INSTITUTE OF ACTUARIES OF INDIA

CT6 – STATISTICAL METHODS

MAY 2009 EXAMINATION

INDICATIVE SOLUTION

1.

(i) No.

The largest element of the first column is 2, but this is not the smallest element in the first row. The largest element in the second column is 3, but this is not the smallest element in the second row. Therefore, there is no saddle point.

(ii) Player A: Maximum loss is
2 under strategy I, and
3 under strategy II.
Maximum loss is minimized with strategy I.

Player B: Maximum loss is
1 under strategy 1, and
2 under strategy 2.
Maximum loss is minimized with strategy 1.

(iii)

Day	Strategy of Player A	Strategy of Player B	Value of game
1	I	1	2
2	II	1	–1
3	II	2	3
4	I	2	–2
5	I	1	2
6	II	1	–1

(iv) The value of the game will not converge; it will rotate (clockwise) indefinitely among the elements of the loss matrix.

(v) Adoption of a randomized strategy would have the following advantages.

- By adopting their respective *pure* minimax strategies, Players A and B have to be ready to accept worst-case losses of 2 and 1, respectively. By adopting a randomized minimax strategy, *each* player can have a *smaller* expected loss, irrespective of the strategy of his opponent.
- By adopting a randomized minimax strategy, each player can ensure that his opponent cannot gain any advantage by knowing his own strategy (randomized strategy is spy-proof).

[10]

2. (i) Let the shape and scale parameters be α and β, respectively. We have mean $\frac{\alpha}{\beta} = 110$ variance $\frac{\alpha}{\beta^2} = 1100$. Therefore, $\alpha = 11$ and $\beta = 0.1$. [2]

(ii) The moment generating function of the prior distribution of λ is

$$M_\lambda(t) = E\left(e^{\lambda t}\right) = \left(\frac{\beta}{\beta - t}\right)^\alpha.$$

The MGF of the marginal distribution of the number of accidents in a year, N, is

$$M_N(t) = E\left(e^{Nt}\right) = E_\lambda\left[E\left(e^{Nt}\middle|\lambda\right)\right] = E\left(e^{\lambda(e^t - 1)}\right) = M_\lambda\left(e^t - 1\right) = \left(\frac{\beta}{\beta - e^t + 1}\right)^\alpha.$$ Thus,

$$M_N(t) = \left(\frac{\beta}{\beta - e^t + 1}\right)^{\alpha} = \left[\frac{\beta/(\beta+1)}{1-\{1-\beta/(\beta-1)\}e^t}\right]^{\alpha}.$$

This expression has the form $\left(\frac{p}{1-qe^t}\right)^r$, which is the MGF of the negative binomial distribution with parameters r and p.

Here, $r = \alpha = 11$, $p = \frac{\beta}{1-\beta} = 0.0909$ and $q = 1 - p = \frac{1}{1-\beta} = 0.9091.$

ALTERNATIVE METHOD:

$$P(N = n) = \int_0^\infty \frac{e^{-\lambda}\lambda^n}{n!} \times \frac{\beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)} d\lambda = \int_0^\infty \frac{\beta^\alpha \lambda^{n+\alpha-1} e^{-(\beta+1)\lambda}}{n!\Gamma(\alpha)} d\lambda.$$

After simplifying this expression by evaluating the gamma integral, we have

$$P(N = n) = \frac{\beta^\alpha \Gamma(n+\alpha)}{(\beta+1)^{n+\alpha} n!\Gamma(\alpha)}.$$

When α is an integer (in the present case it is 11), we can match this expression with the probability function of the negative binomial distribution, as follows.

$$P(N = n) = \binom{n+\alpha-1}{n}\left(\frac{\beta}{\beta+1}\right)^{\alpha}\left(\frac{1}{\beta+1}\right)^{n} = \binom{r+n-1}{n} p^r q^n,$$

where, $r = \alpha = 11$, $p = \frac{\beta}{1-\beta} = 0.0909$ and $q = 1 - p = \frac{1}{1-\beta} = 0.9091.$

(iii) The posterior distribution of λ is proportional to Likelihood $\times$ prior

$$= \left(\prod_{i=1}^{5} \frac{\lambda^{n_i} e^{-\lambda}}{n_i!}\right) \times \left(\frac{\beta^\alpha \lambda^{\alpha-1} e^{-\beta\lambda}}{\Gamma(\alpha)}\right)$$

$$\propto \lambda^{\sum_{i=1}^{5} n_i + \alpha - 1} \times e^{-(5+\beta)\lambda},$$

which can be recognized as the density of gamma$\left(\sum_{i=1}^{5} n_i + \alpha, 5+\beta\right)$, i.e., that of gamma(644, 5.1).

(iv) Posterior mean of $\lambda = \frac{\sum_{i=1}^{5} n_i + \alpha,}{5+\beta} = \frac{644}{5.1} = 126.3.$

Posterior mean $= \frac{5\bar{n} + \beta(\alpha/\beta)}{5+\beta}$, which is in the form of a credibility estimate

$Z\bar{n} + (1-Z)\frac{\alpha}{\beta}$, with

the credibility factor $Z = \frac{5}{5+\beta} = 0.9804$.

[12]

3. (i) The two main types of Proportional reinsurance arrangements are **Quota share** and **Surplus reinsurance**.
Under Quota share reinsurance, a fixed proportion of each risk is ceded to the reinsurer.
Under Surplus reinsurance, the proportion ceded to the reinsurer varies from risk to risk.

(ii) (a) Let the aggregate size of the first n claims be S_n, and the number of claims till time t be $N(t)$.
The premium collected till time t is $1.25E(S_{N(t)}) = 1.25 \times 400 \times 3.65t = 1825t$.

The probability of ruin is

$$P(Ruin) = P(S_{N(t)} - 1825t > 0) = \sum_{n=1}^{\infty} P(S_n > 1825t \mid N(t) = n)P(N(t) = n)$$

$$= P(S_n > 1825t \mid N(t) = 1)P(N(t) = 1) + \sum_{n=2}^{\infty} P(S_n > 1825t \mid N(t) = n)P(N(t) = n)$$

$$= P(S_1 > 1825t \mid N(t) = 1)P(N(t) = 1) + \sum_{n=2}^{\infty} P(S_n > 1825t \mid N(t) = n)P(N(t) = n)$$

$$\leq P(S_1 > 1825t \mid N(t) = 1)P(N(t) = 1) + P(N(t) > 1).$$

$$= P(\text{Size of one claim is more than premium accrued in 10 days})$$

$$\times\ P(\text{One claim arises in 10 days}) + P(\text{More than one claim arises in 10 days}),$$

Here, $t = 10/365$.
S_1 has the exponential distribution with mean 400, and $N(t)$ has the Poisson distribution with mean $E(N(t)) = 3.65 \times (10/365) = 0.1$.
Therefore,

$$P(Ruin) \leq \left(e^{-1825\times(10/365)/400}\right) \times \left(\frac{e^{-0.1}(0.1)}{1!}\right) + 1 - \left(\frac{e^{-0.1}(0.1)^0}{0!}\right) - \left(\frac{e^{-0.1}(0.1)^1}{1!}\right)$$

$$= e^{-0.125} \times e^{-0.1} \times 0.1 + 1 - e^{-0.1} - e^{-0.1} \times 0.1 = 0.0845.$$

(b) The premium charged by the reinsurer is $1.4\lambda\, E(Z)$, where

$$E(Z) = \int_{200}^{600} (x - 200)\frac{e^{-x/400}}{400}dx + \int_{600}^{\infty} 600\frac{e^{-x/400}}{400}dx$$

$$= [-(x-200)e^{-x/400}]\Big|_{200}^{600} + \int_{200}^{600} e^{-x/400}dx + 600e^{-3/2}$$

$$= -400e^{-3/2} - [400e^{-x/400}]\Big|_{200}^{600} + 600e^{-3/2}$$

$$= -200e^{-3/2} + 400e^{-1/2}$$

$$= 197.99$$

Therefore, the premium charged by the reinsurer is Rs 1012.

[12]

4. (i) A delay triangle or run-off triangle is a table used in general insurance, which shows the development of claim amounts or claim numbers in respect of each past accident/underwriting period.
The accident/underwriting periods represent rows and the development periods represent columns.
Statistical methods can be used to complete the table so that estimates of claim number/amount for future years can be obtained.

(ii) Dividing each cell in the first table by the corresponding cell in the second table gives the average incurred cost per claim, by years of accident and development.

Average cost per claim (Rs '000s)

Accident year	Development year			
	0	1	2	3
1997	5.000	5.068	4.954	4.901
1998	5.204	5.277	5.169	
1999	5.636	5.295		
2000	5.200			

Using the basic chain ladder method to complete the claim number and average cost per claim table

Number of claims

Accident year	Development year			
	0	1	2	3
1997	56	74	87	91
1998	49	65	77	80.54
1999	44	61	71.97	75.28
2000	50	67.11	79.18	82.83

[0.25m for each projected cell]

d.f 0-1 : 1.3423
d f 1-2 : 1.1799
d f 2-3 : 1.0460

[0.5m for each dev factor]

Average cost per claim

Accident year	Development year			
	0	1	2	3
1997	5.000	5.068	4.954	4.901
1998	5.204	5.277	5.169	5.114
1999	5.636	5.295	5.181	5.126
2000	5.200	5.134	5.024	4.970

[0.25m for each projected cell]

d.f 0-1 – 0.9874
d f 1-2 - 0.9786
d f 2-3 - 0.9893

[0.5m for each dev factor]

So the ultimate claim amount (in '000 Rupees) from AY 1997 to 2000 is:

(91*4.901) + (80.54*5.114) + (75.28*5.126) + (82.83*4.970) = 1655.

Since claims paid to date amounted to Rs 1323,000, the total reserve required would be Rs 332,000.

ALTERNATIVE METHOD USING GROSSING-UP FACTORS

Dividing each cell in the first table by the corresponding cell in the second table gives the average incurred cost per claim, by years of accident and development.

Average cost per claim (Rs '000s)

Accident year	Development year			
	0	1	2	3
1997	5.000	5.068	4.954	4.901
1998	5.204	5.277	5.169	
1999	5.636	5.295		
2000	5.200			

Grossing-up factors for average cost per claim

Accident year	Development year			
	0	1	2	3
1997	102.02%	103.40%	101.08%	100.00%
1998	101.77%	103.19%	101.08%	100.00%
1999	109.95%	103.30%		100.00%
2000	104.58%			100.00%

Projected ultimate average cost per claim

Accident year	Development year				
	0	1	2	3	Ult
1997	5.000	5.068	4.954	4.901	4.901
1998	5.204	5.277	5.169		5.114
1999	5.636	5.295			5.126
2000	5.200				4.972

Grossing-up factors for number of claims

Accident year	Development year			
	0	1	2	3
1997	61.54%	81.32%	95.60%	100.00%
1998	60.84%	80.71%	95.60%	100.00%
1999	58.43%	81.01%		100.00%
2000	60.27%			100.00%

Projected ultimate number of claims

Accident year	Development year				
	0	1	2	3	Ult
1997	56	74	87	91	91
1998	49	65	77		80.54
1999	44	61			75.30
2000	50				82.96

So the ultimate claim amount (in '000 Rupees) from AY 1997 to 2000 is:

(91*4.901) + (80.54*5.114) + (75.30*5.126) + (82.96*4.972)
= 1656 ('000 Rupees).
Since claims paid to date amounted to Rs 1323,000, the total reserve required would be Rs 333,000.

[10]

5. (i) If X is lognormal with parameters μ and σ^2, then log$X$ has normal distribution with parameters μ and σ^2. [0.5]

$$P(X > M) = P(\log X > \log M) = P\left(Z > \frac{\log M - \mu}{\sigma}\right) = 1 - \Phi\left(\frac{\log M - \mu}{\sigma}\right).$$

(ii) a) Consider first a policyholder at the 0% discount level. If a claim is made, the next two premiums will Rs 800 and Rs 640, giving a total of Rs 1440. If no claim is made, the next two premiums will be Rs 640 and Rs 400 giving a total of Rs 1040. The difference between these two figures is Rs 400
So a policyholder on 0% discount will only make a claim if the amount of damage exceeds Rs 400.
Similarly a policyholder on 20% discount will make a claim if the amount of damage exceeds Rs 640,
and a policyholder on 50% discount will make a claim if the amount of damage exceeds Rs 240.

b) We now need to find the proportion of accidents, which exceed these amounts. Using (i)
0% level: $P(\log X > 400) = 1 - \Phi(0.9957)$
$= 0.15970$

20% level: $P(\log X > 640) = 1 - \Phi(1.2307) = 0.10922$

50% level: $P(\log X > 640) = 1 - \Phi(0.7403) = 0.22956$

So the transition (to higher level) probabilities for the NCD system are obtained by multiplying these figures by 0.2. This leads to the following transition probability matrix.

	0%	20%	50%
0%	0.03194	0.96806	0
20%	0.02184	0	0.97816
50%	0	0.04591	0.95409

c) Let the steady state proportions be π_0, π_{20} and π_{50}, respectively. They satisfy the equations

$\pi_0 + \pi_{20} + \pi_{50} = 1,$

$0.03194\pi_0 + 0.02184\pi_{20} = \pi_0,$

$0.96806\pi_0 + 0.04591\pi_{50} = \pi_{20}.$

$0.97816\pi_{20} + 0.95409\pi_{50} = \pi_{50}.$

From the second equation, we have $\pi_{20} = \dfrac{1-0.03194}{0.02184}\pi_0 = 44.325\pi_0$.

From the fourth equation, we have

$\pi_{50} = \dfrac{0.97816}{1-0.95409}\pi_{20} = 21.306\pi_{20} = 944.392\pi_0.$

Substituting these in the first equation, we have

$(1 + 44.325 + 944.392)\pi_0 = 1$, i.e., $\pi_0 = 0.0010$.

Finally, we have $\pi_0 = 0.0010$, $\pi_{20} = 0.0448$, $\pi_{50} = 0.9542$.

[11]

6. Consider a single policy. Let N denote the number of claims arising from the policy and let X_i denote the amount of the i^{th} claim. Then N has the Poisson (0.2) distribution and X_i has the lognormal (5,2) distribution. We consider the distribution of the aggregate claim amount (S) for each policy.

$E(S) = 0.2E(X) = 0.2e^{\mu+\sigma^2/2} = 0.2e^6 .= 80.686.$

$V(S) = 0.2E(X^2) = 0.2e^{2\mu+2\sigma^2} = 0.2e^{14} . = 490.429^2.$

Let Y_j denote the profit on the j^{th} policy. Then

$Y_j = 125 - S_j.$

$E(Y_j) = 125 - E(S_j) = 125 - 0.2e^6.$

$V(Y_j) = V(S_j) = 0.2e^{14}.$

We need to determine the number of policies m that need to be sold to ensure that

$P\left(\sum_{j=1}^{m} Y_j > 0\right) \le 0.99.$

If m is not too small, we can use the normal approximation for the distribution of $\overline{Y}$, with mean $125 - 0.2e^6$ and variance $0.2e^{14}/m$.

$$P\left(\sum_{j=1}^{m} Y_j > 0\right) = P(\overline{Y} > 0) = P\left(\frac{\overline{Y} - 125 + 0.2e^6}{\sqrt{0.2e^{14}/m}} > \frac{-125 + 0.2e^6}{\sqrt{0.2e^{14}/m}}\right) = 1 - \Phi\left(\frac{-125 + 0.2e^6}{\sqrt{0.2e^{14}/m}}\right).$$

In order to ensure that this quantity is not less than 0.99, we must have

$$\frac{-125 + 0.2e^6}{\sqrt{0.2e^{14}/m}} \le \Phi^{-1}(1 - 0.99) = -2.326.$$

Therefore,

$$\sqrt{m} \ge 2.326 \times \frac{\sqrt{0.2e^{14}}}{125 - 0.2e^6} = 25.746 \text{, i.e., } m \ge 663.$$ **[7]**

7. (i) $P(Y_i = y_i) = p_i^{y_i}(1 - p_i)^{1 - y_i}$

$$= \exp[y_i \ln p_i + (1 - y_i)\ln(1 - p_i)]$$

$$= \exp\left[y_i \ln\left(\frac{p_i}{1 - p_i}\right) + \ln(1 - p_i)\right]$$

$$= \exp[y_i\theta_i - b(\theta_i)],$$

where $\theta_i = \ln\left(\frac{p_i}{1 - p_i}\right)$, i.e., $p_i = \frac{e^{\theta_i}}{1 + e^{\theta_i}}$,

and $b(\theta_i) = -\ln(1 - p_i) = -\ln\left(\frac{1}{1 + e^{\theta_i}}\right) = \ln(1 + e^{\theta_i})$.

Thus, the probability function is a member of the exponential family with $a(\phi_i) = \phi_i$, $\phi_i = 1$ and $c(y_i, \phi_i) = 0$.

The canonical link function is the logit function $\theta_i = \ln\left(\frac{p_i}{1 - p_i}\right)$.

(ii) The expected value is calculated from first principles as follows.
$E(Y_i) = 0 \times (1 - p_i) + 1 \times p_i = p_i$.

From part (i), we have $b(\theta_i) = \ln(1 + e^{\theta_i})$.

So $b'(\theta_i) = \frac{e^{\theta_i}}{1 + e^{\theta_i}} = p_i = E(Y_i)$.

The variance, calculated from first principles, is
$V(Y_i) = E(Y_i^2) - E^2(Y_i) = [0^2(1 - p_i) + 1^2 p_i] - p_i^2 = p_i(1 - p_i)$.

On the other hand, by differentiating $b'(\theta_i)$, we have

$$b''(\theta_i) = \frac{e^{\theta_i} \times (1 + e^{\theta_i}) - e^{2\theta_i}}{(1 + e^{\theta_i})^2} = \frac{e^{\theta_i}}{(1 + e^{\theta_i})^2} = p_i(1 - p_i).$$

Since $a(\phi_i) = 1$, we have

$a(\phi_i)b''(\theta_i) = p_i(1-p_i) = V(Y_i)$.

(iii) The likelihood function is

$$L(p_1, p_2, ..., p_n) = \prod_{i=1}^{n} P(Y_i = y_i) = \prod_{i=1}^{n} p_i^{y_i}(1-p_i)^{1-y_i} .$$

(iv) Here, the canonical link function is the logit function:

$$\theta_i = \ln\left(\frac{p_i}{1-p_i}\right).$$

Since the only covariate is the age (x_i), the regression model is

$$\ln\left(\frac{p_i}{1-p_i}\right) = \alpha + \beta x_i .$$

One can rewrite this relation as

$$\frac{p_i}{1-p_i} = e^{\alpha+\beta x_i} \text{ , i.e., } p_i = \frac{e^{\alpha+\beta x_i}}{1+e^{\alpha+\beta x_i}} .$$

Therefore, the likelihood for the GLM parameters is

$$L(\alpha,\beta) = \prod_{i=1}^{n} p_i^{y_i}(1-p_i)^{1-y_i} = \prod_{i=1}^{n} \frac{e^{(\alpha+\beta x_i)y_i}}{1+e^{\alpha+\beta x_i}}$$

[9]

8. (i) A vector time series is said to be cointegrated if the components are marginally I(1), and there is a linear combination of the components which is stationary. One comes across a cointegrated process when, e.g., one of the components of the vector time series drives the other component(s), or when all of them are driven by another time series.

(ii) An AutoRegressive Conditionally Heteroscedastic (ARCH) model is of the form

$$X_t = \mu + e_t\sqrt{\sum_{k=1}^{p}\alpha_k(X_{t-k}-\mu)^2} ,$$

where e_t is a sequence of independent standard normal random variables. It is seen from the model that, given the past p samples, the current samples have a constant mean, but the variance depends on the observed fluctuations in these past samples. Such models are useful for modeling time series where a period of sudden change is followed by a period of high volatility.

[4]

9. (i) (a) The process can be written as $(1-0.4B-0.2B^2)Y_t = (1+0.025B)Z_t + 0.016$.

The characteristic equation is $1-0.4z-0.2z^2 = 0$.

There is no root having magnitude 1. Therefore, d = 0.

Hence, the process is ARIMA (2,0,1).

(b) $(1-0.4-0.2)E(Y_t) = 0.016$. Therefore, $E(Y_t) = 0.016/0.4 = 0.04$ or 4%.

(c) The two roots of the characteristics equation are $-1\pm\sqrt{6}$, i.e., 1.4495 and –3.4495, both of which have magnitude larger than 1. Hence, the process $\{Y_t\}$ is stationary.

(ii) (a) This AR process has the same characteristic equations and the same roots as in part (iii). The k^{th} order auto-covariance is of the form (given in Core Reading, chapter CT6-12, section 3.4)

$$\gamma_k = A_1(-1+\sqrt{6})^{-k} + A_2(-1-\sqrt{6})^{-k},$$

where A_1 and A_2 are constants. Therefore, the k^{th} order auto-correlation is of the form

$$\rho_k = \frac{\gamma_k}{\gamma_0} = \frac{A_1(-1+\sqrt{6})^{-k} + A_2(-1-\sqrt{6})^{-k}}{A_1 + A_2} = \alpha(-1+\sqrt{6})^{-k} + (1-\alpha)(-1-\sqrt{6})^{-k},$$ w

here α is a constant. We can determine α by calculating ρ_1 directly from the Yule-Walker equation

$$\gamma_1 = 0.4\gamma_0 + 0.2\gamma_1,$$

which implies that $\rho_1 = \frac{\gamma_1}{\gamma_0} = \frac{0.4}{1-0.2} = 0.5$. Equating this value with the general expression for $k = 1$, we have

$0.5 = \alpha(-1+\sqrt{6})^{-1} + (1-\alpha)(-1-\sqrt{6})^{-1}$, and therefore,

$$\alpha = \frac{0.5-(-1-\sqrt{6})^{-1}}{(-1+\sqrt{6})^{-1}-(-1-\sqrt{6})^{-1}} = \frac{2\sqrt{6}+3}{4\sqrt{6}} = 0.8062.$$

It follows that

$$\rho_k = \left(\frac{2\sqrt{6}+3}{4\sqrt{6}}\right)(-1+\sqrt{6})^{-k} + \left(\frac{2\sqrt{6}-3}{4\sqrt{6}}\right)(-1-\sqrt{6})^{-k}$$

$$= 0.8062(0.6899)^k + 0.1938(-0.2899)^k.$$

The ACF values for the first few lags are $\rho_1 = 0.5$, $\rho_2 = 0.4$, $\rho_3 = 0.26$, $\rho_4 = 0.184$.

(Partial credit for determining a few ACF values: 1 mark for correct computation of each value. Maximum partial credit with no general solution is 3.)

(b) Three diagnostic checks are as under (**any two** should fetch full credit).

- *Inspection of the graph of the time-plot of residuals*: Visual inspection might reveal a pattern, such as uneven fluctuations or clusters of only positive / only negative residuals, which indicate inadequate fit.

- I*nspection of the sample autocorrelation functions of the residuals*: Too many ACF or PACF values outside the range $\pm 2/\sqrt{N}$ (N being the sample size) may indicate poor fit or too few parameters.

- *Counting turning points*: The number of turning points (points where the value of the time series is smaller/larger than both neighboring values) for a sequence of independent random variables has average $2(N-2)/3$ and variance $(16N-29)/90$. If the residuals from a particular fit has too few or too many turning points (with reference to a normal distribution with the said mean and variance), then the fit is inadequate.

[12]

10. (i) A truly random number generator cannot generate long sequences of numbers as efficiently as pseudo random number generators. One can mitigate this problem by generating in advance long sequences of truly random numbers for later use, but this generally requires huge storage space/hardware enhancement of the computer.

If truly random numbers are generated at the time of use (i.e., not read out of computer memory), then the sequence cannot be reproduced.

(ii) A Linear Congruential Generator (LCG) is a process of generating pseudo random numbers using an initial integer value, called the seed, and a recursive formula.

A typical recursive formula is of the form
$x_n = (ax_{n-1} + c) \pmod m$,
where a and c are fixed integers smaller than the third integer m. Given a seed x_0, one can go on generating successive integers in the range 0 to m, using this formula. Usually m is chosen as a very large number, and x/m is a pseudo-random number between 0 and $(m-1)/m$.

(iii) The polar method is as follows.

1. Generate two independent uniform (0,1) variates, u_1 and u_2.
2. Calculate $v_1 = 2u_1 - 1$, $v_2 = 2u_2 - 1$ and $s = v_1^2 + v_2^2$.
3. A. If $s > 1$, go to step 1

   B. Otherwise, return $z_1 = \sqrt{\frac{-2\ln s}{s}}\, v_1$ and $z_2 = \sqrt{\frac{-2\ln s}{s}}\, v_2$.

(iv) (a) The density can be written as $f(x) = E[Yf_1(x) + (1-Y)f_0(x)]$, where f_1 and f_0, are the densities of the uniform distribution over [0,1] and [2,3], respectively, and Y is a binary random variable taking the value 1 with probability 0.2 and the value 0 with probability 0.8. Thus, f is the marginal density of a random variable, whose conditional distributions given $Y = 1$ and $Y = 0$ are f_1 and f_0, respectively.
Using this interpretation, one can devise the following simple strategy to generate samples from f.
Generate two independent uniform (0,1) variates, u_1 and u_2.
If $u_1 > 0.2$, return $2 + u_2$, else return u_2.

(b) The given distribution is that of $|X|$, where X is standard normal. Therefore, in order to generate a sample from f, generate normal variates x by Box-Muller or polar method (described in part (iii)), and return the absolute value $|x|$.

(c) The given distribution is a modification of the exponential distribution, where the entire probability mass of the distribution over the range $x > 2$ is removed and put at the point $x = 2$. Therefore, a simple strategy would be to generate a uniform (0,1) variate u, and return $\min\{2, -\ln(u)\}$.

(For parts (a), (b) and (c), other reasonable answers should fetch credit.)

[13]

[Total 100 Marks]

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