SUPER 10
IIT-JEE 2010

SOLUTIONS
of
Mock Test 1
(Paper-1)

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MOCK TEST - 1
(Paper - I)

ANSWERS

CHEMISTRY		MATHEMATICS		PHYSICS	
1.	(B)	21.	(D)	41.	(B)
2.	(B)	22.	(C)	42.	(C)
3.	(C)	23.	(A)	43.	(C)
4.	(B)	24.	(D)	44.	(B)
5.	(D)	25.	(D)	45.	(B)
6.	(B)	26.	(A)	46.	(C)
7.	(C)	27.	(A)	47.	(C)
8.	(D)	28.	(A)	48.	(B)
9.	(A, C)	29.	(A, B, C, D)	49.	(A, B, C)
10.	(A, B, C, D)	30.	(B, C)	50.	(B, D)
11.	(C, D)	31.	(A, B)	51.	(B)
12.	(A, B, D)	32.	(A, C)	52.	(A, B, D)
13.	(B)	33.	(A)	53.	(D)
14.	(A)	34.	(B)	54.	(C)
15.	(B)	35.	(D)	55.	(A)
16.	(B)	36.	(B)	56.	(C)
17.	(C)	37.	(B)	57.	(A)
18.	(B)	38.	(A)	58.	(C)
19.	A – (p, q, t)	39.	A $\rightarrow$ (q, s)	59.	A $\rightarrow$ (p, q, r, t)
	B – (p, r)		B $\rightarrow$ (p)		B $\rightarrow$ (q, r, t)
	C – (p, q, t)		C $\rightarrow$ (r)		C $\rightarrow$ (s)
	D – (s)		D $\rightarrow$ (t)		D $\rightarrow$ (r)
20.	A – (p, r, t)	40.	A $\rightarrow$ (p, r, s, t)	50.	A $\rightarrow$ (q, s, t)
	B – (p, r)		B $\rightarrow$ (q, t)		B $\rightarrow$ (s)
	C – (p, q, r, t)		C $\rightarrow$ (p, q, r, s, t)		C $\rightarrow$ (q, s)
	D – (p, r, s)		D $\rightarrow$ (p, q, r, s)		D $\rightarrow$ (p)

ANSWERS & HINTS

PART - I : CHEMISTRY

1. Answer (B)

Milliequivalents of H_2O_2 = Milliequivalents of $K_2Cr_2O_7$

$$\frac{W}{17} \times 1000 = 10 \times \frac{1}{6} \times 6$$

$$w_{H_2O_2} = 0.17g\ H_2O_2$$

Suppose, weight of sample = x g

$$x \times \frac{68}{100} = 0.17$$

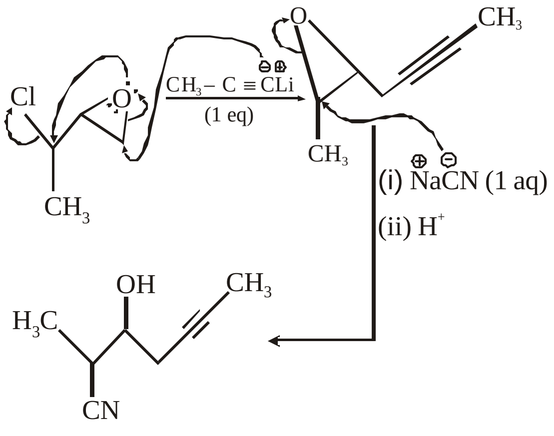
x = 0.25g

2. Answer (B)

$$-\frac{d[I_2]}{dt} = -\frac{1}{2}\frac{d[S_2O_3^{2-}]}{dt} = \frac{d[S_4O_6^{2-}]}{dt}$$

$$= \frac{1}{2}\frac{d[I^-]}{dt} \text{ = Rate of reaction.}$$

3. Answer (C)



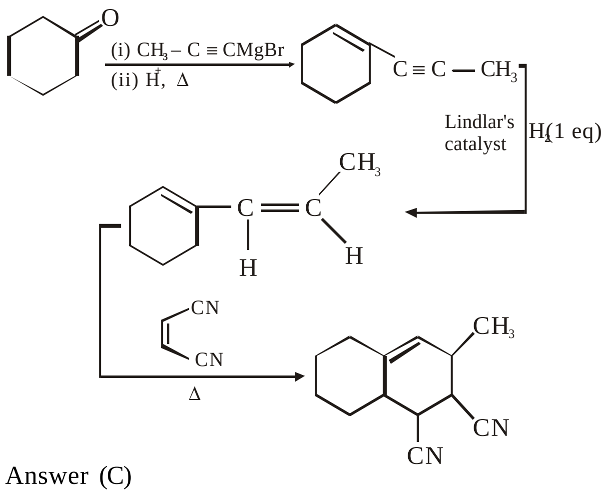
4. Answer (B)

BeO is an amphoteric oxide. $BeSO_4$ is water soluble while $Be(OH)_2$ is water insoluble.

5. Answer (D)

On adding O_2(g), IInd equilibrium shifts in backward direction & 1st equilibrium shifts in forward direction.

6. Answer (B)



7. Answer (C)

If X = 0.2 & Y = 0 then

$\Delta T_f = i \times K_f \times m$

$= 2 \times 1.86 \times 0.2$

$= 0.744°$

If X = 0 & Y = 0.2 then

$\Delta T_f = 3 \times 1.86 \times 0.2$

$= 1.116°$

So, observed range of ΔT_f becomes 0.744° to 1.116°.

8. Answer (D)

On heating, β-graphite is converted into α-graphite & on grinding α-graphite is converted into β-graphite.

9. Answer (A, C)

The n-factor of $K_4[Fe(CN)_6]$ is 3 because $3K^+$ are displaced by Ca^{2+} in one mole.

Now,

Milliequivalent of Ca^{2+} = milliequivalent of $K_4[Fe(CN)_6]$

$$\frac{W}{20} \times 1000 = 30 \times 0.1 \times 3$$

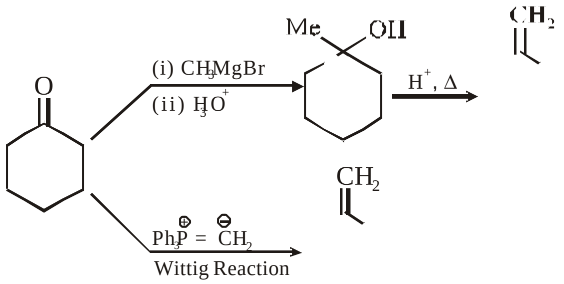
$$W_{Ca} = \frac{600 \times 0.3}{1000} = 0.18g \text{ pure calcium}$$

% purity of calcium ore = $\frac{0.18}{1} \times 100 = 18\%$

10. Answer (A, B, C, D)

Cyclopentadienyl rings have aromatic character. So, it undergoes Friedel Crafts acylation & do not give reactions of dienes.

11. Answer (C, D)



12. Answer (A, B, D)

(A) When a body diagonal plane is placed then 4 corners, 2 edges, 2 faces & one body atoms are removed. Number of Na^+ ions removed

$$= \frac{1}{4}\times 2+1=1\frac{1}{2}Na^+$$

Number of Cl^- ions removed

$$=\frac{1}{2}\times 2+\frac{1}{8}\times 4=1\frac{1}{2}Cl^-.$$

Hence, stoichiometry of NaCl remains same.

(B) When a rectangular plane is placed then 4 edges, 4 faces & one body atoms are removed.

Number of Na^+ ion removed = $\frac{1}{4}\times 4+1=2Na^+$

Number of Cl^- ions removed = $\frac{1}{2}\times 4=2Cl^-$

Hence, stoichiometry of NaCl remains same.

(C) When a body diagonal line is passed then two corners & one body atoms are removed

Number of Na^+ ions removed = $1Na^+$ ion

Number of Cl^- ions removed = $\frac{1}{8}\times 2=\frac{1}{4}Cl^-$ ion

Hence, stoichiometry of NaCl does not same

(D) On passing a tetrad axis, two face atoms & 1 body atom are removed.

Number of Na^+ ions removed = $1Na^+$ ion

Number of Cl^- ions removed = $\frac{1}{2}\times 2=1Cl^-$ ion

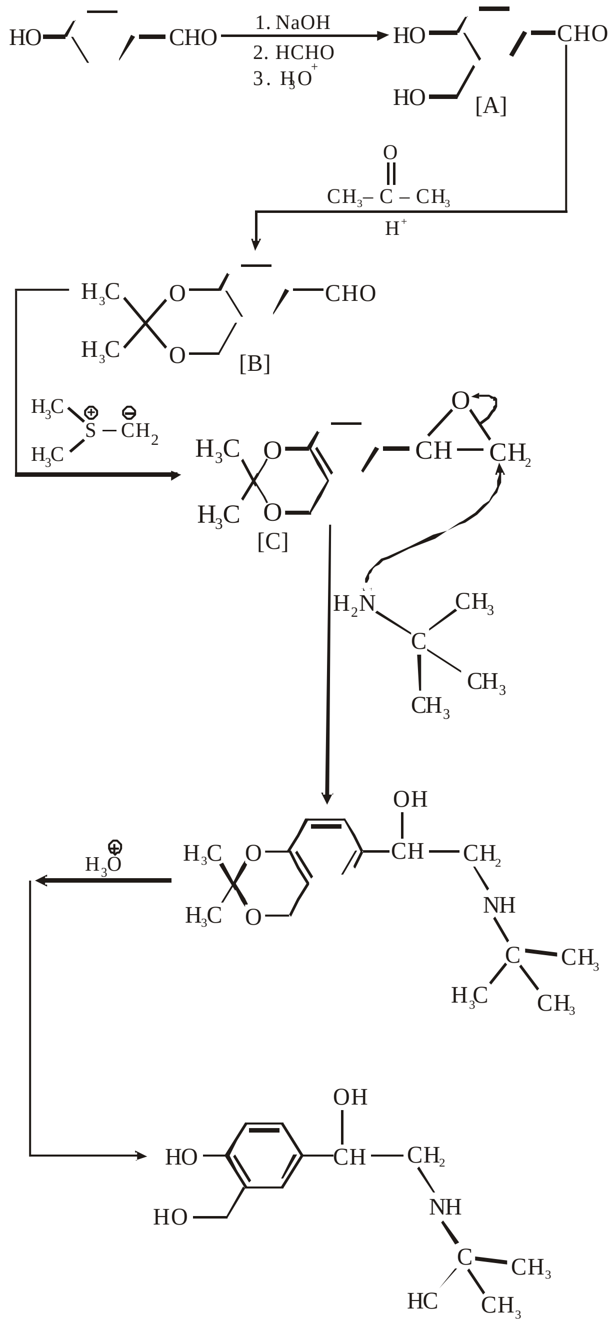
Hence, stoichiometry of NaCl remains same.

13. Answer (B)

14. Answer (A)

15. Answer (B)

Solution of Q13 to Q15



[Albuterol]

16. Answer (B)

Pb_3O_4 exists as $2PbO. PbO_2$.

In (B) option, reaction proceeds as :

$$2PbO+4HCl\xrightarrow{}2PbCl_2+2H_2O$$
(Acid - base reaction)

$$\overset{+4}{Pb}O_2+4HCl\xrightarrow{}\overset{+2}{Pb}Cl_2+2H_2O+\overset{0}{Cl_2}$$
(Redox reaction)

(A) is acid base reaction, (C) is salt displacement reaction & (D) is only redox reaction.

17. Answer (C)

The balanced equation is

$$6H^{+} + IO_3^{-} + 5I^{-} \longrightarrow 3I_2 + 3H_2O$$

So, 1 mole IO_3^- produces 3 mole of I_2.

I_2 reacts with $Na_2S_2O_3$ as follows

$2NaS_2O_3 + I_2 \rightarrow Na_2S_4O_6 + 2NaI$

1 mole $I_2 \equiv 2$ mole $Na_2S_2O_3$

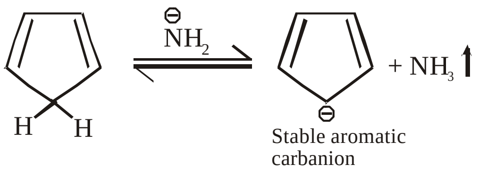
3 mole $I_2 \equiv 6$ mole $Na_2S_2O_3$

18. Answer (B)

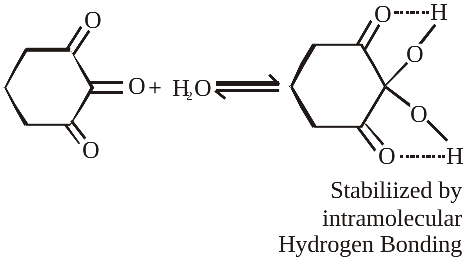
The n-factor of P_4 & FeC_2O_4 both are 3.

19. Answer A(p, q, t), B(p, r), C(p, q, t), D(s)

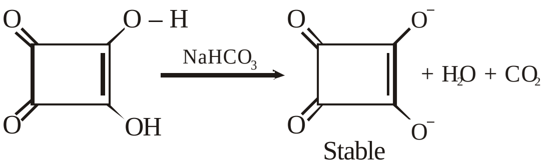
(A)



(B)

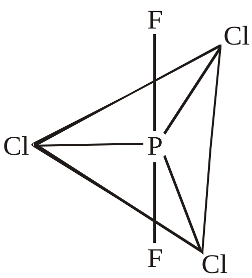


(C)



20. Answer A(p, r, t), B(p, r), C(p, q, r, t), D(p, r, s)

(A) PCl_3F_2



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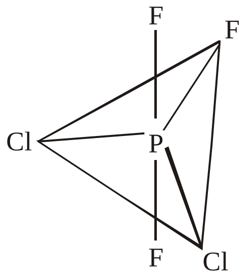
sp^3d hybridisation

Trigonal bipyramidal geometry

dipole moment = Zero

(B) PCl_2F_3



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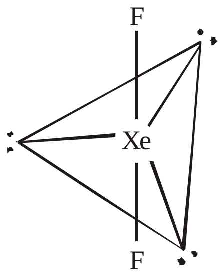
sp^3d hybridisation

Trigonal bipyramidal geometry

(C) XeF_2



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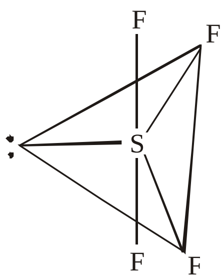
sp^3d hybridisation

Trigonal bipyramidal geometry

linear shape

dipole moment = Zero

(D) SF_4



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sp^3d hybridisation

Trigonal bipyramidal geometry

see saw shape.

PART - II : MATHEMATICS

21. Answer (D)

Putting $y = \frac{x-3}{x+1}$

$\Rightarrow \quad xy + y = x - 3$

$\Rightarrow \quad x = \frac{3+y}{1-y}$

Thus, $f(y) + f\left(\frac{y-3}{y+1}\right) = \frac{3+y}{1-y}$

Similarly for $y = \frac{3+x}{1-x} \Rightarrow x = \frac{y-3}{y+1}$

and $\frac{x-3}{x+1} = \frac{3+y}{1-y}$

We have $f\left(\frac{3+y}{1-y}\right) + f(y) = \frac{y-3}{y+1}$

Adding them, we get

$$\frac{8y}{1-y^2} = 2f(y) + f\left(\frac{y-3}{y+1}\right) + f\left(\frac{3+y}{1-y}\right) = 2f(y) + y$$

$\Rightarrow \quad f(y) = \frac{4y}{1-y^2} - \frac{y}{2}$

$\Rightarrow \quad f(2) = \frac{8}{1-4} - 1 = -\frac{8}{3} - 1 = -\frac{11}{3}$

22. Answer (C)

$x^2 - 4x + 3 < 0 \qquad \Rightarrow x \in (1, 3)$

Let $f(x) = 2^{1-x} + k$, $g(x) = x^2 - 2(k+7)x + 5$

When $1 < x < 3$, the images of $f(x)$ and $g(x)$ are both below the x-axis and since $A \subseteq B$, $f(x)$ is decreasing and $g(x)$ is quadratic function. $A \subseteq B$ iff $f(1) \le 0$, $g(1) \le 0$ and $g(3) \le 0$

$\Rightarrow \quad -4 \le k \le -1$

23. Answer (A)

The domain of the given function ϕ is the set of values of x satisfying

$\sqrt{\log_2 x - 1} + \frac{1}{2}\log_{1/2} x^3 + 2 > 0$

$\Rightarrow \quad \sqrt{\log_2 x - 1} - \frac{3}{2}\log_2 x + \frac{3}{2} + \frac{1}{2} > 0 \qquad$ ...(i)

and $\log_2 x - 1 \ge 0$

by solving (i) we get

$\Rightarrow \quad 0 \le \log_2 x - 1 < 1$

$\Rightarrow \quad 2 \le x < 4$

24. Answer (D)

Let S and T be the mid-points of the sides PR and QR respectively.

Then, $\overrightarrow{XP} + \overrightarrow{XR} = 2\overrightarrow{XS}$

and $2\left(\overrightarrow{XQ} + \overrightarrow{XR}\right) = 4\overrightarrow{XT}$

$\Rightarrow \quad \overrightarrow{XP} + 2\overrightarrow{XQ} + 3\overrightarrow{XR} = 2\left(\overrightarrow{XS} + 2\overrightarrow{XT}\right) = \vec{0}$

$\Rightarrow \quad \overrightarrow{XS}$ and $\overrightarrow{XT}$ are collinear and $\left|\overrightarrow{XS}\right| = 2\left|\overrightarrow{XT}\right|$

$\Rightarrow \quad \frac{ar\Delta PTS}{ar\Delta PB} = \frac{3}{2}$ and $\frac{ar(\Delta PQR)}{ar(PR)} = \frac{3\times 2}{2} = 3$

Alternate Solution

Let the positive vector of P, Q, R with respect to X be $\vec{a}, \vec{b}, \vec{c}$ respectively

$\Rightarrow \quad$ Area of $\Delta PQR = \frac{1}{2}\left|\vec{a}\times\vec{b} + \vec{b}\times\vec{c} + \vec{c}\times\vec{a}\right|$

but $\vec{a} + 2\vec{b} + 3\vec{c} = 0$

$\Rightarrow \quad 2\vec{a}\times\vec{b} = 3\vec{c}\times\vec{a} \qquad$ ...(i)

and $\vec{b}\times\vec{c} = \frac{\vec{b}\times\vec{c}}{2}$

$\Rightarrow$ Area of $\Delta PQR = \frac{1}{2}\left|\frac{3}{2}\vec{c}\times\vec{a}+\frac{1}{2}\vec{c}\times\vec{a}+\vec{c}\times\vec{a}\right|$

$\frac{3}{2}\left|\vec{c}\times\vec{a}\right|$

Similarly area of $\Delta PXR = \frac{1}{2}\left|\vec{c}\times\vec{a}\right|$

ratio = 3

25. Answer (D)

We have $\sqrt{2}+\sqrt{3}>\pi$

$\Rightarrow \quad 0<\frac{\pi}{2}-\sqrt{2}<\sqrt{3}-\frac{\pi}{2}<\frac{\pi}{2}$

$\Rightarrow \quad \sin\sqrt{2}>\sin\sqrt{3}$

and $\quad 0<\sqrt{2}<\frac{\pi}{2},\ \frac{\pi}{2}<\sqrt{3}<\pi$

$\Rightarrow \quad \cos\sqrt{2}>0, \cos\sqrt{3}<0$

$\Rightarrow \quad \cos\sqrt{2}-\cos\sqrt{3}>0$

Hence the curve represented by the equation is an ellipse

Now, $\left(\sin\sqrt{2}-\sin\sqrt{3}\right)-\left(\cos\sqrt{2}-\cos\sqrt{3}\right)$

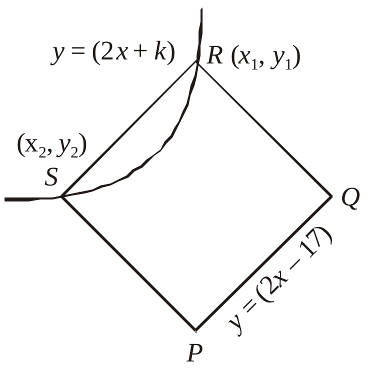
$$=2\sqrt{2}\sin\frac{\sqrt{2}-\sqrt{3}}{2}.\sin\left(\frac{\sqrt{2}+\sqrt{3}}{2}+\frac{\pi}{4}\right)<0$$

$\Rightarrow \quad \sin\sqrt{2}-\sin\sqrt{3}<\cos\sqrt{2}-\cos\sqrt{3}$

$\Rightarrow$ The foci lie on the y-axis

26. Answer (A)

Let $R(x_1, y_1)$ and $S(x_2, y_2)$ lie on the parabola $y = x^2$ and the equation of RS be $y = 2x + k$



$\Rightarrow \quad x^2 - 2x + 1 = 1 + k$

$\Rightarrow \quad x = 1 \pm \sqrt{1+k}$

$$RS^2 = (x_1-x_2)^2+(y_1-y_2)^2 = 5(x_2-x_1)^2 = 20(k+1)$$

Let us pick a point (6, –5) on $y = 2x - 17$ and the distance from the point to the line $y = 2x + k$ is a

$\Rightarrow \quad a = \frac{|17+k|}{\sqrt{5}} = 2\sqrt{5}\sqrt{(k+1)}$

$\Rightarrow \quad k_1 = 3, k_2 = 63$

$\Rightarrow \quad a^2 = 80$ or $a^2 = 1280$

$\Rightarrow \quad$ Min $(a^2) = 80$

27. Answer (A)

Required probability $= \frac{{}^{15}C_3 - 15\times{}^{11}C_1 - 15}{{}^{15}C_3} = \frac{55}{91}$

28. Answer (A)

We have $\phi'(x) = \phi(x)$

$\Rightarrow \quad \phi(x) = ke^x$

$\Rightarrow \quad \phi(0) = 1 = ke^0 \Rightarrow k = 1$

$\Rightarrow \quad \phi(x) = e^x$

$\Rightarrow \quad \psi(x) = x^2 - e^x$

Hence $\int_0^1 \phi(x)\psi(x)\,dx = \int_0^1\left(x^2e^x - e^{2x}\right)dx$

$$=\left[x^2e^x\right]_0^1 - 2\int_0^1 xe^x\,dx - \left[\frac{e^{2x}}{2}\right]_0^1 = e - \frac{e^2}{2} - \frac{3}{2}$$

29. Answer (A, B, C, D)

$f(x)$ is non-negative $\forall\, x \in R$

$\Rightarrow \quad f(x) \ge 0,\ \forall\, x \in R$

$\Rightarrow \quad 64a^2 - 16a \le 0$

$\Rightarrow \quad a(4a - 1) \le 0$

$\Rightarrow \quad a \in \left[0, \frac{1}{4}\right]$

Integral value of $a = 0$

$f(0) < 0 \Rightarrow a < 0$

For distinct roots in (0, 1)

$\Delta > 0 \quad \Rightarrow a(4a - 1) > 0$

If α, β are the roots

then $0 < \dfrac{\alpha+\beta}{2} < 1 \Rightarrow 0 < a < 1$

$f(0) > 0 \quad \Rightarrow a > 0$

$f(1) > 0 \quad \Rightarrow 4 - 7a > 0$

$\Rightarrow a < \dfrac{4}{7}$

Hence $a \in \left(\dfrac{1}{4}, \dfrac{4}{7}\right)$

When $a = \dfrac{1}{4}$, min $f(x) = 0$

30. Answer (B, C)

We have on squaring and adding

$4 + 9 + 12 \sin(\theta + \phi) = 25$

$\Rightarrow \quad \sin(\theta + \phi) = 1 = \sin\dfrac{\pi}{2}$

$\Rightarrow \quad \theta + \phi = (4n+1)\dfrac{\pi}{2}, n \in Z$

When $\theta + \phi = \dfrac{\pi}{2} \Rightarrow \phi = \dfrac{\pi}{2} - \theta$

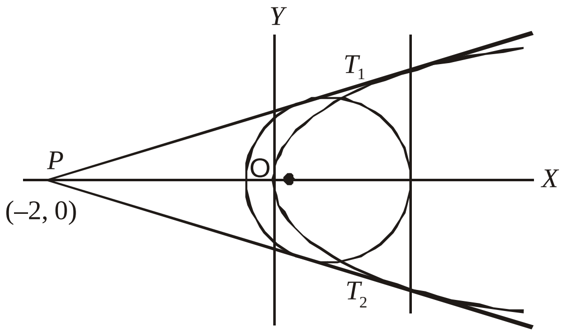
$5 \sin\theta = 3 \Rightarrow \sin\theta = \dfrac{3}{5}$ or $\cos\theta = \dfrac{4}{5}$

Also $\cos\phi = \dfrac{3}{5}$ and $\sin\phi = \dfrac{4}{5}$

Hence $\phi > \theta$

31. Answer (A, B)

Since the point $P(-2, 0)$ lies on the directrix of the parabola, hence the angle between the tangents is $\dfrac{\pi}{2}$ and the equations of PT_1 and PT_2 are $y = x + 2$ and $x + y + 2 = 0$ respectively. The equation of T_1T_2 is $x - 2 = 0$



Let $(a, 0)$ be the centre of the circle and the radius be r.

Then $\dfrac{|a+2|}{\sqrt{2}} = \dfrac{|a-2|}{1} =$ radius of the circle $= r$

$\Rightarrow \quad a^2 + 4a + 4 = 2(a^2 - 4a + 4)$

$\Rightarrow \quad a^2 - 12a + 4 = 0$

$\Rightarrow \quad a = \dfrac{12 \pm 8\sqrt{2}}{2} = 6 \pm 4\sqrt{2}$

radius $= |a - 2| = 4(\sqrt{2} - 1), 4(\sqrt{2} + 1)$

32. Answer (A, C)

The given lines are coplanar if

$\left[2\hat{i} + 5\hat{k} \quad \hat{i} - \hat{j} + \mu\hat{k} \quad \hat{i} + \mu\hat{j} + 2\hat{k}\right] = 0$

$\Rightarrow \quad \begin{vmatrix} 2 & 0 & 5 \\ 1 & -1 & \mu \\ 1 & \mu & 2 \end{vmatrix} = 0$

$\Rightarrow \quad \mu = \dfrac{5+\sqrt{33}}{4}, \dfrac{5-\sqrt{33}}{4}$

33. Answer (A)

34. Answer (B)

35. Answer (D)

Q.33 to Q35 Solution

The equation of the tangents to the given curve $y = \phi(x)$ and $y = \psi(x)$ at points with equal abscissae x are

$Y - \phi(x) = \phi'(x)\ (X - x)$

and $Y - \psi(x) = \psi'(x)\ (X - x)$

which will intersect on y-axis if

$Y - \phi(x) = -x\ \phi'(x)$

$Y - \psi(x) = -x\ \psi'(x)$

$\Rightarrow\ \phi(x) - \psi(x) = x\dfrac{d}{dx}\{\phi(x) - \psi(x)\}$

$\Rightarrow\ \dfrac{d(\phi(x)-\psi(x))}{\phi(x)-\psi(x)} = \dfrac{dx}{x}$

$\Rightarrow\ \ln(\phi(x) - \psi(x)) = \ln cx$

$\Rightarrow\ \phi(x) - \psi(x) = cx$ ...(i)

The equations of normals at the points with equal abscissae x to the given curves are given by

$$Y - \phi(x) = \frac{-1}{\phi'(x)}(X - x)$$

and $Y - \psi(x) = -\dfrac{1}{\psi'(x)}(X - x)$

These normal intersect on x-axis, hence $X = x + \phi(x)\ \phi'(x)$ and $X = x + \psi(x)\ \psi'(x)$

$\Rightarrow\ \phi(x)\phi'(x) = \psi(x)\psi'(x)$

$\Rightarrow\ (\phi(x))^2 - (\psi(x))^2 = k$

$\Rightarrow\ \phi(x) + \psi(x) = \dfrac{k}{cx}$

$\Rightarrow\ \phi(x) = \dfrac{1}{2}\left(cx + \dfrac{K}{cx}\right),\ \psi(x) = \dfrac{1}{2}\left(cx - \dfrac{K}{cx}\right)$

Since curve passes throgh (1, 1) and (2, 3),

Hence $\phi(x) = -x + \dfrac{2}{x}$, $\psi(x) = x + \dfrac{2}{x}$

36. Answer (B)

We have

$$I = \int_0^\infty \frac{\tan^{-1} ax}{x(1+x^2)}dx \Rightarrow \frac{dI}{da} = \int_0^\infty \frac{1}{(1+a^2x^2)(1+x^2)}dx$$

$$= \frac{1}{(1-a^2)}\int_0^\infty \left(\frac{1}{1+x^2} - \frac{a^2}{1+a^2x^2}\right)dx = \frac{\pi}{2(1+a)}$$

$\Rightarrow\ I = \dfrac{\pi}{2}\ln(1+a) + c$, at $a = 0$, $c = 0$

$\Rightarrow\ I = \dfrac{\pi}{2}\ln(1+a)$

37. Answer (B)

$$\frac{dI}{da} = \int_0^\infty \frac{1}{1+a^2x^2}.2ax^2.\frac{1}{1+b^2x^2}dx$$

$$= \frac{2a}{a^2-b^2}\int_0^\infty \left(\frac{1}{1+b^2x^2} - \frac{1}{1+a^2x^2}\right)dx$$

$$= \frac{2ab}{a^2-b^2}\left(\frac{\pi}{2}\right)\left(\frac{1}{b} - \frac{1}{a}\right) = \frac{\pi}{b(a+b)}$$

$$\Rightarrow I = \frac{\pi}{b}\ln(a+b) + c$$

Using $a = 0$, $I = 0 \Rightarrow K = -\dfrac{\pi}{2}\ln b$

Hence $I = \dfrac{\pi}{b}\ln\left(\dfrac{a+b}{b}\right)$

38. Answer (A)

$$I = \int_0^\infty \frac{\sin mx}{x}dx$$

$\Rightarrow \dfrac{dI}{dm} = \displaystyle\int_0^\infty \cos mx\,dx = \left|\frac{\sin mx}{m}\right|_0^\infty$, which is not defined

39. Answer A–(q, s), B–(p), C–(r), D–(t)

A. The no. of shortest routes that must pass through the junction $A = {}^5C_2 \times {}^8C_3$

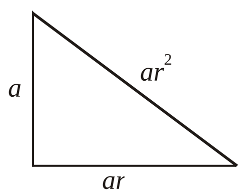
B. The no. of routes that must pass through the street $AB = {}^5C_2 \times {}^7C_3$

C. The no. of routes that pass through junctions A and $C = {}^5C_2 \times {}^4C_1 \times {}^4C_2$

D. When street AB is closed, the number of possible routes $= {}^{13}C_5 - {}^5C_2 \times {}^7C_3$

40. Answer A–(p, r, s, t), B–(q, t), C–(p, q, r, s, t), D–(p, q, r, s)

B. We have



$(ar^2)^2 = a^2 + a^2r^2$ where $a = 2$

$\Rightarrow \quad r^4 - r^2 - 1 = 0$

$$\Rightarrow \quad r^2 = \frac{1 \pm \sqrt{5}}{2}$$

$$\Rightarrow \quad r^2 = \frac{1 + \sqrt{5}}{2}$$

$$= ar^2 = 2\left(\frac{\sqrt{5}+1}{2}\right) = \sqrt{5}+1 = 1+\sqrt{5}$$

$$= a + \sqrt{b} \Rightarrow a = 1,\ b = 5$$

$a^2 + b^2 = 26$

C. We have $a + ar + ar^2 = 70$

$\Rightarrow \quad 10ar = 4a + 4ar^2$

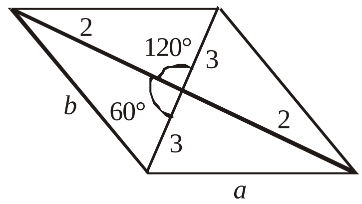
$\Rightarrow \quad 2r^2 - 5r + 2 = 0$

$\Rightarrow \quad (r-2)(2r-1) = 0$

$\Rightarrow \quad r = 2, \frac{1}{2}$, but $r \neq 2$

$\Rightarrow \quad r = \frac{1}{2},\quad a = 40$

D. We have $a^2 = 9 + 4 - 2.2.3.\left(-\frac{1}{2}\right) = 13 + 6 = 19$



$a^2 = 19 \Rightarrow a = \sqrt{19}$

and $\quad b^2 = 4 + 9 - 2.3.2.\frac{1}{2} = 7$

$\Rightarrow \quad b = \sqrt{7}$

Perimeter = $2\left(\sqrt{19} + \sqrt{7}\right)$

$\Rightarrow \quad a + b = 19 + 7 = 26$

PART - III : PHYSICS

41. Answer (B)

By conservation of energy

$$pE = \frac{1}{2}\left(\frac{7}{5}mr^2\right)\omega^2 = \frac{7}{10}mv^2$$

$$\Rightarrow \quad \sqrt{\frac{10\,pE}{7m}} = v$$

42. Answer (C)

$\vec{v}_{\text{cm}} = 4\hat{i} + 8\hat{j} \quad \vec{v}_{A,\text{cm}} = -2\hat{i} - 4\hat{j}$

$\Rightarrow \quad \lambda_{A,\ \text{cm}} = 3\lambda$

43. Answer (C)

$\tan\theta = \frac{4r}{h}$ in critical case and $\tan 37^\circ = \frac{r}{h} \Rightarrow \tan\theta = 3$

44. Answer (B)

Restoring torque on small displacement θ

$= -(kR^2\theta + 0.6\ mgR\theta)$

$$\Rightarrow \quad \alpha = -\left(\frac{k}{m} + \frac{0.6g}{R}\right)\theta = -\omega^2\theta$$

$$\Rightarrow \quad \omega = \sqrt{\frac{k}{m} + \frac{0.6g}{R}} = \sqrt{\frac{100}{0.5} + \frac{0.8 \times 10}{3/28}} = \sqrt{256} = 16\ \text{rad/s}$$

45. Answer (B)

F is equals to gravitational force at different points

46. Answer (C)

$$U = \frac{1}{2}Li^2$$

$$\Rightarrow \quad U' = Li\frac{di}{dt} = iV$$

$$\Rightarrow \quad U' \propto e^{-t/\tau}(1 - e^{-t/\tau})$$

47. Answer (C)

Initial pressure in bubble = $P+\dfrac{4s}{r}$

Final pressure = $8\left(P+\dfrac{4s}{r}\right)$, as volume becomes $\dfrac{1}{8}$th and T is constant.

$\Rightarrow$ Outside pressure = $8\left(P+\dfrac{4s}{r}\right)-\dfrac{4s}{r/2}=8\left(P+\dfrac{3s}{r}\right)$

48. Answer (B)

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{5}-\frac{1}{10}\right)=\frac{1}{20}$$

$\Rightarrow$ $m=\dfrac{v}{u}=2$

$v_{\text{meniscus}}=\sqrt{2gh}=\sqrt{2\times 9.8\times 0.1}$ = 1.4 m/s (down)

$v_{\text{object/meniscus}}$ = 1.4 m/s (upward)

$v_{\text{image/meniscus}}=m^2\, v_{\text{object/meniscus}}$

= 4 × 1.4 = 5.6 m/s (upward)

$\Rightarrow$ v_{image} = 5.6 + 1.4 = 7 m/s

49. Answer (A, B, C)

For refraction at AB, $\mu_1 \sin 90° = \mu_2 \cos\alpha$ ...(i)

For refraction at AC, $\mu_3 \sin 90° = \mu_2 \sin\alpha$ ...(ii)

Eq (i) & (ii)

$\mu_1^2+\mu_3^2=\mu_2^2$

For emergence of ray from AC, $\alpha \le C$ at the face

$\Rightarrow$ $\alpha \le \sin^{-1}\left(\dfrac{\mu_3}{\mu_2}\right)$

for T.I.R $\alpha > C$

$\Rightarrow$ $\mu_1^2+\mu_3^2<\mu_2^2$

50. Answer (B, D)

Orbital speed and orbital angular momentum independent of mass of revolving particle

51. Answer (B)

As $|\vec{v}_1|=|\vec{v}_2|,\ v'=\dfrac{v}{2}$

$\Rightarrow$ Angle between $\vec{v}_1$ and $\vec{v}_2$ = θ = 60°

$\Rightarrow$ $a=\sqrt{3}\dfrac{R}{2}$ and $b=\dfrac{R}{2}$ $\Rightarrow$ $a=\sqrt{3}b$

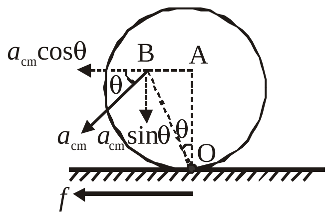
52. Answer (A, B, D)

Linear momentum is conserved if particle strikes at $x=\dfrac{2L}{3}$ as in this case reaction at hinge is zero.

As τ about hinge is zero, $\vec{L}$ = constant about hinge.

53. Answer (D)

FBD of body is as shown.



By $\tau = I\alpha$ about instantaneous axis at O

$\alpha=\dfrac{g}{6R}$

$\Rightarrow$ $a_x=a_{cm}\cos\theta$

$=\alpha.OB\cos\theta$

$=\dfrac{g}{6R}.\sqrt{5}R.\dfrac{2R}{\sqrt{5}R}$

$=\dfrac{g}{3}$

54. Answer (C)

$f=m.a_x=\dfrac{2Mg}{3}$

55. Answer (A)

By conservation of energy

$$2MgR=\frac{1}{2}(4MR^2)\omega^2 \Rightarrow \omega=\sqrt{\frac{g}{R}}$$

56. Answer (C)

For any point P on surface A if $|S_2P - S_1P|$ = constant, locus of P is circle.

For any point P on surface B if $|S_2P - S_1P|$ = constant, locus of P is hyperbola.

57. Answer (A)

For maxima, path difference = $n\lambda$

For maxima path difference = $(2n-1)\dfrac{\lambda}{2}$

58. Answer (C)

$$\vec{v}_{fringe} = -v\hat{j} - \frac{\lambda.v}{3\lambda}\hat{i} = -v\hat{j} - \frac{v}{3}\hat{i}$$

$$\Rightarrow \quad v_{fringe} = \frac{\sqrt{10}v}{3}$$

59. Answer A –(p, q, r, t), B–(q, r, t), C–(s), D–(r)

Use $\vec{\omega} = \dfrac{d\vec{\theta}}{dt}$, $\vec{\tau} = \dfrac{d\vec{L}}{dt}$ and $a_{radial} = \dfrac{v^2}{r}$

60. Answer A–(q, s, t), B–(s), C–(q, s), D–(p)

(p) Energy is lost due to friction

(q) Resistance = $\dfrac{\rho l}{A} = \dfrac{\rho l^2}{\text{volume}}$

(r) Use Bernoull's theorem

(s) Direction of $\vec{\tau}$ (about O) charges as particle moves from A to B

(t) Internal energy increase on dissociation as n becomes double while f becomes $\dfrac{3}{5}$th

$$\Rightarrow \quad U' = \frac{6}{5}U$$

By $PV = nRT$, P increases

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