This question paper contains 4+2 printed pages]

Your Roll No. ....

6183

B.Sc. (Hons.) Computer Science/II Sem. J

Paper 204—PROBABILITY

(Admission of 2001 and onwards)

Time : 3 Hours *Maximum Marks : 75*

(Write your Roll No on the top immediately on receipt of this question paper)

Attempt *All* questions.

All questions carry equal marks.

Use of calculator is allowed.

1. In answering a question on a multiple choice test a student either knows the answer or he guesses Let 'p' be the probability that he knows the answer and '$1 - p$' the probability that he guesses. Assume that a student who guesses the answer will be correct with probability $\frac{1}{m}$, where 'm' is the number of alternatives. What is the conditional probability that a student knew the answer to a question given that he answered it correctly ?

P T O

2. Suppose that each of three men at a party throws his hat into the centre of the room. The hats are first mixed up and then each man randomly selects a hat. What is the probability that none of the three men selects his own hat ?

3. Let $X \sim B(n_1, p)$ and $Y \sim B(n_2, p)$ be independent random variables. Show that $X + Y \sim B(n_1 + n_2, p)$. Also comment on the distribution of $X - Y$.

4. Let X be binomially distributed with parameters n and p. Show that as k goes from 0 to n, $P(X = k)$ increases monotonically, then decreases monotonically reaching its largest value in the case that $(n + 1)p$ is an integer, when k equals either $(n + 1)p - 1$ *or* $(n + 1)p$.

5 Define Moment Generating Function (MGF) Obtain MGF of normal distribution and hence find its mean and variance

6 State and prove central limit theorem.

7. If X and Y are independent Gamma variables with parameters 'μ' and 'ν' respectively, show that variables $U = X + Y$ and $Z = \frac{X}{X + Y}$ are independent Also identify the distribution of U and Z.

8. The joint density of X and Y is given by

$$f(x, y) = \begin{cases} \frac{1}{2} y \, e^{-xy} & , 0 < x < \infty, 0 < y < 2 \\ 0 & , \quad \text{otherwise} \end{cases}$$

What is $E[e^{X/2} \mid Y = 1]$?

9. Let X be epxonential with mean $\frac{1}{\lambda}$, that is :

$$f_X(x) = \lambda e^{-\lambda x}, 0 < x < \infty.$$

Find E $[X \mid X > 1]$.

10. Let X_1 and X_2 be independent geometric random variables with respective parameters p_1 and p_2. Find $P\{|X_1 - X_2| \leq 1\}$

11. Find the mean and variance of the Geometric distribution using the concept of conditional expectation.

12. Define the following :

(i) Markov chain;

(ii) Persistant state;

(iii) Transient state;

(iv) Ergodic state.

13. Three white and three black balls are distributed in two urns in such a way that each contains three balls. We say that the system is in state i, i = 0, 1, 2, 3, if the first urn contains i white balls. At each step, we draw one ball from each urn and place the ball drawn from the first urn into the second and conversely with the ball from the second urn. Let X_n denote the state of the system after the nth step. Explain why $\{x_n, n = 0, 1, ...\}$ is a Markov chain and calculate its transition probability matrix.

14. Explain the approach for simulating exponential random variable with mean 1.

15. Show that the maximum entropy for the ensemble $X = \{(x_1, p_1), (x_2, p_2), \ldots, (x_n, p_n)\}$ is achieved when $p_1 = p_2 = \ldots\ldots = p_n = \frac{1}{n}$.