Name : .....

Roll No. : .....

Invigilator's Signature : .....

CS/B.Tech/SEM-1/M-101/2009-10

2009

MATHEMATICS

Time Allotted : 3 Hours | Full Marks : 70

The figures in the margin indicate full marks.

Candidates are required to give their answers in their own words as far as practicable.

GROUP – A
(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The value of $\lim_{n \to \infty} \left[\frac{\sqrt{1} + \sqrt{2} + \sqrt{3} + ... + \sqrt{n}}{n\sqrt{n}} \right]$ is

a) 0 b) 1

c) $\frac{2}{3}$ d) $\frac{3}{2}$.

ii) Which of the following functions obeys Rolle's theorem in $[0, \pi]$?

a) x b) $\sin x$

c) $\cos x$ d) $\tan x$.

iii) If C is the circle $x^2 + y^2 = 4$, then $\int_C x^2\,dx$ is

a) 0 b) $\frac{1}{3}$

c) 3 d) 1.

iv) The equation $x^2 + y^2 = a^2$, $z = 0$ represents

a) circle b) cylinder

c) sphere d) right circular cylinder.

v) If θ be an angle between the vectors

$\vec{a} = 6\hat{i} + 2\hat{j} + 3\hat{k}$ and $\vec{b} = 2\hat{i} - 9\hat{j} + 6\hat{k}$, then

a) $\theta = \cos^{-1}\left(\frac{12}{77}\right)$ b) $\theta = \sin^{-1}\left(\frac{12}{77}\right)$

c) $\theta = \tan^{-1}\left(\frac{12}{77}\right)$ d) none of these.

vi) If Cauchy's mean value theorem is applicable to the function $f(x)$ x and $g(x) = x^2$, then the value of C is

a) 3/2 b) 0

c) 1/2 d) – 3/2.

vii) If $y^2 = 4ax$ (a is a real constant), then $\frac{d^2y}{dx^2} \cdot \frac{d^2x}{dy^2}$ is

a) $\frac{2a}{y}$ b) $\frac{2a}{y^3}$

c) $-\frac{2a}{y^3}$ d) $-\frac{2a}{y}$.

viii) The law of mean is given by

a) $\frac{f(b)+f(a)}{b-a} = f'(c)$

b) $\frac{f(b)+f(a)}{b+a} = f'(c)$

c) $\frac{f(b)-f(a)}{b-a} = f'(c)$

d) $\frac{f(b)-f(a)}{b-a} = f(c)$.

ix) If $x = r\cos\theta$ and $y = r\sin\theta$, then the value of $\frac{\partial(r,\theta)}{\partial(x,y)}$ is

a) 0 b) r

c) $\frac{1}{r}$ d) $-r$.

x) The series $\sum_{n=1}^{\infty} \frac{2}{e^n}$ is

a) convergent b) divergent

c) oscillatory d) none of these.

xi) The function $f(x) = \begin{cases} x\sin\frac{1}{x}, & x \neq 0 \\ 0, & x = 0 \end{cases}$ is

a) continuous and differentiable at $x = 0$

b) continuous but not differentiable at $x = 0$

c) neither continuous nor differentiable at $x = 0$

d) none of these.

xii) If $f(x, y) = \tan(x / y)$, then $x\frac{\partial f}{\partial x} + y\frac{\partial f}{\partial y}$ is

a) $\tan(x / y)$ b) $\cot(x / y)$

c) 0 d) none of these.

xiii) The moment of inertia of a thin uniform rod of mass M and length $2a$ about an axis perpendcular to the rod at its centre is

a) $\frac{Ma^2}{3}$ b) $\frac{Ma^2}{2}$

c) Ma^2 d) $\frac{Ma^2}{4}$.

xiv) The point of intersection of the line $\frac{x-1}{2} = \frac{y}{3} = \frac{z+1}{-1}$ with the plane $x + 2y - z = 5$ is

a) (1, 1, 1) b) (0, 1, 3)

c) $\left(\frac{5}{3}, 1, \frac{-4}{3}\right)$ d) none of these.

xv) The reduction formula of $I_n = \int_0^{\pi/2} \cos^n x \, dx$ is

a) $I_n = \frac{n-1}{n} I_{n-1}$ b) $I_n = \frac{n}{n-1} I_{n-1}$

c) $I_n = \frac{n-1}{n} I_{n-2}$ d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. If $y = \left(x^2 - 1 \right)^n$, then show that

$$\left(x^2 - 1 \right) y_{n+2} + 2xy_{n+1} - n\left(n + 1 \right) y_n = 0.$$

3. If $\vec{a}$, $\vec{b}$, $\vec{c}$ are three vectors, then show the

$\left[\vec{a} \times \vec{b}, \vec{b} \times \vec{c}, \vec{c} \times \vec{a} \right] = \left[\vec{a}, \vec{b}, \vec{c} \right]^2$, where symbols have their usual meanings.

4. Test the convergence of the series

$$1 + \frac{2^2}{3^2}x + \frac{2^2.4^2}{3^2.5^2}x^2 + \frac{2^2.4^2.6^2}{3^2.5^2.7^2}x^3 + \ldots\ldots \quad (x \neq 1).$$

5. *A*, *B*, *C* and *D* are points (α, 3, – 1), (3, 5, – 3), (1, 2, 3,) and (3, 5, 7) respectively. If *AB* is perpendicular to *CD*, then find the value of α.

6. If $u = \cos^{-1}\left\{ \frac{x+y}{\sqrt{x}+\sqrt{y}} \right\}$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + \frac{1}{2}\cot u = 0$.

7. Verify Rolle's theorem for the function

$$f(x) = |x|, \; -1 \leq x \leq 1 .$$

GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

8. a) Examine continuity and differentiability of $f(x)$ at $x = 0$, when $f(x) = x \sin\left(\frac{1}{x}\right)$; $(x \neq 0)$ and $f(0) = 0$.

b) Show that $f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & \text{for } (x,y) \neq (0,0) \\ 0 & \text{for } (x,y) = (0,0) \end{cases}$

is not continuous at (0, 0)

c) Find the extrema of the function

$f(x, y) = x^3 + 3xy^2 - 3y^2 - 3x^2 + 4.$ 5 + 5 + 5

9. a) Obtain a reduction formula for $\int_0^{\pi/2} \sin^n x \, dx$ and evaluate $\int_0^{\pi/2} \sin^5 x \, dx.$

b) If $z = f(x, y)$ where $x = e^u \cos v$, $y = e^u \sin v$ then show that

$$y\frac{\partial z}{\partial u} + x\frac{\partial z}{\partial v} = e^{2u}\frac{\partial z}{\partial y}.$$

c) Prove that the function $f(x) = |x - 1|$, $0 < x < 2$, is continuous at $x = 1$, but not differentiable there. Is it continuous and derivable at $x = 0$? 5 + 5 + 5

10. a) State Leibnitz's theorem for Alternating Series and test convergence of the series $1 - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \ldots\ldots\ldots$

b) Define absolute and conditional convergence of Series. Also show that the series $\sum_{n=1}^{\infty} \frac{\cos nx}{n^2}$ is absolutely convergent. 6 + 9

11. a) A particle moves on the curve $x = 2t^2$, $y = t^2 - 4t$, $z = 3t - 5$, where t is the time. Find the components of velocity and acceleration at time $t = 1$ in the direction $\hat{i} - 3\hat{j} + 2\hat{k}$.

b) Find the angles between the lines whose direction cosines are given by the equations $l + m + n = 0$ and $l^2 + m^2 - n^2 = 0$.

c) Find the shortest distance between the lines

$$\frac{x-3}{1} = \frac{y-5}{-2} = \frac{z-7}{1} \text{ and } \frac{x+1}{7} = \frac{y+1}{-6} = \frac{z+1}{1}.$$

5 + 5 + 5

12. a) Find the n-th derivative of $y = (ax + b)^m$, m is any number.

b) Test the convergence of the series

$$1 + \frac{x}{2} + \frac{x^2}{5} + \frac{x^3}{10} + \ldots\ldots$$

c) Find :

$div\ \vec{F}$ and $curl\ \vec{F}$, where $\vec{F} = \text{grad}\left(x^3 + y^3 + z^3 - 3xyz\right)$.

5 + 5 + 5

13. a) Find the whole length of the loop of the curve

$9y^2 = (x-2)(x-5)^2$.

b) Evaluate $\int_0^{\frac{\pi}{2}} \int_0^{\pi} \sin(x+y)\, dx\, dy$.

c) State Green's Theorem. 6 + 6 + 3