B4.2-R3: DISCRETE STRUCTURES

NOTE:

1.	**Answer question 1 and any FOUR questions from 2 to 7.**
2.	**Parts of the same question should be answered together and in the same sequence.**

Time: 3 Hours **Total Marks: 100**

1.

a) How many elements does each of these sets have?
P({a, b,{a, b}}), P({Φ ,a,{a},{{a}}}), P(P(Φ)), P(Φ)
Here P(A) represents power set of A.

b) Let f be the function from {a, b, c} to {1, 2, 3} such that f(a) = 2, f(b)=3 and f(c) = 1. Is function f invertible and if it is what is its inverse?

c) State the converse, contra positive and inverse of the following:
"A positive integer is a prime only if it has no divisor other than 1 and itself."

d) How many edges does a graph have if it has vertices of degrees 5, 2, 2,2,2,1?

e) Find the duals of
x(y +0) and $\bar{x}.1 + (\bar{y} + z)$

f) What is the coefficient of $x^3 y^2 z^3$ in $(x + y+ z)^9$.

g) Write a grammar that generates the set $\{0^n 1^{2n} \mid n = 0,1….\}$

(7x4)

2.

a) Show that the relation R on Z X Z defined by (a, b) R (c, d) if and only if a +d = b+ c is an equivalence relation. Write three equivalence classes.

b) Let G be the set of all nonzero real numbers and let
a*b = ab/2
Show that (G,*) is an abelian group.

c) Show that among any group of five (not necessarily consecutive) integers, there are two with the same remainder when divided by 4.

(6+6+6)

3.

a) Consider the following truth table:-

P	Q	R	S
1	1	1	0
1	1	0	0
1	0	1	1
1	0	0	1
0	1	1	0
0	1	0	1
0	0	1	0
0	0	0	0

Construct a Boolean Expression having this table as truth table. Simplify this expression. Also construct a circuit having P, Q, R as input and S as output.

b) Solve the following recursive relations:
$a_{n+1} - 1.5\, a_n = 0$; n >=0
$a_n = 5\, a_{n-1} + 6\, a_{n-2}$; n >= 2 ; $a_0 = a_1 = 3$

c) Let Q(x, y, z) be the statement " x + y = z". What are the truth values of the statements?
∃ ∀x ∀y∃z Q(x, y, z) and ∃z∀x ∀y Q(x, y, z)

(6+6+6)

4.

a) Construct a grammar for the language:

$L = \{w \mid n_a(w) >= n_b(w)\}$. Where $w \in (a+b)^*$ and $n_a(w)$ represents number of a's in W.

b) Find a finite state machine that recognizes the language:

$\{10^n \mid n \geq 0\} \cup \{1\,0^n\,1\,0^m \mid n, m \geq 0\}$

c) Prove that for any a, b in a Boolean algebra B

i) $a + a.\,b = a$

ii) $a.(a+b) = a$

(6+6+6)

5.

a) Show that in any simple connected planar graph, $e \geq 3f/2$ and $e \leq 3n-6$. Here n = number of vertices, e = no. of edges and f = no. of regions.

b) Define a Hamiltonian graph. Define Euler Graph. Give an example of each.

c) Give a simple condition on the weights of a graph that will guarantee that there is a unique minimum spanning tree for the graph.

(6+6+6)

6.

a) If a graph G is not connected, prove that complement of G is connected.

b) Write the assumptions (if any) made in Floyd -Warshall algorithm. Use this algorithm for the graph whose weight matrix is given below:

$$\begin{bmatrix} 0 & 4 & -3 & \infty \\ -3 & 0 & -7 & \infty \\ \infty & 10 & 0 & 3 \\ 5 & 6 & 6 & 0 \end{bmatrix}$$

c) Find the number of solutions of

$e1+e2+e3 = 17$

Where e1, e2 and e3 are non negative integers with $2 \leq e1 \leq 5$, $3 \leq e2 \leq 6$ and $4 \leq e3 \leq 7$.

(6+6+6)

7.

a) Show that $((p \vee \neg q) \wedge (\neg p \vee \neg q)) \vee q$ is a tautology, where p and q are Boolean Variables.

b) Use generating function to find the number of k- combinations of a set with n elements. Assume that the Binomial theorem has already been established.

c) Explain following:

i) Simplification of machines

ii) Pigeon hole principle

iii) Partitioning of set

(6+6+6)