Code: AE01/AC01/AT01 **Subject: MATHEMATICS-I**
Time: 3 Hours **DECEMBER 2010** **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: **(2 × 10)**

a. The value of $\lim\limits_{\substack{x \to 0 \\ y \to 0}} \frac{xy}{x^2 + y^2}$ is

(A) 0 **(B)** 1
(C) –1 **(D)** does not exists

b. If $z = \phi(x+ct) + \psi(x-ct)$, then $\frac{\partial^2 z}{\partial t^2} - c^2 \frac{\partial^2 z}{\partial x^2}$ is equal to

(A) 0 **(B)** 1
(C) c **(D)** c^2

c. The value of $\int_0^1 \int_0^{\sqrt{1-x^2}} dy\,dx$ is equal to

(A) $\frac{\pi}{2}$ **(B)** $\frac{\pi}{3}$
(C) $\frac{\pi}{4}$ **(D)** $\frac{\pi}{8}$

d. The expansion of $e^x \sin y$ in powers of x and y up to first degree terms is

(A) x **(B)** y
(C) x+y **(D)** x–y

e. The differential equation $(x+x^2+ay^2)dx+(y^3-y+bxy)dy=0$ is exact if

(A) a=b **(B)** a=2b
(C) b=2a **(D)** a+b=0

f. The solution of differential equation $\frac{d^2y}{dx^2} + 9y = \cos 3x$ is

(A) $y = (c_1 \cos 3x + c_2 \sin 3x) + \frac{x}{6} \sin 3x$ **(B)** $y = c_1 \cos 3x + c_2 \sin 3x + \frac{x}{6} \cos 3x$

(C) $y = (c_1 + c_2 x)e^{3x} + \sin 3x$ **(D)** $y = (c_1 + c_2 x)e^{-3x} + \cos 3x$

g. The rank of $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 1 & 4 & 2 & 5 \\ 2 & 6 & 5 & 7 \end{bmatrix}$ is

(A) 1 **(B)** 2
(C) 3 **(D)** 4

h. The sum and product of eigen values of $\begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$ are respectively

(A) (3,5) **(B)** (5,3)
(C) (2,4) **(D)** (4,2)

i. The value of $\frac{d}{dx}\left(x^2 J_2(x)\right)$ is

(A) $xJ_0(n)$ **(B)** $x^2J_0(n)$
(C) $xJ_1(n)$ **(D)** $x^2J_1(n)$

j. The value of $\int_{-1}^{+1} P_2(n)dn$ is

(A) 0 **(B)** 1
(C) 2 **(D)** 3

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. State and prove Euler's theorem. **(8)**

b. If $u=\log(x^3+y^3+z^3-3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$ **(8)**

Q.3 a. Find the maximum value of $x^m y^n z^p$, given that $x+y+z=a$ **(8)**

b. Expand $e^x \log_e(1+y)$ in powers of x and y up to second degree terms. **(8)**

Q.4 a. Evaluate $\iint xy(x+y)dxdy$, over the area between $y=x^2$ and $y=x$ **(8)**

b. Solve the differential equation $(1+x^2)\frac{dy}{dx}+2xy=x^2$ **(8)**

Q.5 a. Solve the differential equation $\frac{d^2y}{dx^2}-4\frac{dy}{dx}+4y=e^{2x}+\sin 2x$ **(8)**

b. Use method of undetermined coefficients to solve $\frac{d^2y}{dx^2}-3\frac{dy}{dx}+2y=x^2+e^x$ **(8)**

Q.6 a. Solve the differential equation $x^2\frac{d^2y}{dx^2}+4x\frac{dy}{dx}+2y=x+\sin x$ **(8)**

b. Use elementary row transformations to find inverse of $\begin{bmatrix} 2 & 1 & -1 \\ 0 & 2 & 1 \\ 5 & 2 & -3 \end{bmatrix}$ **(8)**

Q.7 a. Find the values of 'a' and 'b' for which the equations x+ay+z=3, x+2y+2z=b, x+5y+3z=9 are consistent. When will these equations have a unique solution? **(8)**

b. Define Hermitian and Skew-Harmitian matrices. Show that everysquare matrix can be written as the sum of a Hermitian and Skew-Harmitian matrices. **(8)**

Q.8 a. Find a matrix P which transforms the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ to the diagonal form. **(8)**

b. Solve in series the differential equation $\frac{d^2y}{dx^2}+x\frac{dy}{dx}+y=0$ **(8)**

Q.9 a. Show that $J_4(x)=\left(\frac{48}{x^3}-\frac{8}{x}\right)J_1(x)+\left(1-\frac{24}{x^2}\right)J_0(x)$ **(8)**

b. State and prove Radrigues formula. **(8)**