therefore, according to the principle of conservation of mass,

Total net rate of mass outflow in x, y andz direction = rate of change of mass in control volume

$$\left[\frac{\partial}{\partial x}(\rho u)\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial w}(\rho w)\right]dxdydx=-\frac{\partial \rho}{\partial t}dxdydz$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}(\rho u)\frac{\partial}{\partial z}(\rho u)+\frac{\partial}{\partial z}(\rho w)\right]dxdydx=0$$

Since the volume of control volume can not be zero, the above equation, becomes

$$\frac{\partial \rho}{\partial t}+\frac{\partial}{\partial x}(\rho u)+\frac{\partial}{\partial y}(\rho v)+\frac{\partial}{\partial z}(\rho w)=0$$

Equation (.2) is known as *continuity equation.*

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) + \frac{\partial}{\partial z}(\rho w) = 0$$

$$\frac{\partial \rho}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z} + \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\right] = 0$$

$$\frac{D\rho}{Dt} + \rho\left[\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x}\right] = 0$$

$$\frac{1}{\rho}\frac{D\rho}{Dt} + \frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} = 0$$

For incompressible flow, the rate of volumetric dilation per unit volume ($\frac{1}{\rho}\frac{D\rho}{Dt}$) of a fluid element in motion is zero. Then the above equation becomes

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial x} + \frac{\partial w}{\partial x} = 0$$

Equation (13.6) can be written in a vector form as $\Delta.\vec{V} = 0$.

Ans. (b) Let the nozzle exit and the point 4·5 m above the nozzle be designated by point 1, and 2 respectively, as shown in Fig.

Diameter at nozzle $D_1 = 25$ mm = 0·025 m

Velocity of jet at the exit of the nozzle $V_1 = 12$ m/s

Let the diameter of the jet at a point 4·5 m above nozzle be D_2 and velocity of jet at point 2 be V_2 m/s.

Considering the motion of the jet from the exit of the nozzle to point 2, we have $V_2^2 = V_1^2 - 2gh$

where h is the distance between point 1 and 2 (here $h = 4·5$ m)

Putting the values of V_1 and h, we get $V_2^2 = 12^2 - 2 \times 9·81 \times 4·5$ or, $V_2 = 7·46$ m/s

Applying the continuity equations between the exit of the nozzle and the point 2, we have $A_1V_1 = A_2V_2$

or, $\frac{\pi}{4}D_1^2V_1 = \frac{\pi}{4}D_2^2V_2$ or, $D_2^2 = D_1^2\frac{V_1}{V_2} = 0·025^2\frac{12}{7·46}$ or, $D_2 = 0·0317$ m

Ans. (c) The contimuity equation in differential form for a two-diamensional, incompressible flow is–

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

$$u = \frac{x}{x^2 + y^2} \quad v = \frac{y}{x^2 + y^2}$$

$$\frac{\partial u}{\partial x}=\frac{1}{x^2+y^2}-\frac{2x^2}{\left(x^2+y^2\right)^2}$$

$$\frac{\partial v}{\partial y}=\frac{1}{x^2+y^2}-\frac{2y^2}{\left(x^2+y^2\right)^2}$$

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0$$

Velocity field satisfies the continuity equation. Therefore, the flow is continuous.

10. (a) Draw the nature of p – v and T – s plots of a Rankine cycle (with saturated steam at turbine inlet). 4

(b) A lump of steel of 15-kg mass at 557°C is dropped in 120 kg of oil at 25°C. The specific heats of steel and oil are 0.5 kJ/kg-K respectively. Calculate the entropy change of the steel the oil and the universe.

(c) The bodies, each of equal mass m and heat capacity C are of temperature T_1 and T_2 ($T_1 > T_2$) respectively. The first body is used as source of heat for reversible engine and the second body as the sink. Show that the maximum work obtainable from such arrangement is $mC\left(\sqrt{T_1}-\sqrt{T_2}\right)^2$.

Ans. (a) The Rankine cycle is an ideal cycel for vapour cycles. The cycle is shown in Fig. on P – v and T – s diagrams. the Rankine cycle comprises of the following processes :

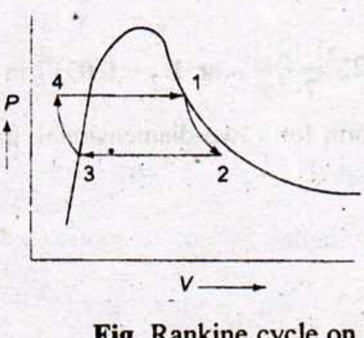
4 – 1 : Reversible (constant pressure) heat addition in a boiler

1 – 2 : Reversible adiabatic expansion in a turbine

2 – 3 : Reversible (constant pressure) heat rejection in a condenser

3 – 4 : Reversible adiabatic compression in a pump

Dry saturated steam enters the turbine and expands reversibly and adiabatically to condenser pressure. The steam is then condensed at constant pressure and temperature to a saturated liquid. The saturated liquid leaving the condenser is then pumped reversibly and adiabatically into the boiler pressure. The compressed liquid is first heated to the saturation temperature at boiler pressure and then evaporated to the state 1 to complete the cycle.



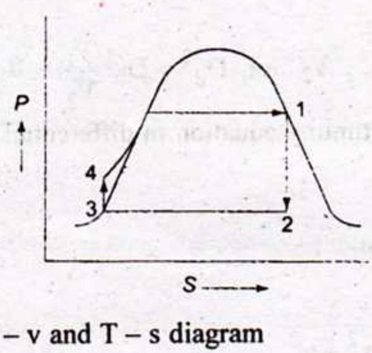


Fig. Rankine cycle on P – v and T – s diagram

Ans. (b) Let the final temperature be T_f

$$15 \times 0.5(557 - T_f) = 120 \times 3.5(T_f - 25)$$

$$T_f = 34.33°C = 307.33K$$

Tntropy change of steel $= \int_{T_1}^{T_f} \frac{\delta Q}{T} = mC_p \, ln \frac{T_f}{T_i} = 15 \times 0 \cdot 5 \, ln \frac{307 \cdot 33}{830} = -7 \cdot 45 \text{ kJ/K}$

Entropy change of oil $= \int_{T_2}^{T_f} \frac{\delta Q}{T} = mC_p \, ln \frac{T_f}{T_i} = 120 \times 3 \cdot 5 \, ln \frac{307 \cdot 33}{298} = 12 \cdot 95 \text{ kJ/K}$

Entropy change of universe = Entropy change of steel + Entropy change of oil = 7·45 + 12·95 = 5.5 kJ/K

Ans. (c) As heat is transferred from the first body and heat is rejected to the second body, the temperature of the first bod will be decreasing and that of the second body will be increasing. When both the bodies attain the same temperature, the heat engine will stop operating.

Let T_f be the final temperature.

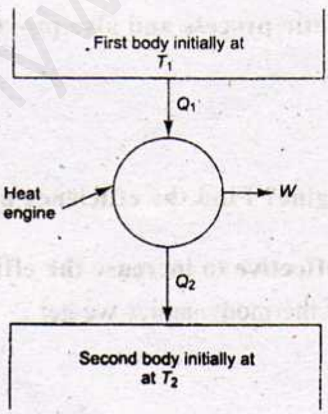
Total heat transfer from the first body, $Q_1 = mC_p(T_1 - T_f)$

Total heat rejected to second body, $Q_2 = mC_p(T_f - T_2)$

Work done by the heat engine, $W = Q_1 - Q_2$

$$W = mC_p(T_1 + T_2 - 2T_f)$$

For minimum value of T_f maximum value of W will be maximum.



Change of entropy of the first body

$$\Delta S_1 = \int_{T_1}^{T_f} \frac{mC_p dT}{T} = mC_p \, ln \frac{T_f}{T_1}$$

Change of entropy of the second body $\Delta S_2 = \int_{T_2}^{T_f} \frac{mC_p dT}{T} = mC_p \, ln \frac{T_f}{T_1}$

We know that the heat engine is a cyclic device, and cyclic integral of any property is zero. Since entropy is a property of a system.

Entropy change for the heat engine, $\Delta S_{HE} = \Delta S_1 + \Delta S_2 + \Delta S_{HE}$

Total entropy change of the universe

$$(\Delta S)_{uni} = \Delta S_1 + \Delta S_2 + \Delta S_{HE} = mC_p \ln\frac{T_f}{T_1} + mC_p \ln\frac{T_f}{T_2} = mC_p \ln\frac{T_f^2}{T_1T_2}$$

From the entropy principle $(\Delta S)_{uni} \geq 0$

$$mC_p \ln\frac{T_f^2}{T_1T_2} \geq 0$$

For minimum value of T_f

$$mC_p \ln\frac{T_f^2}{T_1T_2} = 0$$

$$\ln\frac{T_f^2}{T_1T_2} = 0 = \ln 1$$

$$T_f = \sqrt{T_1T_2}$$

Maximum work obtainable $W_{max} = mC_p\left(T_1 + T_2 - 2\sqrt{T_1T_2}\right) = mC_p\left(\sqrt{T_1} - \sqrt{T_2}\right)^2$

11. (a) Prove that PV^γ in adiabatic process and also prove

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$ 5 + 3

(b) What is a cyclic heat engine? Find the efficiency of heat engine in terms of source and sink temperature.
Which change is more effective to increase the efficiency of the engine. 1+3+3

Ans. (a) From the first law of thermodynamics we get

$$\delta Q - \delta W = dU$$

$$\delta W = -dU$$

$$W_{1-2} = U_1 - U_2$$

For adiabatic process, $\delta Q = 0$

Now, for a non-flow process, $\delta Q = dU + PdV$

For an adiabatic non-flow reversible process,

$$0 = dU + PdV$$

$$0 = m.\, c_v dT + PdV$$

$$dT = -\frac{PdV}{m.c_v}$$

For an ideal gas, PV = m.RT

$$PdV + VdP = mRdT$$

$$PdV + PdV = mR\left(-\frac{PdV}{mC_v}\right)$$

$$PdV + VdP + (C_P - C_v)\frac{PdV}{C_v} = 0$$

$$C_v\,(PdV + VdP) + (C_P - C_v)\,PdV = 0$$

Dividing by C_vPV, we get $\frac{dV}{V} + \frac{dP}{P} + \gamma\frac{dV}{V} - \frac{dV}{V} = 0$

$$\frac{dP}{P} + \gamma\frac{dV}{V} = 0$$

Integrating, we get

$$\text{In } P + \gamma \text{ In } V = \text{In constant}$$

$$PV^{\gamma} = \text{constant}$$

$$P_1V_1^{\gamma} = P_2V_2^{\gamma}$$

$$\frac{T_1}{T_2} = \frac{P_1}{P_2}.\frac{V_1}{V_2} = \left(\frac{V_2}{V_1}\right)^{\gamma}.\frac{V_1}{V_2}$$

$$\frac{T_1}{T_2} = \left(\frac{V_2}{V_1}\right)^{\gamma-1}$$

$$\frac{T_1}{T_2} = \frac{P_1}{P_2}.\frac{V_1}{V_2} = \frac{P_1}{P_2}.\left(\frac{P_2}{P_1}\right)^{\frac{1}{\gamma}}$$

$$\frac{T_1}{T_2} = \left(\frac{P_1}{P_2}\right)^{\frac{\gamma-1}{\gamma}}$$

Ans. (b) The heat engine that operates on the Carnot cycle is called the *Carnot heat engine.* The thermal efficiency of any heat engine is given by

$$\eta_{ther} = \frac{Q_1 - Q_2}{Q_1} = 1 - \frac{Q_2}{Q_1}$$