First Year B.Sc. Degree Examination
Aug/Sept 2009
Directorate of Correspondence Course

MATHEMATICS - I

Time : 3 Hours Max. Marks : 90

Note : Answer any SIX full questions of the following choosing atleast ONE from each part.

PART - A

1. a) i) Find ϕ (48). 2
   ii) Find gcd of 81 and 237. 2
   b) Prove that $3^{50} \equiv 2 \pmod 7$. 5
   c) Prove that the Relation 'congruence modulo n' is an equivalence relation on the set of all integers. 6

2. a) i) Define an equivalence relation. 2
   ii) Let $f : R \to R$ and $g : R \to R$ defined by $f(x)=4x-1$ and $g(x) = \cos x$. Show that $fog \neq got$. 2
   b) Let $f : X \to Y$ be a function and A and B are two subsets of x, then prove that $f(A \cup B) = f(A) \cup f(B)$. 5
   c) Given $f : A \to B$ defined by $f(x) = \frac{x+5}{x+4}$ where $A=R-\{-4\}$ and $B=R-\{1\}$. Show that f is Bijective and find f^{-1}. 6

PART -B

3. a) i) If $f(x) = [x]$ where [x] denotes the greatest integer not greater than x. Evaluate $\lim_{x \to 1} f(x)$. 2
   ii) If $2x^2 - 3xy + 4y^2 = 1$ find dy/dx. 2
   b) Discuss the continuity of the function defined by

$$f(x) = \begin{cases} x^2+2 & \text{when } x>1 \\ 2x+1 & \text{when } x=1 \\ 3 & \text{when } x<1 \end{cases} \quad \text{at } x=1$$ 5

   c) If $y=a \cos(\log x) + b \sin(\log x)$ show that $x^2y_{n+2} + (2n+1)\, xy_{n+1} + (n^2+1)\, y_n = 0$. 6

4. a) i) If $y = a \log \sec\left(\frac{x}{a}\right)$ find $\frac{ds}{dx}$. 2
   ii) Find radius of curvature of the curve $y = 4\sin x - \sin 2x$ at $x = \frac{\pi}{2}$ 2
   b) Show that the pedal equation of the parabola $y^2 = 4a(x+a)$ is $P^2 = ar$. 5
   c) Show that the evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ is $(x+y)^{2/3} + (x-y)^{2/3} = 2a^{2/3}$. 6

PART - C

5. a) i) Find the equation of the plane passing through the point (2,3,4) and parallel to the plane $5x-7y+2z+6=0$. 2

ii) Show that planes $5x+3y-4z+2=0$ and $10x+6y-8z+3=0$ are parallel. 2

b) Find the equation of the place passing through the points (2,–1,3) (4,0,5) and (2,1,7). 5

c) Determine the mutual positions of the lines

ℓ_1 : $x=1-t$, $y=2+t$, $z=2t$

ℓ_2 : $x=3-2s$, $y=4+2s$, $z=6+4s$.

6. a) i) Find the equation of the sphere whose centre is (2,–1,3) and radius is 5. 2

ii) Find the asymptotes parallel to the co-ordinate axes for the curve $xy^3=x^3+a(x^2+y^2)$. 2

b) Find all the asymptotes of the curve $y^3-x^2y+2y^2+4y+1=0$. 5

c) Find the surface area of a hemisphere of radius a. 6

PART - D

7. a) i) Express, the matrix $A=\begin{bmatrix} 8 & 6 \\ 2 & 5 \end{bmatrix}$ as the sum of symmetric and skew symmetric matrices. 2

ii) Find the non-Trivial solution of the system $x+3y-2z=0$, $2x-y+4z=0$, $x-11y+14z=0$. 2

b) Find the Rank of the matrix $A=\begin{bmatrix} 1 & 2 & -1 & 4 \\ 2 & 4 & 3 & 4 \\ 1 & 2 & 3 & 4 \\ -1 & -2 & 6 & -7 \end{bmatrix}$ 5

c) Test the following system for consistency and solve

$x + 2y - z = 3$

$3x - y + 2z = 1$

$2x - 2y + 3z = 2$

8. a) i) Evaluate $\int \frac{dx}{9+4x^2}$ 6

ii) Evaluate $\int_0^{\pi/2} \sin^4 x . \cos^6 x . dx$ 2

b) Evaluate $\int \frac{3\cos x + \sin x}{4\cos x + 3\sin x} dx$ 2

c) Evaluate $\int_0^{\pi} \frac{x \tan x}{\sec x + \cos x} dx$ 5

6

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