INSTITUTE OF ACTUARIES OF INDIA

EXAMINATIONS

11th May 2011

Subject CT6 – Statistical Models

Time allowed: Three Hours (10.00 – 13.00)

Total Marks: 100

INSTRUCTIONS TO THE CANDIDATES

1. ***Please read the instructions on the front page of answer booklet and instructions to examinees sent along with hall ticket carefully and follow without exception***

2. ***Mark allocations are shown in brackets.***

3. ***Attempt all questions, beginning your answer to each question on a separate sheet.***

4. ***In addition to this paper you will be provided with graph paper, if required.***

AT THE END OF THE EXAMINATION
Please return your answer book and this question paper to the supervisor separately.

Q. 1) State two conditions that must hold for a risk to be insurable. List four other risk criteria that would be considered desirable by a general insurer.

[4]

Q. 2) If the ACF's at lags 1 and 2 of an AR(2) process are 0.8 and 0.46, calculate the PACF values at all lags.

[3]

Q. 3) The distribution function of the loss (X) arising from a particular risk is given by

$$P(X \leq x) = 1 - \left(1 + \frac{\alpha}{\beta} x\right)^{-1/\alpha} \quad , \; x \geq 0, \; 0 < \alpha < \frac{1}{2}, \; \beta > 0.$$

(i) Find an expression for the probability density function of X. (2)

(ii) A random sample of 20 values taken from this distribution has mean 0.6 and variance 2. Determine the 'method of moments' estimators of α and β. (6)

[8]

Q. 4) A farmer has the option of growing apples, pears or oranges at his farm next year. He expects to make a profit of Rs.10, 15 and 5 respectively on each kilogram of apples, pears and oranges sold. He estimates the following amounts (in 1000 kilograms) of fruit that he would be able to grow in three possible weather conditions:

	Warm and wet (Θ_1)	Cool (Θ_2)	Warm and dry (Θ_3)
Apples	10	7	9
Pears	4	6	7
Oranges	15	20	10

(i) Determine which fruit the farmer should grow so as to maximize his maximum profit. (1)

(ii) Determine the revised Bayes criterion solution assuming that Θ_1 and Θ_2 are equally likely to occur and that the probability of occurrence of Θ_3 is p. (5)

(iii) Determine the Bayes criterion solution to this problem if each scenario is equally likely to occur. (1)

[7]

Q. 5) Show that the adjustment coefficient (R) for aggregate claims modeled as a compound Poisson process satisfies the following inequalities

$$\frac{1}{M}\log(1+\theta) < R < 2\theta\frac{m_1}{m_2},$$

where m_1 and m_2 are the first and second moments of the claim size distribution, θ is the premium loading, and M is an upper limit to the individual claim size.

[6]

Q. 6) Consider the data given below.

Cumulative claim amounts reported

	Development Year >>>					Earned	Paid
AY	0	1	2	3		Premiums	to-date
2008	500	1500	1700	1800		2000	1690
2009	700	1900	2000			2500	1700
2010	600	1400				3000	800
2011	1200					3300	100

(i) Calculate, by using the chain-ladder method, the estimate of outstanding claims to be paid. (3)

(ii) Using the combined chain-ladder ultimate loss ratio for years more than 90% developed as the initial expected loss ratio for all years; calculate the Bornhuetter-Ferguson estimate of future claim payments. (3)

(iii) Compare the two estimates and comment on them. (3)

[9]

Q. 7) The table below shows the claims paid (in Rs.'000s) over a four year period for an insurer writing three classes of business:

	Year 1	Year 2	Year 3	Year 4
Fire	20	12	25	36
Motor	100	250	175	200
Marine	50	63	70	62

(i) Analyse the data using EBCT Model 1 and calculate the expected claim payments for each class in the next year. (4)

(ii) List the assumptions for EBCT Model 1 in the present context (3)

[7]

Q. 8) The aggregate claims incurred from a portfolio of 'Cash & Cover' health insurance policies has a compound Poisson distribution with Poisson parameter 1000 and individual claim size density $f(x) = \frac{1}{5}\exp[-(x-5)/5], \quad x > 5$. The premium charged by the insurer is calculated by using a premium loading factor of 15%. Excess of loss reinsurance with different retention limits are being considered. The reinsurance premium is calculated by using a premium loading factor 30%. Complete the following table, by determining the relation between retention limit and expected annual profit made by the insurer.

Retention limit	Expected annual profit
8	---
---	1425.32
∞	---

[6]

Q. 9) An insurance contract provides for enhancement of the next installment of premium, depending on the size of each single claim. The insured customer is confused about the amount of enhancement. As a result, he actually claims only a fraction of what he would have ideally claimed. The actual claim amount X has the uniform distribution over the interval $[0,\theta]$, θ being the ideal claim amount. The parameter θ has to be estimated from a single actual claim X, by using the *absolute error* loss function.

(i) Using the prior density $g(\theta) = \theta \exp(-\theta)$ for the ideal claim θ, obtain the Bayes estimator of θ. (4)

(ii) If the estimator is restricted to be of the form kX, obtain a suitable constant k so that this estimator minimizes the overall expected loss (with expectation computed with respect to X and θ), irrespective of the distribution of θ. (4)

(iii) The insurer plans to offer a new contract, which is expected to encourage the customer to claim the ideal amount θ. In order to arrive at a suitable level of premium, the insurer decides to use a credibility estimate of θ. For this purpose, he uses:

- the optimum estimator of part (ii) as an estimate based solely on the single data from the risk itself, and
- the mean computed from the prior density given in part (i) as an estimate based on collateral data.

Examine whether the Bayes estimator of θ obtained in part (ii) be regarded as a credibility estimate, for some value of the credibility factor. (2)

[10]

Q. 10) The number of claims incurred by a general insurer on a single day has a compound Poisson distribution with mean 5. The individual claim amounts have the common value of Rs. n lacs if the day of the claim falls on the n^{th} week of the month. (The first week consists of the days up to and including the first Sunday, and each subsequent week begins with a Monday.) Calculate the probability that the total claim amount incurred by the company on the 20^{th} of a randomly chosen month is less than or equal to 7 lacs.

[6]

Q. 11) Alpha General Insurance Company sells a certain kind of travel insurance policies at a flat annual premium of Rs.100 per policy. It has 1000 such policies on its portfolio. 20% of the policies are expected to have a claim in any one year. Loss amounts are assumed to follow a Lognormal distribution with mean 250 and standard deviation 375.

The insurance company is considering the following two types of reinsurance cover:

- Proportional reinsurance with $100\alpha\%$ of all claims and premiums ceded;
- Excess of loss reinsurance with a retention limit of Rs.300 per claim for a fixed premium of Rs.35,000 for these travel insurance policies on its portfolio.

(i) Determine the value of α, which ensures that the two types of reinsurance cover result in exactly the same average net claim amount for Alpha. (6)

(ii) Compare the amounts of profit (from the portfolio) to Alpha under the two types of reinsurance cover, with α chosen as in part (i). (2)

[8]

Q. 12) A student wants to generate pseudo random values from a statistical distribution that has probability density function

$$f(x) = \begin{cases} \dfrac{\sqrt{3}}{4\left(1 + x^2/2\right)^{3/2}} & \text{for } |x| \le 2, \\ 0 & \text{for } |x| > 2. \end{cases}$$

She plans to use the acceptance-rejection method together with the standard normal density $\phi(x)$ for $-\infty < x < \infty$.

(i) Show that the maximum value of the ratio $\dfrac{f(x)}{\phi(x)}$ is $\dfrac{e^2\sqrt{2\pi}}{12}$. (5)

(ii) Assuming that samples from the uniform distribution over [0,1] can be readily generated, and the inverse of the standard normal distribution function is available, give a clear and specific list of steps to generate one sample from $f(x)$. (4)

(iii) Calculate the expected proportion of accepted samples, according to the method described in part (ii). (1)

[10]

Q. 13) The following data summary is obtained from 200 consecutive points of a realization of an ARMA(1,1,0) process with constant mean.

$$\sum_{i=2}^{200}(x_i - x_{i-1})^2 = 936.49, \quad \sum_{i=2}^{199}(x_i - x_{i-1})(x_{i+1} - x_i) = 587.83.$$

(i) Obtain the usual estimates of the AR coefficient and the innovation variance from the given data summary. (5)

(ii) Given the data points $x_{198} = -2.11$, $x_{199} = 0.82$ and $x_{200} = 1.93$, use the Box Jenkins methodology and the estimated parameters obtained in part (i) to forecast x_{201}. (2)

[7]

Q. 14) The claim size (*Y*) from a particular risk is presumed to have the exponential distribution, with the mean μ depending on a single covariate, age (*X*), through the relation

$$g\left(E(Y|X)\right) = \alpha + \beta X \ ,$$

where g is a suitable link function.

(i) Given *n* pairs of observations, (X_i, Y_i) for $i = 1,2,,,,,n$ from this generalized linear model, write down the log-likelihood function explicitly in terms of α and β, using the canonical link function. (4)

(ii) Obtain two equations that are satisfied by the maximum likelihood estimates of α and β. (2)

(iii) Test whether age has an effect on the claim size, from the data set (40, 250), (50, 1000), (60, 250). (3)

[9]

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