S.E. (Computer - Information Technology) (Sem-III) (Rev) Examination May 2007

1fh-19-5-Ex-Nk-07.79

Con. 2937-07. **ND-1312**

23/5/2007

(REVISED COURSE)

Discrete structures

(3 Hours) **[Total Marks : 100**

N.B. (1) Question No. **1** is **compulsory.**
(2) Attempt any **four** questions out of remaining **six** questions.
(3) Assumptions made should be **clearly stated.**
(4) **Figures** to the **right** indicate **full** marks.

1. (a) Use Induction to show that— **4**

$$1 + 2 + 2^2 + 2^3 + \ldots\ldots + 2^n = 2^{n+1} - 1.$$

(b) Define a pigeonhole principle. Show that if seven colours are used to paint 50 bicycles, at least 8 bicycles will be of same colour. **4**

(c) What is an Universal and existential quantifier ? **4**

(d) Find all solutions of the recurrence relation : **4**

$$a_n = 5a_{n-1} - 6a_{n-2} + 7^n$$

(e) Define the following terms with the example :— **4**
 - (i) Disjoint set
 - (ii) Symmetric difference
 - (iii) Partial order relation
 - (iv) Antisymmetric relation.

2. (a) Define the terms : **6**

(i) Poset (ii) Equivalence relation (iii) Lattice.

Give one example of each

(b) Let $H = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ be a parity check matrix. Determine the (3, 6) group code : $e_4 : B^3 \rightarrow B^6$. **6**

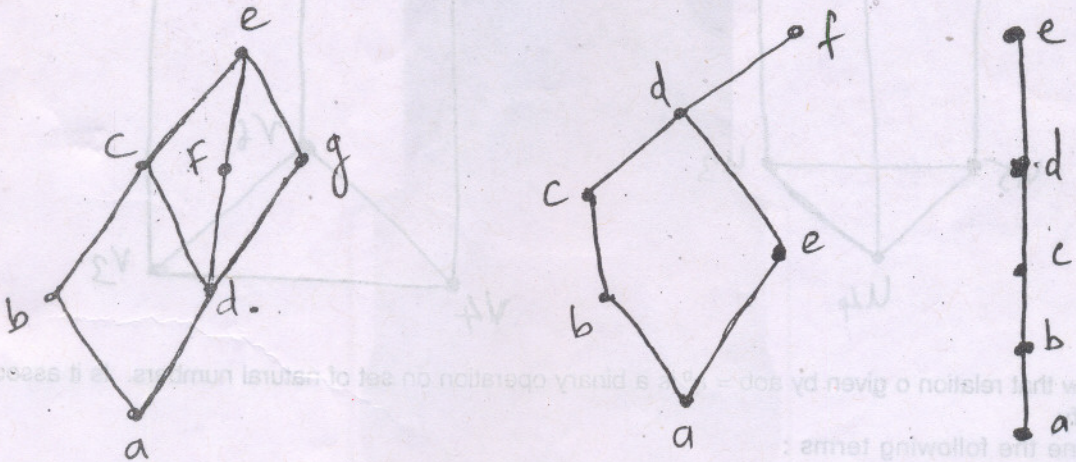
(c) Prove that set A = { 0, 1, 2, 3, 4, 5 } is a Abelian group under addition modulo 6. **4**

(d) Show that (2, 5) Encoding function $e : B^2 \rightarrow B^5$ defined by— **4**

e(0 0) = 00000
e(0 1) = 01110
e(1 0) = 10101
e(1 1) = 11011

is a group code.

3. (a) Determine whether each lattice is distributive, complemented or both. Justify your answer. 6



(b) Define planar graph. What are the necessary and sufficient conditions to exist Euler path, Euler circuit and Hamiltonian circuit ? 6

(c) Let A = { 1, 2, 3, 4 } for the relation R whose matrix is given below. Find the matrix of transitive closure using Warshall algorithm. 4

$$\begin{bmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(d) Let R be the relation on set of real numbers such that aRb if and only if a-b is an integer. Prove that R is an Equivalence relation. 4

4. (a) Show that— 6

$((P \vee Q) \wedge \neg (\neg P \wedge (\neg Q \vee \neg R)) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$

is tautology. (Use laws of logic)

(b) Prove that if (F, +, .) is a field then it is a integral domain. 6

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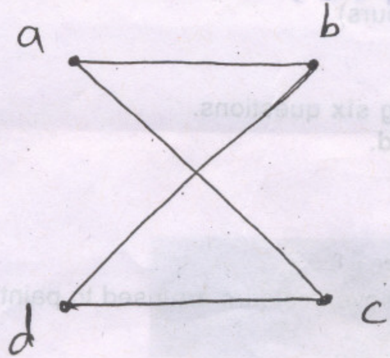
(c) How many paths of length 4 are there from a to d in simple graph shown below : 4



(d) Prove that in any ring (R, + , .) the additive inverse of each ring element is unique. 4

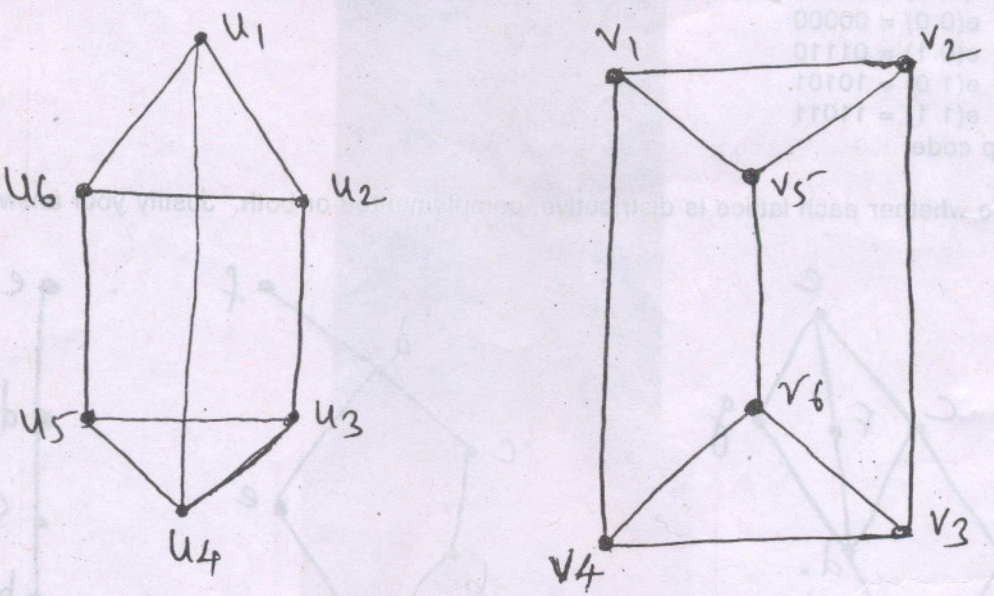
5. (a) In a Survey of 60 people, it was found that :— 6
25 reads Business India
26 reads India Today
26 reads Times of India
11 reads both Business India and India Today
09 reads both Business India and Times of India
08 reads both India Today and Times of India
08 reads none of the three.
(i) How many read all three ?
(ii) How many read exactly one ?

(b) Draw that Hasse diagram for divisibility on the set 6
(i) { 1, 2, 3, 4, 5, 6, 7, 8 } (ii) { 1, 2, 3, 4, 5, 7, 11, 13 }

(c) Define with example Reflexive closure and symmetric closure. 4

(d) A connected planar graph has a 9 vertices having degrees 2, 2, 2, 3, 3,3, 4, 4, 5. How many edges are there ? 4

6. (a) Determine graphs shown in **figure** are isomorphic or not justify your answer : 6



(b) Show that relation o given by aob = a^b is a binary operation on set of natural numbers. Is it associative ? Verify. 6

(c) Define the following terms : 4

(i) Linearly ordered set
(ii) Submonoid
(iii) Normal subgroup
(iv) Ring homomorphism.

(d) Let R is relation on set A then prove that if R is reflexive then R^{-1} is also reflexive. 4

7. (a) Let $H = \begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$ be a parity check matrix. 6

Decode the following words relative to maximum likelihood decoding function.

(i) 0101 (ii) 1010 (iii) 1101

(b) Find the complement of each element in D_{20} and D_{30}. 6

(c) Let L be the bounded distributive lattice. Prove that if complement exist then it is unique. 4

(d) R = { 0, 2, 4, 6, 8 }. Show that R is a commutative ring under addition and multiplication modulo 10. Verify whether it is field or not. 4