# BACHELOR IN COMPUTER APPLICATIONS

## Term-End Examination

## December, 2007

# CS-60 : FOUNDATION COURSE IN MATHEMATICS IN COMPUTING

*Time : 3 hours* *Maximum Marks : 75*

**Note :** *Question No. 1 is* ***compulsory****. Attempt any* ***two*** *questions from Questions No. 2 to 5.*

1. (a) Find without expanding the value of

$$\begin{vmatrix} 1 & a & b+c \\ 1 & b & c+a \\ 1 & c & a+b \end{vmatrix}$$

(b) Find the value of $\underset{x \to \infty}{\text{Lt}} \frac{\sin x}{x}$.

(c) Find the square root of 'i'.

(d) Prove that $[f(x) + f(-x)]$ is an even function and that $[f(x) - f(-x)]$ is an odd function.

(e) Find $\frac{dy}{dx}$, when $y = \cos(x + y)$.

(f) If z is the product of two complex numbers $z_1$ and $z_2$, prove that

$$|z| = |z_1| \, |z_2|$$

and $\text{Arg } z = \text{Arg } z_1 + \text{Arg } z_2$

(g) Prove that

$$\int_{-a}^{+a} f(x)\,dx = 2\int_{0}^{a} f(x)\,dx, \quad \text{if} \quad f(x) = f(-x).$$

(h) Find the derivative of $\tan^{-1}\left(\frac{\sqrt{1+x^2}-1}{x}\right)$ w.r.t. $\tan^{-1} x$.

(i) Find

$$\underset{x\to 0}{\text{Lt}} \ (1 + x)^{1/x}$$

(j) Obtain the equation of the tangent to the circle $x^2 + y^2 = a^2$ at $(x_1, y_1)$.

(k) If the extremities of a focal chord of the parabola $y^2 = 4ax$ are $\left(at_1^2, 2at_1\right)$ and $\left(at_2^2, 2at_2\right)$, then prove that $t_1t_2 = -1$.

(l) Prove that the equation

$$x^2 + 6xy + 9y^2 + 4x + 12y - 5 = 0$$

represents a pair of straight lines.

(m) Find the equation of the plane passing through the point $(3, 2, -1)$ and the intersection of the planes $2x + y + 2z = 9$ and $4x - 5y - 4z = 1$.

(n) For any two sets A and B, prove that

(i) $A \cup A = A$

(ii) $A \cup B = B \cup A$

(o) For a, b, c being non-zero real numbers, prove that

$$(b + c)(c + a)(a + b) > 8abc$$

*3×15=45*

**2.** (a) Find the ranges of x, where the function

$$f(x) = 2x^3 - 15x^2 + 36x + 1$$

increases with x and decreases with x.

(b) Find the distance of the point (1, – 2, 3) from the plane $x - y + z = 5$ measured parallel to the line

$$\frac{x}{2} = \frac{y}{3} = \frac{z}{-6}.$$

(c) For any three complex numbers $z_1, z_2, z_3$, prove that

$$z_1 \, \text{Im}(\bar{z}_2 z_3) + z_2 \, \text{Im}(\bar{z}_3 z_1) + z_3 \, \text{Im}(\bar{z}_1 z_2) = 0$$ *5×3=15*

**3.** (a) Find $\frac{dy}{dx}$, when $y = \tan^{-1} \frac{\cos x + \sin x}{\cos x - \sin x}$.

(b) Evaluate

$$\int \frac{\tan x \, dx}{\sqrt{a + b \tan^2 x}}$$

(c) Solve by Cardano's method the cubic equation

$$x^3 - 3x + 1 = 0$$ *4+5+6=15*

**4.** (a) If $I_n = \int_0^{\pi/2} \cos^n x \, dx$,

then prove that,

$$I_n = \frac{n-1}{n} I_{n-2}$$

(b) Obtain the equation of the pair of straight lines passing through the point (2, 3) and perpendicular to the pair of straight lines represented by

$$3x^2 - 8xy + 5y^2 = 0.$$

(c) Solve the biquadratic equation

$$x^4 - 3x^3 + 3x^2 - 3x + 2 = 0,$$

given that one solution is $x = i$. *5×3=15*

**5.** (a) Find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ in the first quadrant bounded by the co-ordinate axes.

(b) Find the condition that the two spheres

$$x^2 + y^2 + z^2 + 2ux + 2vy + 2wz + d = 0$$

and $x^2 + y^2 + z^2 + 2u'x + 2v'y + 2w'z + d' = 0$

cut each other orthogonally.

(c) Identify the surfaces represented by the following equations and draw rough sketches of the same :

(i) $x^2 + y^2 = 16$

(ii) $y^2 + z^2 = ax$ *5+4+(3×2)=15*