Code: A-01/C-01/T-01 **Subject: MATHEMATICS-I**
Time: 3 Hours **Max. Marks: 100**

NOTE: There are 11 Questions in all.

- • **Question 1 is compulsory and carries 16 marks. Answer to Q. 1. must be written in the space provided for it in the answer book supplied.**
- • **Answer any THREE Questions each from Part I and Part II. Each of these questions carries 14 marks.**
- • **Any required data not explicitly given, may be suitably assumed and stated.**

Q.1 Choose the correct or best alternative in the following: **(2x8)**

a. a. The value of limit $\lim\limits_{(x,y)\to(0,0)} \frac{x^2 y}{x^4 + y^2}$

(A) equals 0. **(B)** equals $\frac{1}{2}$.
(C) equals 1. **(D)** does not exist.

b. b. If $u = x^2 - y^2, v = xy$ then $\frac{\partial x}{\partial u}$ equals

(A) **(A)** $\frac{x}{2(x^2 + y^2)}$. **(B)** $\frac{y}{2(x^2 + y^2)}$.
(C) $\frac{y}{x^2 + y^2}$. **(D)** $\frac{x}{x^2 + y^2}$.

c. The function $f(x, y) = y^2 - x^3$ has

(A) **(A)** a minimum at (0, 0).
(B) **(B)** neither minimum nor maximum at (0, 0).
(C) **(C)** a minimum at (1, 1).
(D) **(D)** a maximum at (1, 1).

d. The family of orthogonal trajectories to the family $y(x - k)^2$, where k is an arbitrary constant, is

(A) $y^{3/2} = \frac{3}{4}(c - x)$. **(B)** $x^{3/2} = (y - c)^2$.

(C) **(C)** $(y-c)^2 = \frac{3}{4}x$. **(D)** $y^2 = \frac{3}{2}(c-x)$.

e. Let y_1, y_2 be two linearly independent solutions of the differential equation $yy'' - (y')^2 = 0$. Then $c_1y_1 + c_2y_2$, where c_1, c_2 are constants is a solution of this differential equation for

(A) **(A)** $c_1 = c_2 = 0$ only. **(B)** $c_1 = 0$ or $c_2 = 0$.
(C) no value of c_1, c_2. **(D)** all real c_1, c_2.

f. If A, B are two square matrices of order n such that AB=**0**, then rank of

(A) **(A)** at least one of A, B is less than n.
(B) **(B)** both A and B is less than n.
(C) **(C)** none of A, B is less than n.
(D) **(D)** at least one of A, B is zero.

g. A 3×3 real matrix has an eigenvalue i, then its other two eigenvalues can be

(A) 0, 1. **(B)** -1, i. **(C)** 2i, -2i. **(D)** 0, -i.

h. The integral $\int_0^{\pi} P_n(\cos\theta)\sin 2\theta \, d\theta$, n>1, where $P_n(x)$ is the Legendre's polynomial of degree n, equals

(A) 1. **(B)** $\frac{1}{2}$. **(C)** 0. **(D)** 2.

PART I

Answer any THREE questions. Each question carries 14 marks.

Q.2 a. Compute $f_{xy}(0,0)$ and $f_{yx}(0,0)$ for the function

$$f(x,y) = \begin{cases} \dfrac{xy^3}{x+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}$$

(6)

b. Let v be a function of (x, y) and x, y are functions of (θ, ϕ) defined by

$$x + y = 2e^{\theta}\cos\phi$$
$$x - y = 2ie^{\theta}\sin\phi$$

where $i = \sqrt{-1}$. Show that $x\dfrac{\partial v}{\partial x} + y\dfrac{\partial v}{\partial y} = \dfrac{\partial v}{\partial \theta}$. **(8)**

Q.3 a. Expand x^y near (1, 1) upto 3rd degree terms by Taylor's series. **(7)**

b. Find the extreme value of $x^2 + y^2 + z^2 + xy + xz + yz$ subject to the conditions $x + y + z = 1$ and $x + 2y + 3z = 3$. **(7)**

Q.4 a. Find the rank of the matrix

$$\begin{bmatrix} 9 & 3 & 1 & 0 \\ 3 & 0 & 1 & -6 \\ 1 & 1 & 1 & 1 \\ 0 & -6 & 1 & 9 \end{bmatrix}$$

(6)

b. Let
$$y_1 = 5x_1 + 3x_2 + 3x_3$$
$$y_2 = 3x_1 + 2x_2 - 2x_3$$
$$y_3 = 2x_1 - x_2 + 2x_3$$

be a linear transformation from (x_1, x_2, x_3) to (y_1, y_2, y_3)

and
$$z_1 = 4x_1 + 2x_3$$
$$z_2 = x_2 + 4x_3$$
$$z_3 = 5x_3,$$

be a linear transformation from (x_1, x_2, x_3) to (z_1, z_2, z_3).

Find the linear transformation from (z_1, z_2, z_3) to (y_1, y_2, y_3) by inverting appropriate matrix and matrix multiplication. **(8)**

Q.5 a. Prove that the eigenvalues of a real matrix are real or complex conjugates in pairs and further if the matrix is orthogonal, then eigenvalues have absolute value 1. **(6)**

b. Find eigenvalues and eigenvectors of the matrix $A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. **(8)**

Q.6 a. Find a matrix X such that $X^{-1}AX$ is a diagonal matrix, where $A=\begin{bmatrix}5 & 4\\ 1 & 2\end{bmatrix}$. Hence compute A^{50}. **(8)**

b. Prove that a general solution of the system

$$8x_1-4x_2+10x_5=1$$
$$x_2+x_4-x_5=2$$
$$x_3-3x_4+2x_5=0$$

can be written as

$$(x_1,x_2,x_3,x_4,x_5)=\left(\frac{9}{8},2,0,0,0\right)+\alpha\left(-\frac{1}{2},-1,3,1,0\right)+\beta\left(-\frac{3}{4},1,-2,0,1\right)$$ where

α,β are arbitrary. **(6)**

PART II

Answer any THREE questions. Each question carries 14 marks.

Q.7 a. Let $\int_0^1\int_1^2 \frac{1}{x^2+y^2}\,dx\,dy+\int_1^2\int_y^2 \frac{1}{x^2+y^2}\,dx\,dy=\iint_R \frac{1}{x^2+y^2}\,dy\,dx$ Recognise the region R of integration on the r.h.s. and then evaluate the integral on the right in the order indicated. **(7)**

b. Compute the volume of the solid bounded by the surfaces $z=\sqrt{4-x^2-y^2}$ and $z=\frac{1}{3}\left(x^2+y^2\right)$. **(7)**

Q.8 a. Let $\mu(x,y)$ be an integrating factor for differential equation Mdx+Ndy=0 and $\Psi(x,y)=0$ is a solution of this equation, then show that $\mu\,G(\Psi)$ is also an integrating factor of this equation, G being a non-zero differentiable function of Ψ. **(6)**

b. Solve the initial value problem $\frac{dy}{dx}=y^2\left(\ell n\,(x)+\frac{1}{x}\right)+y,\ \ y(0)=1$. **(8)**

Q.9 a. Find general solution of differential equation $y'''+y'=\sec x$. **(7)**

b. Solve the boundary value problem

$$x^3y'' - x^2y' + xy = 1,\ y(1) = \frac{1}{4},\ \ y(e) = e + \frac{1}{4e}.$$ **(7)**

Q.10 a. Solve the differential equation $y^{iv} + 32y'' + 256y = 0$. **(5)**

b. Using power series method find a fifth degree polynomial approximation to the solution of initial value problem

$$(x-1)y'' + xy' + y = 0,\ y(0) = 2,\ y'(0) = -1.$$ **(9)**

Q.11 a. Let $J_v(x)$ denote the Bessel's function of first kind. Find the generating function of the sequence $\{J_v(x), v = 0, \pm 1, \pm 2.........\}$. Hence prove that

$$\cos x = J_0(x) - 2J_2(x) + 2J_4(x) - ....$$

$$\sin x = 2J_1(x) - 2J_3(x) + 2J_5(x) - ....$$ **(7)**

b. Show that for Legendre polynomials $P_n(x)$

$$\int_{-1}^{1} xP_n(x)P_{n-1}(x)dx = \frac{2n}{4n^2 - 1},\ n = 1,2,........$$ **(7)**