Code: AE01/AC01/AT01 **JUNE 2010** **Subject: MATHEMATICS-I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 9 Questions in all.

- **Question 1 is compulsory and carries 20 marks. Answer to Q.1 must be written in the space provided for it in the answer book supplied and nowhere else.**
- **Out of the remaining EIGHT Questions answer any FIVE Questions. Each question carries 16 marks.**
- **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or the best alternative in the following: (2 × 10)

a. The value of limit $\lim\limits_{(x,y)\to(0,0)} \dfrac{x+\sqrt{y}}{\sqrt{(x^2+y)}}$ is

(A) limit does not exist **(B)** 0
(C) 1 **(D)** -1

b. If $u = \sin^{-1}\left(\dfrac{x}{y}\right) + \tan^{-1}\left(\dfrac{y}{x}\right)$, then the value of $x\dfrac{\partial u}{\partial x} + y\dfrac{\partial u}{\partial y}$ is

(A) u **(B)** 2u
(C) 3u **(D)** 0

c. The solution of the differential equation $(y+x)^2\dfrac{dy}{dx} = a^2$ is given by

(A) $y + x = a\tan\left(\dfrac{y-c}{a}\right)$ **(B)** $y - x = \tan\left(\dfrac{y-c}{a}\right)$

(C) $y - x = a\tan(y-c)$ **(D)** $a(y-x) = \tan\left(y - \dfrac{c}{a}\right)$

d. The solution of the differential equation $\dfrac{d^2y}{dx^2} - 3\dfrac{dy}{dx} + 2y = e^{3x}$ is

(A) $y = ae^{x} + be^{2x} + \dfrac{1}{2}e^{3x}$ **(B)** $y = ae^{-x} + be^{-2x} + \dfrac{1}{2}e^{3x}$

(C) $y = ae^{x} + be^{-2x} + \dfrac{1}{2}e^{3x}$ **(D)** $y = ae^{-x} + be^{2x} + \dfrac{1}{2}e^{3x}$

e. If 3x+2y+z= 0, x+4y+z=0, 2x+y+4z=0, be a system of equations then

(A) System is inconsistent.
(B) It has only trivial solution.
(C) It can be reduced to a single equation thus solution does not exist.
(D) Determinant of the coefficient matrix is zero.

f. If λ is an eigenvalue of a non-singular matrix A then the eigenvalue of A^{-1} is

(A) $1/\lambda$ **(B)** λ
(C) $-\lambda$ **(D)** $-1/\lambda$

g. The value of $P_n(-1)$ is
(A) -1 **(B)** 1
(C) $(-1)^n$ **(D)** 0

h. The value of integral $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$ is equal to

(A) $\frac{3}{4}$ **(B)** $\frac{3}{8}$
(C) $\frac{3}{5}$ **(D)** $\frac{3}{7}$

i. If $u = f\left(\frac{y}{x}\right)$ then

(A) $x\frac{\partial u}{\partial x} - y\frac{\partial u}{\partial y} = 0$ **(B)** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$
(C) $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 2u$ **(D)** $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1$

j. The value of the integral $\int x^2 J_1(x)dx$ is

(A) $x^2 J_1(x) + c$ **(B)** $x^2 J_{-1}(x) + c$
(C) $x^2 J_2(x) + c$ **(D)** $x^2 J_{-2}(x) + c$

Answer any FIVE Questions out of EIGHT Questions.
Each Question carries 16 marks.

Q.2 a. Find the extreme value of the function f(x,y,z) = 2x + 3y + z such that $x^2+y^2 = 5$ and x + z =1. **(8)**

b. Show that the function $$f(x,y)=\begin{cases}(x+y)\sin\left(\dfrac{1}{x+y}\right), & x+y\neq 0\\ 0, & x+y=0\end{cases}$$ is continuous at (0,0) but its partial derivatives of first order does not exist at (0,0). **(8)**

Q.3 a. Evaluate $\int_0^\infty\int_0^\infty e^{-(x^2+y^2)}dxdy$ by changing to polar coordinates. Hence show that $\int_0^\infty e^{-x^2}dx=\sqrt{\pi}/2$. **(8)**

b. Show that the approximate change in the angle A of a triangle ABC due to small changes $\delta a,\ \delta b,\ \delta c$ in the sides a, b, c respectively, is given by $\delta A=\dfrac{a}{2\Delta}(\delta a-\delta b\cos C-\delta c\cos B)$ where Δ is the area of the triangle. Verify that $\delta A+\delta B+\delta C=0$ **(8)**

Q.4 a. If $x+y=2e^\theta\cos\phi$ and $x-y=2ie^\theta\sin\phi$. Show that $$\frac{\partial^2 u}{\partial\theta^2}+\frac{\partial^2 u}{\partial\phi^2}=4xy\frac{\partial^2 u}{\partial x\partial y}$$ **(8)**

b. Using the method of variation of parameter method, find the general solution of the differential equation $y''+16y=32\sec 2x$. **(8)**

Q.5 a. Find the general solution of the equation $y''-4y'+13y=18e^{2x}\sin 3x$. **(8)**

b. Find the general solution of the equation $x^3\dfrac{d^3y}{dx^3}+5x^2\dfrac{d^2y}{dx^2}+5x\dfrac{dy}{dx}+y=x^2+\ln x$. **(8)**

Q.6 a. Solve $(1+y^2)dx=(\tan^{-1}y-x)dy$ **(8)**

b. If $A=\begin{bmatrix}2+i & 3 & -1+3i\\ -5 & i & 4-2i\end{bmatrix}$, show that AA* is a Hermitian matrix, where A* is the conjugate transpose of A. **(8)**

Q.7 a. Show that the matrix A is diagonalizable. $A = \begin{pmatrix} 3 & 1 & -1 \\ -2 & 1 & 2 \\ 0 & 1 & 2 \end{pmatrix}$. If so, obtain the matrix P such that $P^{-1}AP$ is a diagonal matrix. **(8)**

b. Investigate the values of λ for which the equations

$$(\lambda - 1)x + (3\lambda + 1)y + 2\lambda z = 0,$$
$$(\lambda - 1)x + (4\lambda - 2)y + (\lambda + 3)z = 0,$$
$$2x + (3\lambda + 1)y + 3(\lambda - 1)z = 0$$

are consistent, and hence find the ratios of x:y:z when λ has the smallest of these values. **(8)**

Q.8 a. Use elementary row operations to find inverse of $A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ **(5)**

b. Find the first five non-vanishing terms in the power series solution of the initial value problem $(1-x^2)y'' + 2xy' + y = 0,\ y(0) = 1,\ y'(0) = 1.$ **(11)**

Q.9 a. Show that $J_{5/2}(x) = \sqrt{\dfrac{2}{n\pi}}\left[\dfrac{1}{x^2}(3-x^2)\sin x - \dfrac{3}{x}\cos x\right]$ **(8)**

b. Show that $\displaystyle\int_{-1}^{1} P_m(x)P_n(x)dx = \begin{cases} 0, & m \neq n \\ \dfrac{2}{2n+1}, & m = n \end{cases}$ **(8)**